PP-wave string interactions from
\( n \)-point correlators of BMN operators

Chong-Sun Chu, Valentin V. Khoze and Gabriele Travaglini

Centre for Particle Theory, University of Durham, Durham, DH1 3LE, UK

Email: chong-sun.chu, valya.khoze, gabriele.travaglini@durham.ac.uk

Abstract

We show that the BMN operators close under operator product expansion and form a sector in the \( \mathcal{N} = 4 \) supersymmetric Yang-Mills theory. We then identify short-distance limits of general BMN \( n \)-point correlators, and show how they correspond to the pp-wave string amplitudes. We also show that instantons do not contribute to the pp-wave/SYM correspondence.
1 Introduction

Recently Berenstein, Maldacena and Nastase (BMN) [1] put forward a remarkable proposal of a correspondence between certain operators in $\mathcal{N} = 4$ supersymmetric $SU(N)$ Yang-Mills theory (SYM) and massive states in string theory in a pp-wave background [2]

$$\frac{1}{\sqrt{J N^{J/2+1}}} \text{Tr} Z^J \longleftrightarrow |0, p^+\rangle, \quad (1)$$

$$\frac{1}{\sqrt{J N^{J/2+1}}} \sum_{l=0}^{J} \text{Tr} [\phi^3 Z^l \phi^4 Z^{J-l}] e^{\frac{2\pi n l}{J}} \longleftrightarrow a_n^\dagger a_{-n}^\dagger |0, p^+\rangle. \quad (2)$$

It is often said that the BMN operators form a sector in SYM in the double scaling limit

$$N \to \infty, \quad J \sim \sqrt{N} \quad \text{with } g_{\text{YM}} \text{ fixed}. \quad (3)$$

In this paper, we will give a more precise meaning to this statement using the operator product expansion (OPE). We will argue that, in the double scaling limit, the OPE of BMN operators does not give rise to non-BMN operators. We will use this short OPE to analyse certain short distance limits of general $n$-point correlators of the BMN operators, and find a precise correspondence with the structure of the string amplitudes in the pp-wave background.

The BMN correspondence holds in the double scaling limit (3). In this limit the 't Hooft coupling $\lambda = g_{\text{YM}}^2 N$ is infinite and perturbative calculations in gauge theory are not under control. BMN instead concentrated on a class of 'near-BPS' operators with large $R$-charge $J$, e.g. as in (2). For these operators the coupling is effectively

$$\lambda' = \frac{g_{\text{YM}}^2 N}{J^2} = \frac{1}{(\mu p^+ \alpha')^2}, \quad (4)$$

which is finite in the large $N$ limit (3) and can be taken small. The scaling dimensions $\Delta$ of the BMN operators are finite in limit (3), and are related to the masses of the corresponding string states via

$$\Delta - J = H_{\text{lc}} / \mu, \quad (5)$$

where $H_{\text{lc}}$ is the lightcone string Hamiltonian and $\mu$ is the scale of the pp-wave metric. Written in terms of gauge theory parameters, the string theory gives a prediction for the conformal dimension of the BMN operators

$$\Delta - J = \sqrt{1 + \frac{g_{\text{YM}}^2 N n^2}{J^2}}. \quad (6)$$

Since all the states in the perturbative spectrum of the string theory are already accounted for by the full set of BMN operators, it is a central part of the BMN proposal that
the scaling dimensions of the other (non-BMN) operators become infinite\(^1\) in this limit (3), and hence the non-BMN operators play no rôle in the perturbative pp-wave/SYM correspondence.

The field theory side of the pp-wave/SYM correspondence was recently discussed in [3–6]. Non-planar diagrams in the BMN limit (3) were first studied in [3] and in [4] and were found to be important and governed by \(J^2/N^2\). It follows from the double scaling limit (3) that in addition to \(\lambda'\) defined in (4), there is a second dimensionless constant

\[
\frac{J^2}{N} = 4\pi g_s (\mu p^+ \alpha')^2
\]

which plays the rôle of the genus counting parameter for the SYM Feynman diagrams [3, 4]. Anomalous dimensions were computed in [3–5, 7]. It was proposed in [4] that the coefficient of the three-point function of BMN operators in SYM is related to the three-string interactions in the pp-wave background. Planar three-point functions of BMN operators in free field theory were calculated in [4] and in the first nontrivial order of \(\lambda'\) in [6]. The proposal of [4] states that the matrix element of the lightcone Hamiltonian is related to the coefficient \(C_{ijk}\) of the three-point function in field theory via

\[
\langle i | P^- | j, k \rangle = \mu (\Delta_i - \Delta_j - \Delta_k) C_{ijk}
\]

in the leading order in \(\lambda'\). Checks of this in the free field limit were performed in [4, 8, 9]. See [10] for further aspects about string interactions in pp-wave background. Another form of this proposal (which is insensitive to the prefactor) [11] relates the ratio of the three-string amplitudes with those of the field theory three-point function coefficients

\[
\frac{\langle \Phi_1 | \langle \Phi_2 | \langle \Phi_3 | V \rangle \rangle}{\langle 0_1 | \langle 0_2 | \langle 0_3 | V \rangle \rangle} = \frac{C_{123}}{C_{123}^{(vac)}}
\]

Here \(\langle \Phi_1 | \langle \Phi_2 | \langle \Phi_3 | V \rangle \rangle\) is the three-string scattering amplitude in the string field theory formalism [8], \(\langle 0_1 | \langle 0_2 | \langle 0_3 | V \rangle \rangle\) is the vacuum amplitude and \(V\) is the lightcone three-string vertex. In [6] the field theory results for the three-point function and the corresponding string theory prediction were derived and found to be in precise agreement, thus confirming (9) up to and including the order \(\lambda'\) corrections.

The next important problem to understand is how the higher point SYM correlation functions manifest themselves on the string theory side of the correspondence. This issue will be addressed in the present paper. The plan of the paper is as follow. In section 2, we show that the OPE of the BMN operators is closed in the double scaling limit. We also show that this short OPE has a very natural interpretation in string theory. Using this short OPE, we establish and extend in section 3 the correspondence between SYM correlators and pp-wave string amplitudes. We show that in a certain short distance limit involving a hierarchy of multi-pincherings, generic BMN correlators reduce to expressions

\(^1\)Indeed it appears so in perturbation theory.
written in terms of the three-point function coefficients and the anomalous dimensions. These expressions have a form that corresponds precisely to tree amplitudes of strings in pp-wave background. We also briefly discuss how string loop amplitudes can be extracted from the BMN correlators. In section 4, we show that two- and three-point functions of BMN operators are protected from instanton corrections. This, together with the relation between string amplitudes and SYM correlators, developed in section 3, shows that there are no D-instanton correction to pp-wave string amplitudes. This is consistent with the apparent absence of D-instanton solutions in pp-wave background.

Other relevant aspects of the correspondence have been studied in [12], where it was emphasised that the worldsheet model is exactly solvable in the lightcone gauge. Questions of holographic relation in the pp-wave context were considered in [13].

2 Short OPE of BMN operators

We first briefly recall the structure of operator product expansion (OPE). In a general QFT, the OPE is the statement that, in the short distance limit, the product of two local operators can be expressed in terms of a sum over local operators in the theory

$$\mathcal{O}_I(0)\mathcal{O}_J(x) = \sum_K c^K_{IJ}(x)\mathcal{O}_K(0).$$  \hspace{1cm} (10)

This is an operator relation. For generic correlators $\langle \mathcal{O}_I(0)\mathcal{O}_J(x)\prod_k A_k(y_k) \rangle$, the relation (10) holds only when $|x| \ll |y_k|$. For conformally invariant unitary theories, one can always choose to work with a basis of operators which do not mix with each other and have definite conformal dimensions. The OPE takes then a simple form

$$\mathcal{O}_I(0)\mathcal{O}_J(x) = \sum_K \frac{C_{IJK}}{|x|^{|\Delta_I + \Delta_J - \Delta_K|}} \mathcal{O}_K(0).$$  \hspace{1cm} (11)

Conformal invariance of the theory implies that the two-point and three-point functions can be written in the form

$$\langle \mathcal{O}_I(x_1)\mathcal{O}_J(x_2) \rangle = \frac{\delta_{IJ}}{(4\pi^2x_{12}^2)^{\Delta_I}};$$  \hspace{1cm} (12)

$$\langle \mathcal{O}_{I_1}(x_1)\mathcal{O}_{I_2}(x_2)\mathcal{O}_{I_3}(x_3) \rangle = \frac{C_{I_1I_2I_3}}{(4\pi^2x_{12}^2)^{\frac{\Delta_1+\Delta_2-\Delta_3}{2}}(4\pi^2x_{13}^2)^{\frac{\Delta_1+\Delta_3-\Delta_2}{2}}(4\pi^2x_{23}^2)^{\frac{\Delta_2+\Delta_3-\Delta_1}{2}}},$$  \hspace{1cm} (13)

where $x_{IJ}^2 := (x_I - x_J)^2$.

We will now consider the OPE of two BMN operators. It is known [3, 4] that the original BMN operators (for example, the LHS of (2)) mix at the nonplanar level. Hence
the original BMN operators do not have well defined conformal dimensions, and one has to define a new basis where the operators do not mix [3]. This redefinition has to be implemented to all orders in $\lambda'$ and $g_2$. In what follows, we will work with this diagonalized basis of BMN operators.

As we mentioned earlier, the scaling dimension $\Delta_K$ of non-BMN operator becomes infinite in the double scaling limit, and hence they do not appear in the OPE. As a result, the sum in (11) is reduced to BMN operators only,

$$O_I(0)O_J(x) = \sum_{K \in \text{BMN}} \frac{C_{IJK}}{|x|^{\Delta_I + \Delta_J - \Delta_K}} O_K(0). \quad (14)$$

To make this statement more precise, let us consider a $n$-point function $\langle O_1(0)O_2(x)\prod_i A_i(y_i) \rangle$ where $O_1$ and $O_2$ are BMN, and $A_i$ are some local operators. Using (11) with fixed $|y_i| \gg |x|$, we obtain

$$\langle O_1(0)O_2(x)\prod_i A_i(y_i) \rangle = \sum_{K} \frac{C_{12K}}{|x|^{\Delta_1 + \Delta_2 - \Delta_K}} \langle O_K(0)\prod_i A_i(y_i) \rangle$$

$$= \frac{1}{|x|^{\Delta_1 + \Delta_2}} \sum_K C_{12K} \left| \frac{x}{y_1} \right|^{\Delta_K} f_K(y_i)$$

$$= \frac{1}{|x|^{\Delta_1 + \Delta_2}} \sum_{K \in \text{BMN}} C_{12K} \left| \frac{x}{y_1} \right|^{\Delta_K} f_K(y_i) \quad (15)$$

where we have defined $\langle O_K(0)\prod_i A_i(y_i) \rangle := |y_1|^{-\Delta_K} f_K(y_i)$. In the last line of (15), we have used $|x/y_1| \ll 1$ and the fact that $\Delta_K \to \infty$ unless $O_K$ is a BMN operator. This demonstrate the shortening of the OPE of the BMN operators in the double scaling limit.

The short OPE has a natural interpretation in string theory. To see this, let us rewrite (14) in the form

$$|x|^{\Delta_1 + \Delta_2 - J} O_1(0)O_2(x) = \sum_{K \in \text{BMN}} C_{12K} |x|^{\Delta_K - J} O_K(0), \quad x \to 0, \quad (16)$$

where $J := J_1 + J_2$ is the conserved R-charge. The factor $C_{12K}$ corresponds to the 3-string interaction vertex [4], see (9). In view of (5), $|x|^{\Delta_K - J}$ then corresponds to the integrand of the string propagator (without ghosts)

$$\frac{1}{L_0 + L_0 - 2} = \int \frac{d^2q}{|q|^2} q^{L_0 - 1} \bar{q}^{\bar{L}_0 - 1}, \quad (17)$$

with modulus $|q|^2$ mapped to $|x|^{1/\mu}$. Equation (16) has a suggestive diagramic representation, figure 1. We remark that the correspondence in figure 1 relies on the fact that non-BMN operators do not appear in the OPE (16). We will use this fundamental relation to uncover the higher point string amplitudes from the short distance limits of the BMN correlators.
Figure 1: Short OPE of BMN operators and its string interpretation.

3 n-string amplitudes from BMN correlators

It is well known that the only vertex in type IIB string field theory is the three-string vertex \([14]\), describing the joining and splitting of closed strings. At the same time the form of three-point functions in \(\mathcal{N} = 4\) SYM is uniquely determined by conformal invariance. Hence, it is natural to expect that the \(x\)-independent coefficient \(C_{I_1I_2I_3}\) of the three-point function is directly related to the three-string interaction. The analysis carried out in \([4]\) and more recently in \([6]\) confirms that this is indeed the case. On the other hand, general \(n\)-point functions \((n > 3)\) have a non-trivial space-time dependence and their form is not fully determined by conformal invariance. A question then arises of what is the meaning of these \(n\)-point functions of BMN operators on the string theory side. In this section we will argue that the short OPE introduced in the last section leads to a natural correspondence between the short distance limits of multi-BMN correlators and higher string amplitudes.

We first consider the four-point correlation function \(\langle O_1(x_1)O_2(x_2)O_3(x_3)O_4(x_4)\rangle\) of BMN operators. The four-string amplitude in the s-channel corresponds to a specific double OPE of this correlator, such that \(x_{12} \to 0\) and \(x_{34} \to 0\), as depicted in figure 2.
More precisely, consider the following expression

\[
|x_{12}|^{\Delta_1+\Delta_2}|x_{34}|^{\Delta_3+\Delta_4} \langle O_1(x_1)O_2(x_2)O_3(x_3)O_4(x_4) \rangle
\]

\[
= \sum_{K \in \text{BMN}} C_{12K} C_{34K} \left[ \frac{x_{12}x_{34}}{x_{23}^2} \right]^\Delta K, \quad x_{12} \to 0 \text{ and } x_{34} \to 0. \quad (18)
\]

Here we took a specific double pinching limit of a conformally invariant expression involving the four-point correlator of BMN operators. The choice of pinching determines which operators we are expanding in the short OPE. Hence the RHS of (18) is obtained from the short OPE of \( O_1(x_1)O_2(x_2) \) and of \( O_3(x_3)O_4(x_4) \). The skeleton diagram of this double OPE is shown in figure 2 and it corresponds to the s-channel of the string amplitude. As before, \( C_{12K} \) and \( C_{34K} \) correspond to two 3-string interaction vertices, and \( \left( |x_{12}| |x_{34}| / |x_{23}|^2 \right)^{\Delta_K - J} \) corresponds to the integrand of the string propagator. Here \( J = J_1 + J_2 = -J_3 - J_4 \) is \( K \)-independent and \( |x_{12}| |x_{34}| / |x_{23}|^2 \) is a conformally invariant cross-ratio in the double pinching limit. The double pinching limit in (18) is understood as a power-series expansion in the small quantity \( |x_{12}| |x_{34}| / |x_{23}|^2 \). In other words, we keep only finite \( \Delta_K \). The sum over the BMN operators in (18) corresponds precisely to the sum over physical intermediate string states in the s-channel.

The two other channels of the four-point string amplitude similarly arise from the remaining two double pinching limits of the four-point BMN correlator:

\[
t - \text{channel} : \lim_{x_{13} \to 0, x_{24} \to 0} |x_{13}|^{\Delta_1+\Delta_3}|x_{24}|^{\Delta_2+\Delta_4} \langle O_1(x_1)O_2(x_2)O_3(x_3)O_4(x_4) \rangle, \quad (19)
\]

\[
u - \text{channel} : \lim_{x_{14} \to 0, x_{23} \to 0} |x_{14}|^{\Delta_1+\Delta_4}|x_{23}|^{\Delta_2+\Delta_3} \langle O_1(x_1)O_2(x_2)O_3(x_3)O_4(x_4) \rangle. \quad (20)
\]

The double pinching limits in the equations above are to be interpreted as before.

Note that so far we have been concerned with reproducing the integrand of the string amplitude in different channels. To get the full string amplitude, one has to integrate over the string moduli and sum over the different channels in string field theory. Our method, based on the analysis of short distance limits of BMN correlators, does not give information about the integration region over the string moduli. It would be interesting to understand better how and if this arises from the SYM point of view.

We now discuss the generalization to higher-point string amplitudes. The general approach is similar to the analysis above, but there is an important novel feature – a hierarchy of pinchings. To illustrate this we consider an example of seven-point string amplitude depicted in figure 3. As it should be clear from the structure of this string amplitude, the corresponding field theory correlator should be taken in the short-distance limit involving the pinching of \( x_{12}, x_{34}, x_{56} \to 0 \). This has to be followed by a second
pinching of $x_{13}, x_{57} \to 0$. More specifically we consider

$$|x_{12}|^{\Delta_1+\Delta_2} |x_{34}|^{\Delta_3+\Delta_4} |x_{56}|^{\Delta_5+\Delta_6} |x_{57}|^{\Delta_7} \prod_{I=1}^{7} \langle \mathcal{O}_I(x_I) \rangle$$

(21)

$$\sum_{K_1,K_2,K_3} C_{12K_1} C_{34K_2} C_{56K_3} |x_{12}|^{\Delta_{K_1}} |x_{34}|^{\Delta_{K_2}} |x_{56}|^{\Delta_{K_3}} \langle \mathcal{O}_{K_1}(x_1) \mathcal{O}_{K_2}(x_3) \mathcal{O}_{K_3}(x_5) \mathcal{O}_7(x_7) \rangle$$

(1)

$$\sum_{K_{1,2,3,L}} C_{12K_1} C_{34K_2} C_{56K_3} C_{K_{1,2,3}} |x_{12}|^{\Delta_{K_1}} |x_{34}|^{\Delta_{K_2}} |x_{56}|^{\Delta_{K_3}} |x_{13} x_{57}|^{\Delta_L} \frac{x_{12}}{x_{13}} \frac{x_{34}}{x_{13}} \frac{x_{56}}{x_{57}} \frac{x_{13} x_{57}}{x_{15}} ,$$

where the limit (1) denotes the first hierarchical pinching $|x_{12}|, |x_{34}|, |x_{56}| \to 0$, and the limit (2) denotes the second pinching $|x_{13}|, |x_{57}| \to 0$. As before, the three-point coefficients $C_{IJK}$ correspond to the three-string vertices, and the $x$-ratios are identified with the moduli of the corresponding string propagators. The sum over all such inequivalent skeleton diagrams in field theory, appropriately integrated over, corresponds to the full string amplitude.

In the above, we have discussed how, in a certain short distance limit that involves a hierarchy of multi-pinching, an $n$-point BMN correlator corresponds to a tree $n$-string amplitude in a specific channel. Now we discuss how to obtain string loop amplitudes from the BMN correlators. The analysis again hinges on the use of the short OPE, but there is yet another important new feature of the procedure – cluster decomposition. To illustrate the idea, it is sufficient to consider the simple case of a one-loop two-point string amplitude drawn in figure 4. To reproduce this two-point amplitude, we have to consider a six-point BMN correlator.
Consider the following quantity $I$ and its limits

\[
I := |x_{12}|^{2\Delta_1} |x_{46}|^{2\Delta_2} \sum_{r,s \in \text{BMN}} \langle O_1(x_1) O_r(x_2) O_s(x_3) O_r(x_4) O_s(x_5) O_2(x_6) \rangle |x_{23}|^{2\Delta_r + \Delta_s} |x_{45}|^{2\Delta_s + \Delta_r} \tag{1}
\]

\[
\rightarrow |x_{12}|^{2\Delta_1} |x_{46}|^{2\Delta_2} \sum_{r,s \in \text{BMN}} C_{rsa} C_{rsb} |x_{23}|^{\Delta_r + \Delta_s} |x_{45}|^{\Delta_s + \Delta_b} \langle O_1(x_1) O_a(x_2) O_b(x_4) O_2(x_6) \rangle \tag{22}
\]

Here the limit (1) denotes the short distance limit $|x_{23}| \to 0$, $|x_{45}| \to 0$. After this limit, $I$ becomes a function of $x_1, x_2$ and $x_4, x_6$. The second limit (2) is a large distance cluster limit where we group $x_1, x_2$ and $x_4, x_6$ into two independent clusters and send them far away from each other. Due to the cluster decomposition principle which holds in a general QFT, the four-point function in the second line of (22) factorizes in this limit as

\[
\langle O_1(x_1) O_a(x_2) O_b(x_4) O_2(x_6) \rangle = \frac{\delta_{1a}}{|x_{12}|^{2\Delta_1}} \frac{\delta_{2b}}{|x_{46}|^{2\Delta_2}}, \tag{23}
\]

and the last line in (22) follows. As before, the three-point coefficients $C_{IJK}$ correspond to the three-string vertices. $x_{23}$ and $x_{45}$ are identified with the moduli of the corresponding string propagators. Note that only physical degree of freedom propagate in the string loop in the lightcone gauge. The last line of (22), when appropriately integrated over, corresponds to the full one-loop two-point string amplitude.

It should be clear from the above analysis how to generalize to the higher loop case. Generally, to obtain an $n$-point $h$-loop string amplitude for a specific string field theory diagram, one has to start with a $3v$-point BMN correlator where $v$ is the number of vertices in the string diagram. Short distance limit (like (1) above) generates the string propagators. Then it is followed by a large distance clustering limit which separates the $v$ vertices. The resulting expression is in direct correspondence with the string loop amplitude.
Due to the very nature of the Penrose limit, which relies on the existence of null geodesics, the pp-wave metric cannot be Euclideanized and stay real. This suggests that there are no D-instantons in a pp-wave background. For the pp-wave/SYM correspondence to hold, this means that there should be no Yang-Mills instanton corrections to the pinched $n$-point correlators of BMN operators. In general, there is no reason to expect that instantons do not contribute to generic SYM correlators since $g^2_{YM}$ is fixed in the double scaling limit (3). According to the analysis in the previous sections, it is sufficient to show the absence of instanton corrections to the two- and three-point functions of BMN operators. We will prove this now.

We start by examining two-point functions. Our analysis is similar to that in [15], which showed that instanton corrections to extremal correlators in $\mathcal{N} = 4$ SYM vanish. Recall that in the case of $\mathcal{N} = 4$ SYM with gauge group $SU(N)$, the Dirac operator in the adjoint representation in the background of an instanton of winding number $k$ has $8kN$ zero modes. However, only 16 of them are exact zero modes, since the remaining ones are lifted by the presence of a fermion quadrilinear, which is induced by the Yukawa term in the instanton action [16]. All the considerations in this section apply to instantons of arbitrary charge. For full details of the ADHM instanton calculus we refer the reader to the review [17]. The exact zero modes can be generated acting with supersymmetry and superconformal transformation on the instanton, and take the form

$$\lambda^I(x) = \frac{1}{2} F_{\mu\nu} \sigma^{\mu\nu} \zeta^I(x).$$

Here $\zeta^I(x) = \xi^I + \bar{\eta}^I \sigma_\mu (x - x_0)^\mu$ and $I = (0, i)$, with $i = 1, 2, 3$. $\xi^I, \tilde{\eta}^I$ are eight constant Weyl spinors, and $x_0$ is the centre of the instanton configuration. Solving the equations of motion for the three complex scalar fields $\phi^i$ (and $\bar{\phi}_i$) of $\mathcal{N} = 4$ SYM, we obtain

$$\phi^i = \frac{1}{2} \xi^0 F_{\mu\nu} \sigma^{\mu\nu} \zeta^i,$$

$$\phi_i^\dagger = \epsilon_{ijk} \xi^j F_{\mu\nu} \sigma^{\mu\nu} \zeta^k.$$  

We can choose for example $\phi^1$ (resp. $\phi^2, \phi^3$) to be the instanton components of the field $Z$ (resp. $\Phi, \Psi$).

A BMN operator, such as

$$\mathcal{O}_{n,-n}^J = \frac{1}{\sqrt{JN^{J/2+1}}} \sum_{l=0}^{J} \text{Tr} [\Phi Z^l \Psi Z^{J-l}] e^{\frac{2\pi inl}{J}},$$

contains $J \sim \sqrt{N}$ insertions of the field $Z$, and possibly $p$ insertions of “impurities” $\Phi, \Psi$ ($p = 2$ in (27)). Unless all the fermion zero modes are exactly integrated out,
instanton corrections to the correlator $\langle \mathcal{O}_{BMN}(x) \bar{\mathcal{O}}_{BMN}(0) \rangle$ vanish. We now show that this is precisely the case. Notice that the spinor $\zeta^0$ appears only in the expression (25) for $\phi^i$, but not in $\bar{\phi}^i$. It then follows immediately from the property $(\zeta(x))^n = 0$ for $n \geq 3$, that the four zero modes associated to $\zeta^0$ can never be saturated. In conclusion we have

$$\langle \mathcal{O}_{BMN}(x) \bar{\mathcal{O}}_{BMN}(0) \rangle_{\text{instanton}} = 0$$

for any value of $k$ and $N$.

Similarly it is easy to see that it is impossible to saturate exact fermion zero modes for all flavours $i = 1, 2, 3$ for three-point functions of BMN operators. Therefore we conclude that they receive no instanton contributions.

The operator $\mathcal{O}^I_{n,-n}$ is non-BPS for $n \neq 0$. When $n = 0$ it becomes a protected operator, and its two- and three-point functions do not receive either perturbative or nonperturbative corrections [18,19]. Turning on phase factors gives rise to perturbative corrections, but as we have shown, instanton corrections are still absent.

**Acknowledgements**

We would like to thank Patrick Dorey, Dan Freedman, Simon Ross and Rodolfo Russo for useful discussions. CSC thanks the Department of Physics, National Tsing Hua University and the National Center of Theoretical Science, Taiwan for hospitality. We acknowledge grants from the Nuffield foundation and PPARC.

**References**


