Leptonic $CP$ Asymmetries in Flavor-changing $H^0$ Decays

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ABSTRACT

Leptonic flavor-changing $H^0$ decays with branching ratios of the order of $10^{-5} - 10^{-6}$ may constitute an interesting framework when looking for large $CP$-violating effects. We show that leptonic $CP$ asymmetries of an intermediate $H^0$ boson can be fairly large in natural scenarios of the minimal Standard Model (SM) with right-handed neutrinos, at a level that may be probed at future $H^0$ factories.

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1. Introduction

Higgs physics has become the subject of many recent theoretical studies [1,2,6]. It was recently argued that the lepton-flavor-violating decays of an intermediate $H^0$ can be relatively large in the context of the SM with one right-handed neutrino field per family [2]. On the contrary, ordinary see-saw models predict vanishingly small flavor-changing decay rates for the $H^0$ or $Z^0$ particle [3] relying on the strong assumption that the Dirac and Majorana mass matrices $m_D$ and $m_M$, appearing in the Yukawa sector, can be simultaneously brought to a diagonal form. This scenario is equivalent to the single-family case, where the mass of the light Majorana neutrino $m_\nu$ and its mixing angle with the heavy one $\xi_{\nu N}$ are

\[ m_\nu \approx m_D^2 m_N, \quad \xi_{\nu N} \approx \frac{m_D}{m_M} \approx \sqrt{\frac{m_\nu}{m_N}} \]  

Consequently, one gets very heavy Majorana neutrinos with $m_N \gtrsim 10^7$ GeV for neutrinos that are consistent with cosmological constraints and extremely suppressed mixing angles $\xi_{\nu N} \lesssim 10^{-6}$, if one approximates the Dirac mass matrix $m_D$ by the quark or charged lepton mass matrix, as dictated by many GUT models (i.e. $m_D \sim m_{\text{leptons}} \sim 1$ GeV). Similar conclusions are obtained, even if one right-handed neutrino with Majorana interactions will be added in the three generation model [4]. This situation, however, changes drastically in a two or three generation model [5,6]. In general, the Yukawa sector containing both Dirac and Majorana terms can be represented in an appropriate Majorana basis of the neutrino fields as follows:

\[ -\mathcal{L}_M^\nu = \frac{1}{2} (\bar{\nu}_L^0, \bar{\nu}_R^{0C}) \begin{pmatrix} 0 & m_D \\ m_D^T & m_M \end{pmatrix} \begin{pmatrix} \nu_L^{0C} \\ \nu_R^0 \end{pmatrix} + \text{h.c.} \]  

where one can always assume $m_M$ to be a real diagonal $n_R \times n_R$ matrix and $m_D$ an arbitrary non-hermitian $n_L \times n_R$ matrix. The parameter $n_L(n_R)$ indicates the number
of left(right)-handed neutrino fields. Actually, it was demonstrated in [5] that e.g.
for 
\( n_L = n_R = 2 \), matrices of the form

\[
m_D \simeq \begin{pmatrix}
a & b \\
c & bc/a
\end{pmatrix}, \quad m_M \simeq A \begin{pmatrix}
1 & 0 \\
0 & -b^2/a^2
\end{pmatrix}
\]

(3)

lead automatically to two approximately massless neutrinos. If, for instance, \( a = b = c \),
we then obtain the two generation version of a democratic family mixing model. The two
acossiated heavy neutrinos have masses

\[
m_{N_1} \approx A \quad m_{N_2} \approx \frac{b^2}{a^2} A
\]

(4)

The only constraints on the free parameters \( a, b, c, A \) are imposed by phenomenology.
In a global analysis [7] based on charge-current universality, neutral current effects etc.
it was found, for example, that the allowed maximum value for \( (\xi^\dagger)_{ij} \) is of the order of
0.01–0.13, where the larger value refers to the heavy–light mixings of the systems \( e - \tau \)
or \( \mu - \tau \) and the lower value to mixing in the \( e - \mu \) sector.

In this work we wish to investigate whether \( CP \) asymmetries defined as

\[
A_{CP}^{ij} = \frac{\Gamma(H^0 \rightarrow l_i \bar{l}_j) - \Gamma(H^0 \rightarrow \bar{l}_i l_j)}{\Gamma(H^0 \rightarrow l_i \bar{l}_j) + \Gamma(H^0 \rightarrow \bar{l}_i l_j)}
\]

(5)
can, \emph{in principle}, be observed at \( LHC \) or \( SSC \) energies, with \( l_i \) being charged leptons.
We will show that such \( CP \) effects are indeed sizeable for the \( e - \tau \) or \( \mu - \tau \) system, if
the minimal \( SM \) is extended by three right-handed neutrino fields, and would be experi-
mentally measurable when a high efficiency in the \( \tau \)-lepton identification is achieved.
2. The SM with right-handed neutrinos

Although the case of an equal number of left-handed and right-handed neutrino states may seem more aesthetical, in general the number of right-handed neutrinos that can be added in the SM is arbitrary [8]. Thus, the symmetric neutrino mass matrix $M^\nu$ given in eq. (2) can generally be diagonalized by a $(n_L + n_R) \times (n_L + n_R)$ unitary matrix $U^\nu$ in the following way:

$$ U^\nu T M^\nu U^\nu = \hat{M}^\nu $$

This gives $n_L$ light neutrino mass eigenstates ($\nu_i$) and $n_R$ heavy ones ($N_i$) which are related to the weak eigenstates via

$$ \begin{pmatrix} \nu^0_L \\ \nu^0_R \end{pmatrix} = U^\nu \begin{pmatrix} \nu_R \\ N_R \end{pmatrix}, \quad \begin{pmatrix} \nu^0_L \\ \nu^0_R \end{pmatrix} = U^{\nu*} \begin{pmatrix} \nu_L \\ N_L \end{pmatrix} $$

Denoting the neutrino mass eigenstates by $n_i$ (i.e. $n_i \equiv \nu_i$ for $i = 1, 2, \ldots, n_L$ and $n_i \equiv N_i - n_L$ for $i = n_L + 1, \ldots, n_L + n_R$), we can write down the relevant Lagrangians that describes the interactions between Majorana neutrinos $n_i$ and $W^\pm$ or $H^0$ bosons [6]. One has

$$ \mathcal{L}^W_{\text{int}} = -\frac{g_W}{2\sqrt{2}} W^{-\mu l_i} B_{i,j} \gamma_\mu (1 - \gamma_5) n_j + \text{h.c.} $$

$$ \mathcal{L}^H_{\text{int}} = -\frac{g_W}{4 M_W} H^0 \bar{n}_i \left[ (m_{n_i} + m_{n_j}) \text{Re}(C_{ij}) + i \gamma_5 (m_{n_j} - m_{n_i}) \text{Im}(C_{ij}) \right] n_j $$

where $B$ and $C$ are $n_L \times (n_L + n_R)$ and $(n_L + n_R) \times (n_L + n_R)$ dimensional matrices, respectively, which are defined as

$$ B_{i,j} = \sum_{k=1}^{n_L} V^l_{i,k} U^\nu_{kj} \quad \text{with} \quad j = 1, \ldots, n_L + n_R $$

$$ C_{ij} = \sum_{k=1}^{n_L} U^\nu_{ki} U^{\nu*}_{kj} \quad \text{with} \quad i, j = 1, 2, \ldots, n_L + n_R $$

$V^l$ in eq. (10) is the relevant $n_L \times n_L$ Cabbibo-Kobayashi-Maskawa (CKM) matrix. For completeness, we also give the charged current Lagrangian, which describes the coupling
of the Majorana neutrinos \( n_i \) with the unphysical Goldstone bosons \( \chi^\pm \) in the Feynman–t
Hooft gauge [6].

\[
\mathcal{L}_{\text{int}}^\chi = -\frac{g_W}{2\sqrt{2}M_W} \chi^- i [m_L B_{l,i}(1 - \gamma_5) - B_{l,j}(1 + \gamma_5) m_{n_j}] n_j + \text{h.c.}
\]

(12)

Some comments on the structure of \( U^\nu \) are in order. A sufficient condition for the \( n_L \)
light neutrinos to be approximately massless at the tree level is

\[
m_D m^{-1}_M m^T_D = 0
\]

(13)

As already mentioned in the introduction, eq. (13) cannot be satisfied by ordinary see-saw
models for finite Majorana mass terms (i.e. \( n_R = 1 \)). This restriction can naturally be
realized by more than one generation. Especially, one can prove that once condition (13)
is valid, \( M^\nu \) can be diagonalized by a unitary matrix \( U^\nu \) of the form

\[
U^\nu = \begin{pmatrix}
(1 + \xi^* \xi^T)^{-\frac{1}{2}} & \xi^*(1 + \xi^T \xi^*)^{-\frac{1}{2}} \\
-\xi^T(1 + \xi^* \xi^T)^{-\frac{1}{2}} & (1 + \xi^T \xi^*)^{-\frac{1}{2}}
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
0 & V^N
\end{pmatrix}
\]

(14)

where \( \xi = m_D m^{-1}_M \) and \( V^N \) is a unitary \( n_R \times n_R \) matrix that diagonalizes the following
symmetric matrix:

\[
M_M = \frac{1}{2}(1 + \xi^\dagger \xi)^{-\frac{1}{2}} m_M(1 + \xi^T \xi^*)^{\frac{1}{2}} + \frac{1}{2}(1 + \xi^\dagger \xi)^{\frac{1}{2}} m_M(1 + \xi^T \xi^*)^{-\frac{1}{2}}
\]

(15)

Note that the square root of a matrix \( A \) is defined as the matrix \( B \) obeying the equality:
\( BB = A \), where all elements \( B_{ij} \) are taken to be on the first sheet. It should be also noted
that the representation (14) in terms of the matrix-valued parameter \( \xi \) is only possible
for \( \| \xi \| < 1 \). This inequality is obviously consistent with phenomenological constraints.

The matrices \( B \) and \( C \) satisfy a number of useful identities. Below we list the most
relevant for us, i.e.

\[
\sum_{k=1}^{n_L} B_{l,k} B_{l,k}^* = C_{ij}
\]

(16)
\[
\sum_{i=1}^{n_L+n_R} m_{n_i} B_{l_{1i}} B_{l_{2i}} = 0 \quad (17)
\]
\[
\sum_{i=1}^{n_L+n_R} m_{n_i} B_{l_{i}} C_{ij}^* = 0 \quad (18)
\]
\[
\sum_{i=1}^{n_L+n_R} B_{l_{i}} C_{ij} = B_{ij} \quad (19)
\]
\[
\sum_{i=1}^{n_L+n_R} B_{l_{1i}} B_{l_{2i}}^* = \delta_{l_{1i}l_{2i}} \quad (20)
\]

Eq. (20), for example, is the generalized form of the unitarity condition for \( n_L \) charged leptons and \( n_L + n_R \) neutrinos. Since the explicit form of \( B \) and \( C \) is rather complicated, we can, as usual, approximate \( B \) and \( C \) in terms of the mixing parameters \( \xi_{\nu_N} \). Up to leading order in \( \xi \), these matrices are given by

\[
B_{l_{\nu}} = V_{l_{\nu}}^l , \quad B_{l_{IN}} = (V^l \xi V^{N*})_{lN} \quad (21)
\]
\[
C_{\nu\nu} = 1 , \quad C_{\nu N} = (\xi V^{N*})_{\nu N} , \quad C_{NN} = (V^N \xi^* V^{N*})_{NN} \quad (22)
\]

Let us now count the number of \( CP \) phases in \( SU(2)_L \otimes U(1)_Y \) theories with arbitrary number of right-handed neutrinos. It is convenient to discuss this problem in a weak basis where \( m_M \) is real and diagonal and the charged lepton fields are rotated so as to coincide with their masseigenstates, i.e.

\[
l_{L_i}^0 = V_{lj}^l l_{L_j} \quad (23)
\]

At the same time, the neutrino fields have to be transformed as

\[
\nu_{L_i}^0 = V_{lj}^l \nu_{L_j}^0 \quad (24)
\]

so that the charged current sector is diagonal in this basis. The number of \( CP \) phases equals the number of non-trivial independent phases existing in the non-hermitian \( n_L \times n_R \) Dirac mass matrix \( m_D \). Thus, \( m_D \) possesses \( n_L n_R \) phases from which only \( n_L \) phases coming from \( \nu_{L_i}^0 \) can be redefined away, since the phases of \( \nu_{R_i}^0 \) are fixed by the above
assumptions. As a result, the net number of \( CP \) phases in these theories will be given by the relation
\[
N_{CP} = n_L(n_R - 1), \quad \text{for} \quad n_R \geq 1
\]
(25)
The same conclusion can be derived by following line of arguments similar to that of ref. [9]. From eq. (25) it is obvious that models with one right-handed neutrino and arbitrary number of left-handed ones [4] cannot account for possible \( CP \)-violating phenomena in the leptonic sector. In other words, at least two or more right-handed neutrinos are required to explain possible \( CP \) asymmetries in the leptonic \( H^0 \) decays.

In the next section we will present the analytical calculation and numerical results for the off-diagonal leptonic \( H^0 \) decays and their associated \( CP \) phenomena. We will discuss these effects in two illustrative scenarios, where two and three right-handed neutrinos are present, respectively.

3. Leptonic \( CP \) asymmetries in \( H^0 \) decays

As is well known, \( CP \) violation requires the existence of at least two amplitudes with different absorptive parts as well as different relative weak phases that cannot be rotated away. In this context, numerous studies on possible \( CP \) mechanisms that may take place at high energies and can give rise to large \( CP \)-violating phenomena have been presented in the literature over the last years [10-13]. The above requirement leads to the additional restriction that at least one heavy Majorana neutrino \( N_1 \) with mass
\[
m_{N_1} \simeq 100 \text{ GeV}
\]
(26)
must be present. Then, the effective \( l_1 - l_2 - H^0 \) coupling (see fig. 1) – with \( l_1, l_2 \) being
two different charged leptons – acquire absorptive contributions for $m_{N_1} < M_H$, which result from on-shell $\nu_i N_1$ intermediate states as shown in the diagrams 1a and 1b. Here, of course, we have implicitly assumed that we are dealing with an intermediate mass Higgs boson (i.e. $100 < M_H \leq 140$ GeV), whose branching ratio into $b\bar{b}$ pairs is of the order one. In addition, in order to avoid excessive complication in our calculations, we make the following realistic assumptions for the vertex function:

$$m_{l_1} = 0, \quad \frac{m_{l_2}^2}{M_W^2} \ll 1 \quad (27)$$

In fact, the matrix element of the amplitude $H^0 \to \bar{l}_1 l_2$ can be parametrized as follows:

$$T(H^0 \to \bar{l}_1 l_2) = \frac{g_W \alpha_W}{16\pi} m_{l_2} \left[ F_{\text{dis}}(M_H^2) + iF_{\text{abs}}(M_H^2) \right] \bar{u}_{l_2}(1 - \gamma_5)v_{l_1} \quad (28)$$

where $F_{\text{dis}}(q^2)$ and $F_{\text{abs}}(q^2)$ are complex form factors, which are related to each other by a subtracted dispersion relation of the form

$$F_{\text{dis}}(M_H^2) = F_{\text{dis}}(q^2 = 0) + \frac{M_H^2}{\pi} \mathcal{P} \int_0^\infty dq^2 \frac{F_{\text{abs}}(q^2)}{q^2(q^2 - M_H^2)} \quad (29)$$

In what follows, we will focus our attention on two class of models, which have different phenomenological features.

### 3.1 The model with $n_R = 2$

In this first scenario we assume the presence of two right-handed neutrinos in the Yukawa sector, i.e. $n_R = 2$. In particular, to satisfy condition (26), we assume that the resulting two heavy neutrinos $N_1$ and $N_2$ have masses $m_{N_1} \simeq 100$ GeV and $m_{N_2} > 2M_W$. Since in this model $\det M^\nu = 0$, one of the light neutrinos will be massless (at the tree level), while the other two neutrinos can be taken to be extremely light by taking condition (13) to be approximately valid.
The number of mixing parameters $B_{ij}$ and $C_{ij}$ in these theories seems to be rather large. However, employing identities (17) and (18) we can derive some helpful relationships among them such as

$$ |B_{l_2 N_2} B_{l_1 N_2}^*| = \frac{m_{N_1}}{m_{N_2}} |B_{l_2 N_1} B_{l_1 N_1}^*| \tag{30} $$

$$ B_{l_2 N_2} C_{N_2 N_2} B_{l_1 N_2}^* = \frac{m_{N_1}^2}{m_{N_2}^2} B_{l_2 N_1} C_{N_1 N_1} B_{l_1 N_1}^* \tag{31} $$

$$ |B_{l_2 N_1} C_{N_1 N_2} B_{l_1 N_2}^*| = |B_{l_2 N_2} C_{N_2 N_1} B_{l_1 N_1}^*| = \frac{m_{N_1}}{m_{N_2}} |B_{l_2 N_1} C_{N_1 N_1} B_{l_1 N_1}^*| \tag{32} $$

from which we can estimate the order of magnitude for the different $CKM$-mixing combinations. Actually, we find that expressions of the form (31) and (32) arising from the graphs (1a) and (1b) are $\xi^4$-suppressed and will thus not be considered here. Eq. (30) shows a direct relation between heavy neutrino masses and $CKM$ mixings which must also be taken into account. As a consequence, the form factor $F^{\text{dis}}$ evaluated at $q^2 = 0$ [2] can be cast into the following form:

$$ F^{\text{dis}}(q^2 = 0) = \frac{1}{2} \sum_{i=1}^{n_B-2} B_{l_2 N_i} B_{l_1 N_i}^* \lambda_i \left\{ -\frac{3}{2} \frac{1}{1 - \lambda_i} + \frac{3}{2} \frac{1}{(1 - \lambda_i)^2} + \frac{3}{(1 - \lambda_i)^3} + \frac{M_H^2}{M_W^2} \left( \frac{\lambda_i \ln \lambda_i}{(1 - \lambda_i)^3} - \frac{1}{2} \frac{\lambda_i^2 \ln \lambda_i}{(1 - \lambda_i)^3} \right) \right\} \tag{33} $$

where

$$ \lambda_i = \frac{m_{N_i}^2}{M_W^2}, \quad \text{for} \quad i = 1, 2 \tag{34} $$

To make use of eq. (29), we need the analytical form of $F^{\text{abs}}(q^2)$, which has been calculated by applying the usual Cutkosky rules [14]. Keeping terms of order $\xi^2$, we get

$$ F^{\text{abs}}(q^2) = \sum_{i=1}^{n_B-2} B_{l_2 N_i} B_{l_1 N_i}^* \lambda_i \left\{ -I(q^2, m_{N_i}^2, 0) + K(q^2, m_{N_i}^2, 0, M_W^2) \right. $$

$$ - 4F(q^2, m_{N_i}^2, 0, M_W^2) + 2F(q^2, M_W^2, M_W^2, m_{N_i}^2) $$

$$ + 2 \left( 2 + \frac{q^2}{M_W^2} \right) G(q^2, M_W^2, M_W^2, m_{N_i}^2) - 2K(q^2, M_W^2, M_W^2, m_{N_i}^2) $$

$$ + \frac{M_H^2}{M_W^2} \left[ F(q^2, M_W^2, M_W^2, m_{N_i}^2) - K(q^2, M_W^2, M_W^2, m_{N_i}^2) \right] \right\} \tag{35} $$

9
where the functions $I$, $F$, $K$, $G$ are given below

$$I(q^2, m_1^2, m_2^2) = \theta(q^2 - (m_1 + m_2)^2) \frac{\pi}{2q^2} \lambda^{1/2}(q^2, m_1^2, m_2^2)$$

with

$$\lambda(x, y, z) = (x - y - z)^2 - 4yz$$

$$K(q^2, m_1^2, m_2^2, M^2) = \frac{M_W^2}{q^2} I(q^2, m_1^2, m_2^2) + \frac{M^2 - m_1^2}{q^2} K(q^2, m_1^2, m_2^2, M^2)$$

$$G(q^2, m_1^2, m_2^2, M^2) = \frac{M_W^2}{M^2} \left[ F(q^2, m_1^2, m_2^2, M^2) - F(q^2, m_1^2, m_2^2, 0) \right]$$

Finally, the absorptive part of $F(M_H^2)$ for $H^0$-mass values being above the $N_1$ mass and below the threshold of real $WW$ production is obtained by

$$F^{\text{abs}}(M_H^2) = B_{l_2N_1} B_{l_1N_1}^* \frac{m_{N_1}^2}{M_W^2} \left[ - I(M_H^2, M_{N_1}^2, 0) + K(M_H^2, M_{N_1}^2, 0, M_W^2) \right] - 4F(M_H^2, m_{N_1}^2, 0, M_W^2)$$

Being optimistic in our numerical considerations, we use the value 0.13 for the CKM-mixing combination $|B_{l_2N_1} B_{l_1N_1}^*|$ which is about the maximal experimentally allowed value for $l_2 = \tau$ and $l_1 = e$ or $\mu$ [15]. So, after performing numerically the integration in eq. (29), we evaluate the partial width of the $H^0$ decay into two different leptons through the expression

$$\Gamma(H^0 \rightarrow \bar{l}_1l_2 \text{ or } l_1\bar{l}_2) = \frac{\alpha_W^3}{128\pi^2} \frac{m_{l_2}^2}{M_W^2} M_H \left[ |F^{\text{dis}}(M_H^2)|^2 + |F^{\text{abs}}(M_H^2)|^2 \right]$$

Eq. (41) also enable us to check quantitatively the correctness of our formulas for sufficient small values of the ratio $M_H^2/4M_W^2$. We have found them to be in agreement with the
We are now in the position to investigate numerically the observability of the off-diagonal leptonic decay modes of $H^0$ by studying the ratio

$$R_{l_1l_2} = \frac{\Gamma(H^0 \to \bar{l}_1l_2) + \Gamma(H^0 \to l_1\bar{l}_2)}{\Gamma(H^0 \to b\bar{b})} \quad (42)$$

The dominant decay channel of $H^0$ to $b\bar{b}$ pairs, after including QCD corrections, has been estimated in [16]. The results of [16] can be summarized as follows:

$$\Gamma(H^0 \to b\bar{b}) = \frac{3\alpha_W}{8} M_H \frac{m_b^2}{M_W^2} \left( 1 + 1.8\alpha_s + 2.95\alpha_s^2 \right) \left( 1 + \mathcal{O}\left( \frac{\bar{m}_b^2}{M_W^2} \right) \right) \quad (43)$$

where $\bar{m}_b$ is the running b-quark mass given by

$$\bar{m}_b = \hat{m}_b \left( \frac{23\alpha_s}{6\pi} \right) \left( 1 + 0.374\alpha_s + 0.15\alpha_s^2 \right) \quad (44)$$

with $\hat{m}_b = 8.23$ GeV, for $\Lambda_{\overline{MS}} = 150$ MeV. The QCD coupling $\alpha_s$ and running mass $\bar{m}_b$ are evaluated at scale $M_H$.

In this model the CP-asymmetry parameter $A_{CP}$ defined in eq. (5) turns out to be

$$A_{CP} \simeq 2\sin\delta_{CP} \frac{F_{abs}(M_H^2) F_{dis}(N_2^2)}{|F_{abs}(M_H^2)|^2 + |F_{dis}(N_2^2)|^2} \quad (45)$$

where $\delta_{CP}$ is a CP-odd rephasing-invariant angle between the CKM expressions $B_{l_2N_1}B_{l_1N_1}^*$ and $B_{l_2N_2}B_{l_1N_2}^*$. Unfortunately, eq. (31) implies that $\delta_{CP} = 0$ and hence $A_{CP} = 0$. In eq. (45) $F_{dis}(N_2^2)$ stands for the $i$th summation term in eq. (33). Because of relation (30) $F_{dis}(N_1^2)$ will give the biggest contribution to $F_{dis}$. Therefore the branching ratio will be rather independent of the heavy neutrino mass $N_2$. In table 1 we present the numerical results for the branching ratio $R$ as function of the $N_2$ mass. On the other hand, as has been discussed extensively in [17], one expects that LHC or SSC colliders
will produce $10^6 - 10^7$ $H^0$ particles a year provided $M_H \leq 140$ GeV. Therefore, in the $SM$ with two right-handed neutrinos, flavor-changing decays of $H^0$ in the leptonic sector could, in principle, be observed if a next generation luminosity upgrade of a factor of 10 is assumed.

### 3.2 The model with $n_R = 3$

Let us now consider the case where each left-handed neutrino $\nu^0_{L,i}$ has a right-handed partner $\nu^0_{R,i}$. This scenario is more symmetric than the previous one and may hence represent a rather realistic situation. In particular, we assume that the one heavy Majorana neutrino $N_1$ is relatively light, i.e. $m_{N_1} \simeq 100$ GeV, while the other two neutrinos are non-degenerate (i.e. $m_{N_1} \ll m_{N_2} < m_{N_3}$), but their masses lie in the TeV range.

From eq. (18) we have the following useful approximations:

$$C_{N_3 N_3} \simeq \frac{m_{N_3}^2}{m_{N_3}^2} \frac{|B_{l_2 N_2}|^2}{|B_{l_2 N_3}|^2} C_{N_2 N_2}$$  \hspace{1cm} (46)

$$C_{N_3 N_2} \simeq - \frac{m_{N_2}}{m_{N_3}} \frac{B_{l_2 N_2}^*}{B_{l_2 N_3}^*} C_{N_2 N_2}$$  \hspace{1cm} (47)

As a result of eqs (46) and (47), the contribution to the form factor $F^{\text{dis}}(q^2)$ coming from the mixing angle quantity $|B_{l_2 N_2} C_{N_2 N_2} B_{l_1 N_2}^*|$ may become the most dominant one. To leading order in an expansion of $1/\lambda_2$ we get [2]

$$F^{\text{dis}}(q^2 = 0) \simeq \frac{3}{4} B_{l_2 N_2} C_{N_2 N_2} B_{l_1 N_2}^* \lambda_2$$  \hspace{1cm} (48)

For the phenomenologically compatible values of $C_{N_2 N_2} \simeq 0.1$ we readily see that the other CKM-mixing terms appearing in eq. (48) can safely be neglected provided $m_{N_{2,3}} \gtrsim 1$ TeV. In this scenario the ratio $R_{l_1 l_2}$ behaves like

$$R_{l_1 l_2} \simeq 12 \left( \frac{\alpha_W}{32\pi} \right)^2 \frac{m_{l_2}^2}{m_B^2} \lambda_2^2 |B_{l_2 N_2} C_{N_2 N_2} B_{l_1 N_2}^*|^2$$  \hspace{1cm} (49)
while the $CP$ asymmetry can be simply obtained by
\[
A_{CP} \simeq \frac{8}{3} \frac{\sin \delta_{CP}}{|B_{l_2 N_2 C_{N_2 N_2} B_{l_1 N_2}^*}|} \frac{F^{\text{abs}}(M_{\nu_2}^2)}{\lambda_2} \tag{50}
\]

Before proceeding with the presentation of the numerical results, we wish to comment on the dramatic enhancement of the quantity $R$ by increasing the heavy neutrino mass in eq. (49). For definiteness, when $m_{l_2} = m_\tau$, we find that
\[
R^{\text{max}}_{e\tau}, R^{\text{max}}_{\mu\tau} \simeq 10^{-2} \tag{51}
\]
for $m_{N_2} = 10 \text{ TeV}$ and $B_{l_2 N_2} C_{N_2 N_2} B_{l_1 N_2}^* \simeq 0.01$. Here two different constraints must be taken into account: the requirement of perturbative unitarity and consistency with the existing data on lepton-flavor-changing $Z^0$ decays. The former restriction gives roughly an upper bound
\[
\xi^2_{\nu N} \leq \frac{8}{\alpha_W} \frac{M_W^2}{m_N^2} \simeq 0.03 \tag{52}
\]
for $m_{N_2} \simeq 10 \text{ TeV}$ by imposing the inequality condition $\Gamma_N/m_N \leq 1/2$, where $\Gamma_N$ is the width of the heavy neutrino $N$. Nevertheless, to the best of our knowledge, constraints from leptonic $Z^0$ decays have not been considered as yet in this class of models. Adapting the results for $Z^0 \to b\bar{s}$ of ref. [18], we get a first estimate for the $Z^0 \to l_1 \bar{l}_2$ matrix element:
\[
T(Z^0 \to l_1 \bar{l}_2) \sim \frac{g_W \alpha_W}{32\pi \cos \theta_W} B_{l_2 N_2} C_{N_2 N_2} B_{l_1 N_2}^* \frac{m_{N_2}^2}{M_W^2} u_{l_2} \gamma_\mu (1 - \gamma_5) v_{l_1} \varepsilon^\mu_Z \tag{53}
\]
In order to reconcile the experimental value of [19]
\[
\text{BR}(Z^0 \to e\tau \text{ or } \mu\tau) \leq 10^{-5} \tag{54}
\]
with the theoretical prediction, we must constrain mixing-angles and heavy neutrino masses to the following range:
\[
|B_{l_2 N_2} C_{N_2 N_2} B_{l_1 N_2}^*|^2 \frac{m_{N_2}^4}{M_W^4} \leq 10^3 \tag{55}
\]
Consequently, in order to be consistent with eqs (52) and (55) in our numerical estimates, we use the optimistic value of $\sim 0.001$ for $B_{l_2N_2}C_{N_2N_2}B_{l_1N_2}^{\ast}$. From table 2 we see that this scenario yields rather encouraging branching-ratio values of the order of $10^{-4} - 10^{-5}$ and $CP$ asymmetries of the order of one ($\sim 40\%$).

4. Conclusions

Both flavor-violating signals and $CP$ violation in leptonic $H^0$ decays may arise in the Standard Model with three right-handed neutrinos, at a level that may be tested at future $H^0$ factories. Such class of models can naturally provide, tiny masses for the light neutrinos, relatively light masses (e.g. of the order of 100 GeV) for the heavy ones, as well as large light-heavy neutrino mixings (i.e. $\xi_{\nu N} \sim 0.1$), contrary to the traditional see-saw models. We find that these models may reveal attractive phenomenological features at high energy colliders such as $SSC$ or $LHC$. As has been shown, branching ratios of the order of $10^{-4} - 10^{-5}$ and $CP$ asymmetries of order one in leptonic off-diagonal $H^0$ decays can, in principle, be detected in these planned collider machines. More precisely, we suspect to be able to analyze $10^2 - 10^3$ events for the decays $H^0 \rightarrow e\tau$ or $\mu\tau$. Then, the contributing background to the above decays will be about $10^4$ events – in which the Higgs invariant mass has been reconstructed by using leptonic subsequent decays of $t\bar{t}$ pairs [20] – which is 10–10$^2$ times bigger than the desired signal. However, the simultaneous knowledge of large CP asymmetries of the order of $10^{-1} - 10^{-2}$ (after including the background mentioned above) makes it theoretically possible to establish such small and very interesting effects in the leptonic $H^0$ decays. Finally, special attention has been paid to the fact that the resulting values of the phenomena discussed in this work are consistent with all the existing information arising from neutrino-mixing effects, the validity of
pertubative unitarity and data of rare $Z^0$ decays at $LEP$.

In conclusion, in the simplest model which predicts heavy Majorana neutrinos, i.e. the Standard Model with right-handed neutrinos, we have explicitly shown that flavor nonconservation and $CP$ violation in leptonic $H^0$ decays can indeed be sizeable and are generally not suppressed by the usual see-saw mechanism. The attractive theoretical aspects of this minimal class of models may also lead to further phenomenological investigations in the gauge sector of the $SM$ or/and in the kaonic system [21].

Note added. The decay processes $Z^0 \rightarrow l_1\bar{l}_2$ have recently been analyzed in [22] in the context of the models discussed here, giving branching ratios closed to those that are qualitatively obtained by eqs (53) and (55).
References

G. Altarelli, talk given at the Int. Workshop on Electroweak Physics, València, Spain, Oct. 2–5, 1991, CERN TH.6317/91;


    The above two theoretical groups have first studied $CP$ nonconservation at high energy colliders
    by considering flavor-changing $Z^0$ decays. The mechanism of generating $CP$ violation in leptonic
    $H^0$ decays is similar to them.

    which are induced by particle widths (e.g. the top width) has been discussed;


[15] see, for instance, table VII in ref. [7].


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Figure and Table Captions

Fig. 1: Feynman graphs responsible for the flavour-changing lepton–lepton–$H^0$ vertex in the Feynman–‘t Hooft gauge. The dashed lines indicate on-shell contributions from the intermediate states.

Tab. 1: Numerical results of the ratio $R_{\text{max}}$ for the decay modes $H^0 \to e\tau$ or $\mu\tau$ in the SM with two right-handed neutrino fields ($n_R = 2$). In our numerical estimates we have used the following set of parameters: $M_W = 80.6$ GeV, $M_Z = 91.161$ GeV, $m_\tau = 1.784$ GeV, $\alpha_{\text{em}}(M_W^2) = 1/128$, $B_{l_2N_1}B_{l_1N_1}^* \simeq 0.13$.

Tab. 2: Numerical results for $R_{\text{max}}$ and $A_{\text{CP}}$ in a model with three right-handed neutrinos. Here, we have $m_{N_2} < m_{N_3}$ and $B_{l_2N_2}C_{N_2N_2}B_{l_1N_2}^* \sim 0.001$. 

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