Rapidity spectra analysis in terms of non-extensive statistic approach

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We provide description of rapidity spectra of particles produced in $p\bar{p}$ collisions using anomalous diffusion approach to account for their non-equilibrium character. In particular, we exhibit connection between multiproduction processes and anomalous diffusion described through the nonlinear Focker-Planck equation with nonlinearity given by the nonextensivity parameter $q$ describing the underlying Tsallis $q$-statistics and demonstrate how it leads to the Feynman scaling violation in these collisions. The $q$ parameter obtained this way turns out to be closely connected to parameter $1/k$ converting the original poissonian multiplicity distribution to its observed Negative Binomial form. The inelasticity of reaction has been also calculated and found to slightly decrease with the increasing energy of reaction.

1. INTRODUCTION

In description of multiparticle production processes one often uses statistical methods and concepts which follow the classical Boltzmann-Gibbs (BG) approach. However, it was demonstrated recently that to account for the long range correlations and for some intrinsic fluctuations in the hadronizing system one should rather use the nonextensive Tsallis statistics [1], in which one new parameter $q$ describes summarily the possible departure from the usual BG case (which is recovered in the $q \to 1$ limit) [2]. Here we shall provide detail description of the rapidity spectra of particles (mostly pions) produced in $p\bar{p}$ collisions using the anomalous diffusion approach to this problem and in this way accounting for their non-equilibrium character.

This method originates from diffusion model approach to nuclear multiparticle production collisions developed in [3], which has been further (successfully) applied to the recent RHIC data in [4]. The first attempt to extend it also to the case of anomalous diffusion (which corresponds to nonextensive $q \neq 1$ case) has been presented recently for nuclear collisions [5]. The non-linear Fokker-Planck (FP) equation used in this case has form $(f= f(y, t)$ with $y$ being rapidity and $t$ time variable):

$$\frac{\delta}{\delta t} f^\mu = \frac{\delta}{\delta y} \left\{ J(y) f^\mu + D \frac{\delta}{\delta y} f^\nu \right\},$$

(1)

$D$ and $J$ are diffusion and drift coefficients respectively. The hadronization process is visualized here as diffusion in the rapidity space starting with rapidities (in cms frame) $Y_{\text{max}}^{(\pm)} \simeq \pm \ln \sqrt{s}/m_T$ (where $\sqrt{s}$ is invariant energy of reaction and $m_T = \sqrt{m^2 + \langle p_T \rangle^2}$ mean transverse mass kept here as given. In the approaches presented in [3] and [4] linear or constant drift coefficients has been used and in effect obtained double gaussian-like form of rapidity spectra\(^1\). As was demonstrated in [6,5] for drift proportional to the longitudinal momentum of the particle (i.e., $J(y) \sim \sinh y$) one gets the thermal (Boltzmann) distribution (for linear FP equation, i.e., for $q = \nu = 1$) whereas for nonlinear-FP equation the time dependent solution has the specific power-like form the norm of which is conserved only for $\mu = 1$, therefore $\nu = 2 - q$ and our distribution is given

\(^1\)Interestingly enough, such spectra for $dN/dy$ were already postulated and used on purely phenomenological grounds as simple parametrizations of results of string models allowing for fast numerical calculations of cosmic ray cascades [7].
by the following formula:

\[ f_q(y) = \left[ 1 - (1 - q) \frac{m_T}{T} \cosh (y - y_m) \right]^{\frac{1}{1-q}}. \]  

(2)

2. RESULTS

We show now results of fitting experimental data on rapidity distributions of secondaries produced in \( p\bar{p} \) collisions \([8]\) by using formula (2) in the following form:

\[ f_q(y) = f_q^+(y) + f_q^-(y) \]  

(3)

where

\[ f_q^{\pm}(y) = \left[ 1 - (1 - q) \frac{m_T}{T} \cosh (y \pm y_m) \right]^{\frac{1}{1-q}}. \]  

(4)

with \( T, y_m \) and \( q \) being the (energy dependent) parameters (the values of transverse mass \( m_T \) was in this case taken as fixed and equal to \( m_T = 0.4 \) GeV, its possible energy dependence (for example such as used in (7) below) is then hidden in the energy dependence of the parameter \( T \)). The results are shown in Fig. 1 (for UA5 \([9]\) and Tevatron \([10]\) data) where the following energy dependences of \( T, y_m \) and \( q \) were used:

\[ T(\sqrt{s}) = a_1 + b_1 \cdot \ln (\sqrt{s}), \]  

\[ y_m(\sqrt{s}) = a_2 + b_2 \cdot \ln (\sqrt{s}), \]  

\[ q(\sqrt{s}) = a_3 + b_3 \cdot \ln (\sqrt{s}). \]  

(5)

The values of coefficients are: \( a_1 = -0.178 \pm 0.099, \ b_1 = 0.267 \pm 0.016, \ a_2 = 0.451 \pm 0.129, \ b_2 = 0.239 \pm 0.023, \ a_3 = 0.91 \pm 0.012, \ b_3 = 0.072 \pm 0.002. \) It turns out that the same parameters can also fit data from P238 \([11]\) and UA7 \([12]\) groups obtained for different rapidity regions at energy 630 GeV, cf. Fig. 2. It is interesting that although our \( y_m \) is not supposed to be \( a \) priori connected with the values of the maximal possible rapidities at given energy, \( Y_{max} \), nevertheless we have found that ratio \( y_m/Y_{max} \) changes only weakly with energy varying between 53 and 1800 GeV from 0.265 to 0.285. Notice also that for small values of \( |y - y_m| \) Eq.(3) can be approximated by double quasi-gaussian form (which becomes pure double gaussian function in the limit of \( q \to 1 \)).

It is well known that the charged particle multiplicity distributions may well be fitted by a negative binomial distribution (NB) \([8]\). It has two parameters: mean charged multiplicity \( \bar{n} \), and the parameter \( k (k \geq 1) \) affecting its shape (width) (for \( k \to 1 \) NB approaches geometrical distribu-
tions whereas for $k^{-1} \to 0$ it approaches Poisson distribution. The observed widening of the normalized multiplicity distribution with increasing energy implies then that parameter $k$ decreases with energy. Following ideas expressed in [2] we would like to bring ones attention to the fact that the value of parameter $k^{-1}$ may be understood as the measure of fluctuations of mean multiplicity. When one starts with Poisson multiplicity distribution and then allows $\bar{n}$ to fluctuate according to gamma distribution with normalized variance given by $D(\bar{n})$ then it is easy to show [2] that as result one gets NB distribution with $k^{-1} = D(\bar{n}) = \sigma^2(\bar{n})/(\langle \bar{n} \rangle^2) = q - 1$. Namely:

$$
P(n) = \int_0^\infty \frac{e^{-\bar{n}} \cdot \bar{n}^n}{n!} \cdot \frac{\gamma^k \cdot \bar{n}^{k-1} \cdot e^{-\gamma \bar{n}}}{\Gamma(k)} d\bar{n} = $$

$$
= \frac{\Gamma(k+n)}{\Gamma(1+n) \cdot \Gamma(k)} \cdot \frac{\gamma^k}{(\gamma + 1)^{k+n}}, \quad (6)
$$

where $\gamma = k/(\langle \bar{n} \rangle)$. It is worth to notice that, indeed, the energy variation of parameter $q$ is almost the same as $k^{-1}$ (see Fig. 3). The small discrepancies observed there, namely the fact that $q - 1$ is consistently larger than $k^{-1}$, can be explained by realizing that our $q$ has been obtained from fitting the experimental distributions $dN/dy$ and therefore it contains in addition to the above mentioned fluctuations in $\bar{n}$ also fluctuations in inelasticity $K$, not discussed here. In any case it is at this moment tempting to assume that the nonextensive parameter $q$ (parametrizing already nonlinearity of our FP equation used to describe data) also describes those fluctuations.

In Fig. 4 we show the total inelasticity $K = K(s)$ obtained by integrating spectra given by eq. (3) with parameters obtained from the fit to the corresponding rapidity distributions

$$
K = \frac{2}{\sqrt{s}} \int_{Y_{\text{min}}}^{Y_{\text{max}}} \frac{3}{2} \cdot f(y) \cdot m_T \cosh y \, dy \quad (7)
$$

as function of the invariant energy of the reaction $\sqrt{s}$ both for the energies considered here and extrapolated to the LHC energy. In this case the values of transverse mass $m_T$ for a given energy was obtained by using simple interpolating formula: $m_T = 0.3 + 0.044 \ln(\sqrt{s}/20)$. As one can see in Fig. 4 such inelasticity $K(s)$ is decreasing with the energy.

3. SUMMARY AND CONCLUSIONS

We have provided here description of rapidity spectra of particles produced at CERN and Fermilab energies treating their formation as diffusion process in rapidity space. To account for the
anomalous character of such diffusion, the non-
linear form of Focker-Planck equation has been
used with nonlinearity described by parameter
$q$, the same as the nonextensivity parameter de-
scribing the underlying Tsallis statistics. As seen
in Figs. 1 and 2, very good agreement with data
has been obtained with apparently three param-
eters: the “temperature” $T$, position of the peak
at rapidity $y_m$ and parameter $q$ - all logaritmi-
cally dependent on the energy of reaction $\sqrt{s}$.
However, after closer inspection it turns out that
parameter $q$, which according to [2] can be re-
garded as a measure of fluctuations existing in
the physical system under consideration, follows
essentially the fluctuations of multiplicity of parti-
cles produced at given energy. The small differ-
ences noticed in Fig. 3 are, in our opinion, caused
entirely by the fluctuations in the inelasticity of
the reaction, which makes the initial energy avail-
able for the production of secondaries a fluctuat-
ing quantity $3$. Similarly, parameter $y_m$ seems
to be closely connected with the maximal available
rapidity $Y_{max}(\sqrt{s})$ $4$. This shows that the only
parameter which is entirely ”free” is the limiting
temperature $T$ $5$. Therefore we can say that what
we are proposing here is essentially one-parameter
fit successfully describing data on rapidity distri-
butions.

The resultant inelasticity $K$ (defined by the for-
gula (7)) turns out to be decreasing function of
energy, as seen in Fig. 4, confirming our previous
findings in that matter $13$. We would like to
bring ones attention to the fact that in our ap-
proach we can fit equally well (using the same set
of parameters) data obtained by P238 [11] and
UA7 [12] collaborations which cover different re-
gions in rapidity. This makes extrapolation of our
formula to higher rapidity region more credible
and therefore puts more weight on the obtained
energy behaviour of inelasticity.

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\[ \frac{dN}{dy} \]

- \( E_{cm} = 53 \text{ GeV} \)
- \( E_{cm} = 200 \text{ GeV} \)
- \( E_{cm} = 540 \text{ GeV} \)
- \( E_{cm} = 900 \text{ GeV} \)
- \( E_{cm} = 1800 \text{ GeV} \)
- \( E_{cm} = 5000 \text{ GeV} \)