In this paper we summarize the results of the theory working group dedicated to the analysis of $B_c$ production at hadron colliders.
1 Introduction

Bound states of a $b$ and $\bar{c}$ quark pair (the $B_c$ mesons) have never been observed. They represent interesting objects for QCD because their properties are expected to be calculable much in the same way as properties of $c\bar{c}$ and $b\bar{b}$ states have been studied theoretically in the last two decades by making use, e.g., of potential models or QCD sum rules.

A first reason of interest in the study of $B_c$ physics arises from the possibility to test quark potential models for systems made up by quarks of different flavours as well as to compare these predictions with other theoretical approaches. A second reason of interest arises from $B_c$ decays. These decays are described at the quark level by three classes of diagrams: annihilation diagrams, $c$ quark spectator diagrams (e.g. $B_c \rightarrow \psi \mu \nu$ or $B_c \rightarrow \psi \pi$) and $b$ quark spectator diagram (e.g. $B_c \rightarrow B_s X$). The first class of diagrams would allow, in principle, to measure the leptonic decay constant $f_{B_c}$. Note that this measurement would be enhanced at least by the factor $|V_{cb}/V_{ub}|^2 \approx 10^2$ as compared to the analogous measurement of $f_{B_d}$. Furthermore, the existence of two kinds of spectator diagrams is a novel feature as compared to other heavy meson decays.

For all these reasons, as well as to test QCD-like predictions for $B_c$ production, among the future high energy high luminosity hadron colliders a well defined place should be reserved to the study of $B_c$ properties, also because the production of such mesons at these future machines is expected to be rather abundant.

2 The properties of $B_c$ mesons

A possible approach to the study of $b\bar{c}$ meson properties: masses and leptonic decay constants, is provided by the quark potential models [1, 2, 3, 4, 5]; another approach is represented by QCD sum rules [6]. In this Section we review the predictions of both these methods.

An extensive calculation of $b\bar{c}$ meson properties has been performed in [3] by using the Martin’s potential [7]:

$$V(r) = -8.064 + 6.869r^{0.1}, \quad m_c = 1.8 GeV, \quad m_b = 5.174 GeV,$$

where the parameters of the potential are in GeV.

By solving the Schroedinger equation with the potential (2.1) one can obtain the $B_c$ mass spectrum and the value of wave function of $B_c$ at the origin. This method also predicts masses for radial and orbital excitations; they are reported in Table 1 together with the predictions of [1, 2] that are based on flavour dependent potentials. The values of Table 1 for the mass of the pseudoscalar low lying state (the $0^-$ $B_c$ meson) also agree with the result of a relativistic potential model [4] and with a QCD sum rules calculation [8], within theoretical uncertainties. We take as an indicative range of values for the $B_c(0^-)$ mass:

$$m_{B_c} = 6250 \pm 100 \text{ MeV}. \quad (2.2)$$

As for the mass difference between the $1^-$ and $0^-$ states, potential models give $m_{B_c^*} - m_{B_c} \approx 80\text{MeV}$, which could be an overestimate since $m_{B_c^*} - m_B \approx 50\text{MeV}$. Indeed we
should remember that non relativistic potential models have been tested only for the equal mass case and therefore, in the extrapolation to $\bar{c}b$ mesons, sizeable corrections could be introduced.

Note, that making use of $\Psi_{1s}(0) = 0.369 \text{ GeV}^{3/2}$ (see [3]), one can calculate the constant $f_{Bc}$ of weak decay of $B_c(0^-)$ meson: $f_{Bc} = \sqrt{\frac{12}{M}} |\Psi(0)|^2 \approx 570 \text{ MeV}$ where $f_{Bc}$ is defined by:

$$<0|\bar{c}\gamma_\mu\gamma_5 b|B_c(p)> = if_{Bc}p_\mu . \quad (2.3)$$

On the other hand QCD sum rules [6] give: $f_{Bc} = 360 \pm 60 \text{ MeV}$. We cannot solve this discrepancy at the moment and we limit ourselves to quote a rather broad range of values for $f_{Bc}$:

$$f_{Bc} = (300 \div 600) \text{ MeV}. \quad (2.4)$$

### 3 Hadronic production of $B_c$

At the present there are some estimates of the cross section for $B_c$ production based on the parton picture and perturbative QCD. As a lower bound of the cross section one should consider the $B_c$ meson pair production [8].

$$R_{Bc} = \frac{\sigma(B_c\bar{B}_c)}{\sigma(bb)} \simeq (2 \div 3) \cdot 10^{-4} \quad (\sqrt{s} = 100 \text{ GeV}). \quad (3.1)$$

The uncertainty in the prediction is mainly related to the value of $f_{Bc} (= 570 \text{ MeV}$ in this calculation), present in each vertex connecting $b\bar{c}$ with $B_c$.

For inclusive $B_c$ production (i.e. for $B_c\bar{b}c$ final state) the above value of the cross section may be increased by about an order of magnitude. Let us call $f_{(b\rightarrow Bc)}$ the fraction of $b$ antiquarks that will evolve into a $B_c$ meson. No complete calculation of this parameter is available yet, but partial estimates have been carried out [8, 9]. Reasonable estimates of this fundamental parameter are in the range:

$$f_{(b\rightarrow Bc)} = 1 \div 5 \times 10^{-3}, \quad (3.2)$$

combining both perturbative and non-perturbative contributions. Other estimates based on the Monte Carlo codes (HERWIG, version 5.0) are given in [3].

For the UNK energy range: $\sqrt{s} \sim 2 \text{ TeV}$ and with a luminosity: $\mathcal{L} \sim 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$ the expected annual yield of $B$-mesons is $\sim 10^{10}$, which, according to our estimates, corresponds to $\sim 10^7 B_c$ produced per year.

### 4 $B_c$ decays

From the study of inclusive decays of $B_c$ in the framework of quark potential models [10, 11] one gets that the contribution to the total rate from different decay mechanisms is 37% from $\bar{c}$-spectator decays, 45% from $b$-spectator decays and 18% from $b\bar{c}$-annihilation. From QCD sum rules [3] the corresponding values are 48%, 39% and 13% respectively. The $B_c$ lifetime computed by potential models is $\tau_{Bc} \simeq 5 \cdot 10^{-13} \text{ s}$, i.e. $\Gamma_{tot}(B_c) \simeq 1.3 \cdot 10^{-3} \text{ eV}$. 
The corresponding values from QCD sum rules are $\tau_{B_c} \simeq 9 \cdot 10^{-13}$ s, i.e. $\Gamma_{\text{tot}}(B_c) \simeq 7.4 \cdot 10^{-4}$ eV.

Let us now consider specific decay channels. A very clear signature for $B_c$ production could be given by the decay $B_c \to J/\psi \mu^- \bar{\nu}_\mu$ (three leptons coming from the same secondary vertex). The estimated branching ratio for this channel is [14, 11, 12, 6]:

$$BR(B_c \to J/\psi \mu^- \bar{\nu}_\mu) \simeq (1 \div 4) \cdot 10^{-2}.$$  \hspace{1cm} (4.1)

where the lower value corresponds to QCD sum rules prediction and the upper one to potential models.

We can also consider hadronic decays, whose estimates follow assuming the vacuum insertion approximation in effective nonleptonic Hamiltonian. Predictions for these decays from different quark models are summarized in Tables 2 and 3.

As a general characteristic, in $B_c$ decays the presence of $J/\psi$ in final states is rather frequent. The inclusive $B_c \to J/\psi + X$ rate can be evaluated approximately in the large $N_c$ limit, where the resulting branching ratio for $B_c \to J/\psi + (\text{light quarks or leptons})$ is $19\% < Br(B_c \to J/\psi + X) < 24\%$ [11].

In conclusion present theoretical investigations have not reached the accuracy required to get very accurate predictions for the $B_c$ decays. Table 4 summarizes predictions for the decay channels that are more interesting from the experimental point of view.

5 Prospects for $B_c$ Discovery at CDF

From the point of view of the detection, the two most important parameters are the production rate and the branching ratios (BR) into accessible decay modes. For the production rate we shall use the estimate for $f(b \to B_c)$ reported in Section 3. For the BR’s we shall take the ranges of values reported in Table 4. Notice that also the lifetime, as we have seen, has a rather large range of values. This is a critical parameter in view of the possibility to reduce the background to the decay modes via the presence of a secondary vertex.

In order to get a crude estimate of the possible signal at CDF, we will normalize to the number of observed exclusive $B$ decays. The best decay channel that allows full reconstruction is $B_c \to \psi \pi$. We will compare this channel with the observed $B_u \to \psi K^+$, assuming equal acceptance and reconstruction efficiency. Notice however that the efficiency for the $B_c$ decay is expected to be higher; in fact, the larger mass of the $B_c$ w.r.t the $B^+$ and the smaller mass of the pion w.r.t. the kaon will give a larger impact parameter and a higher transverse momentum to the decay pion.

Under the assumption of equal efficiency, we can write the following equation for the number of expected reconstructed decays (the subsequent $\psi \to \mu^+ \mu^-$ decay is understood):

$$\frac{(B_c \to \psi \pi^\pm)}{(B^+ \to \psi K^\pm)} = \frac{f(b \to B_c)}{f(b \to B_u)} \times \frac{BR(B_c \to \psi \pi^\pm)}{BR(B^+ \to \psi K^\pm)} \approx 6 f(b \to B_c),$$  \hspace{1cm} (5.1)

where we used the value of the BR given in Table 4 and $f(b \to B_u)=35\%$. Using the number of currently reconstructed $B \to \psi K^\pm$ (72 events in 9pb$^{-1}$), we obtain $N(B_c \to \psi \pi^\pm) \approx$
430 f_{(b \rightarrow B_c)}. This corresponds to 0.5–2 events, using the range of estimates for f_{(b \rightarrow B_c)}. Considering the levels of the combinatorial background, this signal could be detected with a larger integrated luminosity only in the upper range of f_{(b \rightarrow B_c)}, unless the detection efficiency is significantly higher for this mode than for B \rightarrow \psi K.

One could also hope to establish the presence of a B_c signal by looking at the inclusive B_c \rightarrow \psi \ell \nu decays, observing the presence of the third lepton coming from the same secondary vertex as the \psi. In this case we can write:

$$\frac{(B_c \rightarrow \psi e(\mu)X)}{(B \rightarrow \psi X)} = \frac{f_{(b \rightarrow B_c)}}{f_{(b \rightarrow B)}} \times \frac{2BR(B_c \rightarrow \psi \ell X)}{BR(B \rightarrow \psi X)} \approx 20 f_{(b \rightarrow B_c)}. \quad (5.2)$$

This indicates that of the order of 2–10 % of the \psi’s from B decays could be accompanied by an additional lepton (e or \mu). The requirement that this lepton come from the same vertex as the \psi and with a large p_t relative to the direction of the B should reduce significantly the possible background, but accurate feasibility studies are still lacking. In equation (5.2) we used the Potential Model (PM) estimate of the inclusive B_c \rightarrow \ell \nu decay. The QCD Sum Rules (SR) estimate would give a smaller, and perhaps unobservable, signal.
References


Table Captions

**Table 1** Masses of $B_c$ mesons (in GeV) calculated using various potential (\* are the level masses, calculated without relativistic corrections).

**Table 2** Two-body nonleptonic $b$-spectator decay rates (in units of $10^{-6} \text{ eV}$), calculated in potential models: BSW [13] and ISGW [14]. Values of the parameters are: $m_{B_c} = 6.27$, $m_{B_s} = 5.39$, $m_{B_s^*} = 5.45$, $m_B = 5.27$, $m_{B^*} = 5.33$, $m_\pi = 0.140$, $m_\rho = 0.77$, $m_K = 0.495$, $m_{K^*} = 0.86$, $f_\pi = 0.133$, $f_\rho = 0.216$, $f_\omega = 0.195$, $f_K = 0.162$, $f_{K^*} = 0.216 \text{ GeV}$.

**Table 3** Decay rates (in units of $10^{-6} \text{ eV}$) for some two body nonleptonic $c$-spectator decays, calculated with ISGW [14] form factors.

**Table 4** Values of some parameters of interest for the decay of the $B_c$. The expectations of different models (QCD sum rules (SR) or potential models (PM): BSW [13]; ISGW [14]), together with uncertainty estimates, are included whenever available.
<table>
<thead>
<tr>
<th>State</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1S *</td>
<td>6.315</td>
<td>-</td>
<td>6.301</td>
</tr>
<tr>
<td>2S *</td>
<td>7.009</td>
<td>-</td>
<td>6.893</td>
</tr>
<tr>
<td>3S *</td>
<td>-</td>
<td>-</td>
<td>7.237</td>
</tr>
<tr>
<td>1P *</td>
<td>6.735</td>
<td>-</td>
<td>6.728</td>
</tr>
<tr>
<td>2P *</td>
<td>-</td>
<td>-</td>
<td>7.122</td>
</tr>
<tr>
<td>3P *</td>
<td>-</td>
<td>-</td>
<td>7.395</td>
</tr>
<tr>
<td>1D *</td>
<td>7.008</td>
<td>-</td>
<td>7.145</td>
</tr>
<tr>
<td>2D *</td>
<td>-</td>
<td>-</td>
<td>7.308</td>
</tr>
</tbody>
</table>

Table 1

<table>
<thead>
<tr>
<th>Mode</th>
<th>BSW</th>
<th>ISGW</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_c^+ \to B_s + \pi^+$</td>
<td>47.8</td>
<td>67.7</td>
</tr>
<tr>
<td>$B_c^+ \to B_s^* + \pi^+$</td>
<td>39.4</td>
<td>53.4</td>
</tr>
<tr>
<td>$B_c^+ \to B_s + K^0$</td>
<td>3.1</td>
<td>6.7</td>
</tr>
<tr>
<td>$B_c^+ \to B_s + K^{*0}$</td>
<td>3.4</td>
<td>3.1</td>
</tr>
<tr>
<td>$B_c^+ \to B_s + \pi^+$</td>
<td>1.49</td>
<td>2.9</td>
</tr>
<tr>
<td>$B_c^+ \to B_s^* + \pi^+$</td>
<td>2.42</td>
<td>2.</td>
</tr>
<tr>
<td>$B_c^+ \to B_s^* + \pi^0$</td>
<td>0.05</td>
<td>0.1</td>
</tr>
<tr>
<td>$B_c^+ \to B_s^* + \omega$</td>
<td>0.04</td>
<td>0.09</td>
</tr>
<tr>
<td>$B_c^+ \to B_s^* + \rho^0$</td>
<td>0.48</td>
<td>0.48</td>
</tr>
<tr>
<td>$B_c^+ \to B_s^* + K^+$</td>
<td>3.35</td>
<td>5.</td>
</tr>
</tbody>
</table>

Table 2

<table>
<thead>
<tr>
<th>Mode</th>
<th>BSW</th>
<th>ISGW</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_c^+ \to J/\psi + \pi^+$</td>
<td>2.68</td>
<td></td>
</tr>
<tr>
<td>$B_c^+ \to J/\psi + \pi^+$</td>
<td>2.75</td>
<td></td>
</tr>
<tr>
<td>$B_c^+ \to J/\psi + \rho^0$</td>
<td>0.195</td>
<td>0.2</td>
</tr>
<tr>
<td>$B_c^+ \to J/\psi + K^+$</td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>

Table 3

<table>
<thead>
<tr>
<th>Lifetime</th>
<th>0.5 $\div$ 1.5 $\times 10^{-12}$ sec (PM) , 0.9 $\times 10^{-12}$ sec (SR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{B_c}$</td>
<td>$500 \pm 50$ MeV (PM) , $360 \pm 60$ MeV (SR)</td>
</tr>
</tbody>
</table>

Table 4

<table>
<thead>
<tr>
<th>BR($B_c \to \psi + X$)</th>
<th>$24 \pm 10%$ (PM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BR($B_c \to \psi \ell \nu$)</td>
<td>$3 \pm 1%$ (ISGW,BSW) , $0.8%$ (SR)</td>
</tr>
<tr>
<td>BR($B_c \to \psi \ell \nu + X$)</td>
<td>$4.7%$ (ISGW,BSW)</td>
</tr>
<tr>
<td>BR($B_c \to \psi \pi$)</td>
<td>$0.2%$ (ISGW,BSW)</td>
</tr>
</tbody>
</table>