Direct vs. resolved photon: an exercise in factorization

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Abstract Direct and resolved photon interactions are shown to be intimately related through the factorization mechanism. It is argued that in theoretically consistent analysis of jet production in $\gamma p$ and $ep$ collisions the LO resolved $\gamma$ contribution must be considered together with the NLO direct $\gamma$ component. Recent data from HERA therefore do not provide a direct evidence for the former component, but should rather be interpreted as a manifestation of the $O(\alpha^2 \alpha_s^2)$ term in $ep$ interactions.

In recent months an increasing flow of fresh data from HERA at DESY has brought several interesting new results on the structure of photon. Among them, the interpretation [1, 2] of data on two-jet events an as evidence for the resolved $\gamma$ contribution to $\gamma p$ interactions has attracted considerable attention. I consider this interpretation as premature. Due to close relation between direct and resolved $\gamma$ components, based on the general factorization arguments, there is in fact no principal difference between these two contributions. I shall argue that in theoretically consistent treatment of $\gamma p$ interactions, the usual resolved $\gamma$ component should always be considered simultaneously with the NLO direct one, which, in fact, leads to kinematically very similar final state configurations. For the case of quarks inside the photon, the separation of $\gamma p$ interactions into “direct” and “resolved” components is theoretically ambiguous and doesn’t allow a unique interpretation of observed jet correlations as an evidence for the “resolved” photon.

Let me start with an important comment on the photon structure functions. To the order $O(\alpha \alpha_s)$ the evolution equations for quark (of flavour $i$ and charge $e_i$), gluon and photon distribution functions inside the real photon read

$$\frac{dD_{\gamma/\gamma}(x, M)}{d \ln M} = 0, \quad D_{\gamma/\gamma}(x, M) = \delta(1 - x) \quad (1)$$

$$\frac{dD_{g/\gamma}(x, M)}{d \ln M} = \frac{\alpha_s}{\pi} \sum_{j=1}^{2n_f} \int_{x}^{1} \frac{dy}{y} D_{q_j/\gamma}(y, M) P_{g/q}^{(0)} \left( \frac{x}{y} \right) + \frac{\alpha_s}{\pi} \int_{x}^{1} \frac{dy}{y} D_{g/\gamma}(y, M) P_{g/g}^{(0)} \left( \frac{x}{y} \right) \quad (2)$$

$$\frac{dD_{q_i/\gamma}(x, M)}{d \ln M} = \frac{\alpha}{\pi} k_i(x) + \frac{\alpha_s}{\pi} \int_{x}^{1} \frac{dy}{y} D_{q_i/\gamma}(y, M) P_{q/q}^{(0)} \left( \frac{x}{y} \right) + \frac{\alpha_s}{\pi} \int_{x}^{1} \frac{dy}{y} D_{g/\gamma}(y, M) P_{q/g}^{(0)} \left( \frac{x}{y} \right) \quad (3)$$
where \( k_i(x) = 3e_i^2(x^2 + (1 - x)^2) \) is the LO QED branching function corresponding to the vertex in Fig.1a and \( P_{a/b}^{(0)}(z) \) are the analogous LO QCD branching functions. In (1-3) the argument (not written out explicitly) of the strong coupling constant \( \alpha_s \), the so-called factorization scale \( M \), is the same as the scale of the various distribution functions. In the above equation the term “quarks” denotes both quarks and antiquarks and correspondingly the summation in (2) runs from 1 to \( 2n_f \).

In many papers, e.g. [3-7], dealing with the photon distribution functions one finds the claim that the resolved \( \gamma \) contribution, described by Feynman diagrams like those of Fig.2a, is of the order \( O(\alpha \alpha_s) \), despite the presence of two strong interaction vertices. This claim is based on incorrect analysis of the behaviour of \( D_{q/\gamma}(x, M) \) in the limit \( \alpha_s \to 0 \), which has lead to the wrong conclusion that it behaves like \( O(\alpha/\alpha_s) \), the \( \alpha_s \) in the denominator cancelling one power of \( \alpha_s \) from the vertices. To show this, let us investigate, in LO QCD, the behaviour of \( D_{q/\gamma} \) in weak coupling region. In the simplest case of the generic nonsinglet quark distribution function, \( D_{NS}(x, M) \), the appropriate evolution equation reads [8]

\[
\frac{dD_{NS}(x, M)}{d\ln M} = \frac{\alpha}{\pi} k_{NS}(x) + \frac{\alpha_s}{\pi} \int_x^1 \frac{dy}{y} D_{NS}(y, M) P_{q/q}^{(0)} \left( \frac{x}{y} \right),
\]

where

\[
\alpha_s(M) = \frac{4\pi}{\beta_0 \ln(M^2/\Lambda^2_{LO})}.
\]

Switching off QCD already at this stage, i.e. setting \( \alpha_s = 0 \) on the r.h.s. of (4), we get

\[
D_{NS}^{QED}(x, M) = \frac{\alpha}{\pi} k_{NS}(x) \ln \frac{M}{M_0},
\]

where \( M_0 \) is an arbitrary positive constant (\( M \) is by definition positive). Eq. (6) is, in fact, what we get by integrating the pole part

\[
\frac{\alpha}{2\pi} k_{NS}(x) \frac{1}{-t}
\]

of the corresponding Feynman diagrams in Fig.2b, in the interval \( |t| \in (M_0^2, M^2) \).

The claim that \( D_{NS}(x, M) = O(\alpha/\alpha_s) \) or, in general, \( D_{q/\gamma}(x, M) = O(\alpha/\alpha_s) \), is based on the existence of the so-called “asymptotic pointlike” (or “anomalous”) solution to the full evolution equation (4), which is explicitly calculable in perturbative QCD

\[
D_{NS}^{\alpha_s}(x, M) = \frac{4\pi}{\alpha_s(M)} a_{NS}(x),
\]

where \( a_{NS}(x) \) is given as a solution to the equation

\[
a_{NS}(x) = \frac{\alpha}{2\pi\beta_0} k_{NS}(x) + \frac{2}{\beta_0} \int_x^1 \frac{dy}{y} P_{q/q}^{(0)} \left( \frac{x}{y} \right) a_{NS}(y),
\]

and which indeed behaves like \( O(\alpha/\alpha_s) \)!. However, as we shall see the existence of this solution provides no justification for the claim that the full solution of (4) behaves like...
$O(\alpha/\alpha_s)$. Let me first recall the important fact [8] that this asymptotic pointlike solution doesn’t exist alone but is, together with the solution of the corresponding homogeneous equation (describing the “hadronic” part of the photon), embedded in the general solution of the evolution equation (4). Converting both these components into moments for easier handling, we get for it the expression [8]:

$$D_{NS}(n, M) = \frac{4\pi}{\alpha_s(M)} \left[ 1 - \left( \frac{\alpha_s(M)}{\alpha_s(M_0)} \right)^{1-2P(0)(n)/\beta_0} \right] a_{NS}(n) + \frac{\alpha_s(M)}{\alpha_s(M_0)} D_{NS}(n, M_0)$$

(10)

which has been obtained as the sum of the asymptotic pointlike solution (8) and the general solution to the corresponding homogeneous equation

$$D_{NS}^{\text{had}}(n, M) = D_{NS}(n, M_0) \left[ \frac{\alpha_s(M)}{\alpha_s(M_0)} \right]^{-2P(0)(n)/\beta_0},$$

(11)

taking into account (9). $M_0$ in the above equations denotes the scale at which the boundary condition on $D_{NS}(x, M_0)$ is specified.

In the weak coupling limit, i.e. for $\Lambda_{\text{QCD}} \to 0$, the second term in (10) goes over into $M$-independent $D_{NS}(x, M_0)$, i.e. just the initial condition, while the dominant first term behaves as

$$\frac{4\pi}{\alpha_s(M)} \frac{(1 - 2P(0)(n)/\beta_0) \ln(M/M_0)}{\ln(M/\Lambda_{\text{QCD}})} a_{NS}(n) \to \left( 1 - 2P(0)(n)/\beta_0 \right) \frac{\alpha_s}{\pi} k_{NS}(n) \ln \frac{M}{M_0}$$

(12)

and is thus very close to the simple expression (6), expected solely on the ground of QED vertex in Fig.1, with no QCD effects at all. The only difference with respect to (6) rests in the numerical factor $(1 - 2P(0)(n)/\beta_0)$ containing the term proportional to $P(0)(n)$, which is a consequence of the LO QCD corrections (like those of Fig.1b) to the basic QED vertex of Fig.1a. As the limit (12) is, as noticed already in [9], finite, there is no justification to write $D_{NS}(x, M) = O(\alpha/\alpha_s)$. The same is of course true for the singlet quark distribution function and for $D_{q/\gamma}$ as well. The preceding considerations clearly show how important it is to consider the asymptotic pointlike solution always in conjunction with the hadronic component of the photon. Recall, as another example, that $D_{ap}^{\text{had}}(x, M)$ in (8) diverges badly at $x = 0$. Also in this case the problem is cured [8] in the sum (10).

The crucial aspect of the above procedure is obviously the way the weak coupling limit is constructed. Let us compare the three above mentioned expressions (6,8,10) for $D_{NS}(x, M)$ in this limit, obtained by sending $\Lambda_{\text{QCD}} \to 0$. They differ in two aspects: the place where the limiting procedure was carried out and the interplay with the hadronic component. In the case of (6), $\alpha_s$ was set to zero in the evolution equation itself, leading to a well-defined, finite result for all $M$, which, however, carries no trace of the QCD corrections, present for $\alpha_s \neq 0$. For the asymptotic pointlike solution (8), considered separately from the hadronic part (11), the limit $\Lambda_{\text{QCD}} \to 0$ leads to the result $D_{NS}^{\text{ap}}(x, M) \to \infty$ for any $x, M$. The hadronic component (11), on the other hand, approaches a finite limit, given, as expected, by the initial distribution function $D_{NS}^{\text{had}}(x, M_0)$! This is another signal that $D_{NS}^{\text{ap}}(x, M)$ has little physical meaning of its own but makes sense only in combination with the hadronic
part of the photon. In other words, taking first the limit $\Lambda_{QCD} \to 0$ and then adding the two components leads to ill-defined result.

If, on the other hand, we first add in (10) the two mentioned components and only then take the limit $\Lambda_{QCD} \to 0$ we get not only a well-behaved result (12), but this finite result contains (in the term proportional to $P^{(0)}(n)$) also a clear trace of the presence of higher order QCD corrections. Note that this finite limit, coming from the first term on the r.h.s. of (10), is actually independent of the parametrization of the hadronic part $D_{NS}^{\text{had}}(x, M)$.

In summary, the photon distribution function $D_{q/\gamma}$ is clearly of the order $O(\alpha)$ and consequently the resolved photon contribution to, for instance, jet production in $\gamma p$ collisions of the order $O(\alpha^2 \alpha_s)$ and not, as claimed in [3-7], of the order $O(\alpha \alpha_s)$. It should therefore be added to the NLO direct $\gamma$ contribution, which is of the same order.

The close relation between the LO resolved and NLO direct $\gamma$ contributions is in fact central to the very idea of factorization of parallel singularities. It also turns out that the factorization of parallel singularities provides another clear evidence that $D_{q/\gamma}(x, M) = O(\alpha)$. To demonstrate this assertion, let me discuss, at the NLO, the factorization of parallel singularities into the structure functions of beam particles in three distinct cases:

\begin{align*}
\text{p} + \text{p} &\to 2 \text{ jets} + \text{anything} \\
\gamma + \text{p} &\to 2 \text{ jets} + \text{anything} \\
\gamma + \text{p} &\to W + \text{anything}
\end{align*}

(13)  
(14)  
(15)

Consider the basic QED vertex of Fig.1. Assuming that it is the lower, quark, leg which enters the interaction vertex in Fig.2b, we encounter a singularity of the form $1/t$. Its factorization then amounts to dividing the whole integration range into the lower part $|t| \in (t_{\text{min}}, M^2)$ ($t_{\text{min}}$ is essentially an infrared regulator), put into the quark distribution function $D_{q/\gamma}(x, M)$ and used in the LO resolved $\gamma$ contribution of Fig.2a, and the upper one, $|t| \in (M^2, t_{\text{max}})$, retained in the NLO hard scattering cross-section of the direct $\gamma$ subprocess of Fig.2b. It seems obvious that due to the arbitrariness in the choice of the boundary value $M^2$ between what is included in the “resolved” and what is left in “direct” $\gamma$ component, these components have to be of the same order and thus $D_{q/\gamma}(x, M)$ of the order $O(\alpha)$.

The general idea of factorization of parallel singularities [10] implies the following structure of the cross-section for the process (13), described by Feynman diagrams in Fig.3:

\begin{equation}
\sigma(\text{pp} \to 2 \text{ jets}) = \sum_{a,b} \int_0^1 \text{d}x_1 \int_0^1 \text{d}x_2 D_{a/p}(x_1, M)D_{b/p}(x_2, M)\sigma_{ab}^{\text{hard}}(S, x_1, x_2, M, \mu), \tag{16}
\end{equation}

where the hard scattering cross-section $\sigma_{ab}^{\text{hard}}$ for the parton level subprocess $a+b \to c+d+\cdots$ admits perturbation expansion in powers of $\alpha_s(\mu)$ at the hard scattering scale $\mu$, generally different from the factorization scale $M$:

\begin{equation}
\sigma_{ab}^{\text{hard}}(S, x_1, x_2, M, \mu) = \left[ \alpha_s^2(\mu)\kappa_{ab}^{\text{LO}}(S, x_1, x_2) + \alpha_s^3(\mu)\kappa_{ab}^{\text{NLO}}(S, x_1, x_2, M, \mu) \right]. \tag{17}
\end{equation}

In Figures 3-5 only examples of typical Feynman diagrams, relevant to the discussed point, are shown.
where $S$ is the total pp CMS energy, the sum in (16) runs over all parton pairs $a, b$ in the incoming hadrons. The functions $\kappa^{\text{LO}}, \kappa^{\text{NLO}}$ are assumed to contain all appropriate $\delta$-functions defining the kinematics of the final state jets in (13). Note that $\kappa^{\text{LO}}(S, x_1, x_2)$ is finite, $M$-independent function of these variables. In the above equations the factorization scale $M$ is a free parameter, which separates short distances (large $|t_1|, |t_2|$ of the incoming partons in Fig.3) from large distances, i.e. small virtualities, when both incoming partons $a, b$ are close to their mass shell. The formal invariance of the factorization procedure with respect to the choice of $M$ is guaranteed by the cancellation mechanism which, in the above process, works in such a way that the $M$-dependence of the LO contribution to (16), contained in the $M$-dependent distribution functions $D_{a/p}(x, M)$ and described by Feynman diagrams of Fig.3a, is cancelled to the NLO by the explicit dependence on $M$ of $\kappa^{\text{NLO}}$, corresponding to diagrams of Fig.3b.

In the case of the photoproduction of jets (14) the cancellation mechanism is modified by the presence of the inhomogeneous term in the evolution equation for quark distribution function $D_{q/\gamma}(x, M)$. Let us consider, as an example, the two gluon final state of Fig.4a and, moreover, fix the lower parton leg, i.e. the parton coming from the target proton, to be quark of a particular flavour. The NLO QCD corrections to this subprocess are described, in the case of real emissions, by diagrams of Fig.4b. They cancel that part of the $M$-dependence of the LO diagrams of Fig.4a, which is induced by the homogeneous part of the evolution equation for $D_{q/\gamma}(x, M)$, i.e. the one involving the convolutions with the quark or gluon distribution functions. They, however, don’t cancel the dependence of $D_{q/\gamma}(x, M)$ on $M$ induced by the inhomogeneous term in (3). This additional dependence on $M$ clearly needs a “direct” photon in the initial state and is therefore cancelled by the “direct” $\gamma$ contribution, corresponding to the diagram in Fig.4c. The cross-section of the $2 \to 3$ subprocess $\gamma+q\to q+g+g$ has a parallel singularity, arising from the configuration where the quark-antiquark pair originating from the incoming photon is parallel to it, which has to be subtracted. Instead of (16) we thus have

$$\sigma(\gamma+p \to 2 \text{ jets}) = \sum_b \int_0^1 \! dx_2 D_{b/p}(x_2, M) \sigma_{\gamma b}^{\text{hard}}(S, x_2, M, \mu)$$

$$+ \sum_{a,b} \int_0^1 \! dx_1 \int_0^1 \! dx_2 D_{a/\gamma}(x_1, M) D_{b/p}(x_2, M) \sigma_{ab}^{\text{hard}}(S, x_1, x_2, M, \mu) \quad (18)$$

where the direct $\gamma$ hard scattering cross-section

$$\sigma_{\gamma b}^{\text{hard}}(S, x_2, M, \mu) = \left[ \alpha_s^2(\mu) \kappa_{\gamma b}^{\text{NLO}}(S, x_1, x_2) + \alpha_s^2(\mu) \kappa_{\gamma b}^{\text{NLO}}(S, x_1, x_2, M, \mu) \right] \quad (19)$$

contains the $M$-independent LO contribution, corresponding to diagram in Fig.4d, and $M$-dependent NLO part, which cancels the rest of the $M$-dependence of the LO “resolved” $\gamma$ contribution induced by the inhomogeneous term in (3)! Thus the inclusion of the NLO

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2Similar procedure can be formulated for the case of final state collinear singularities. These are either put into the appropriate fragmentation functions, or cancelled, via KLN theorem, in the sum over all indistinguishable partonic final states.
The direct $\gamma$ contribution is necessary for a theoretically consistent description of jet production in $\gamma p$ collisions to the order $O(\alpha_\gamma^3 s)$. As far as $k_N^{\text{NLO}}$ is concerned, one has to be careful as this quantity depends on the choice of the NLO branching function $P^{(1)}(z)$. Moreover, we cannot just subtract from the integrand of the appropriate Feynman diagrams the singular part, corresponding to the $1/t$ term, as the $M$-dependence enters through the upper limit on the interval of $|t|$, where this subtraction is carried out. This question is discussed in more detail in [13].

At the $O(\alpha_\gamma s)$ order the factorization scale dependence doesn’t cancel, but it is clear that the $M$-dependent NLO direct $\gamma$ term has to be included along with the LO resolved $\gamma$ one, as they are both of the same order. For instance, in a recent analysis [16] of the hadronic properties of the photon the total cross-section of $\gamma p$ interactions is written as a sum of three terms, the first two of them corresponding to the “hadronic” and asymptotic pointlike components of the resolved $\gamma$ contribution, while the third one is the LO direct $\gamma$ contribution. However, as emphasized above, any time the LO resolved $\gamma$ contribution is taken into account, so must also be, for theoretical consistency, the NLO direct $\gamma$ one! Moreover, as in [16] the scale $M$ of $D_{\text{NS}}^p(x, M)$ is taken very small ($\approx 0.5 \text{ GeV}$), the NLO direct $\gamma$ contribution evaluated at this scale may be quite large and its final state practically indistinguishable from the resolved $\gamma$ one. Although including this NLO direct $\gamma$ component means adding another positive contribution and thus further complicating the situation in [16], there is no justification for neglecting it.

Finally, at the $O(\alpha_\gamma)$ order, only the $M$-independent direct $\gamma$ component contributes, which has no parallel singularity once the minimal transverse momentum of the produced jets is specified. The $q\bar{q}$ pair originating from the photon in Fig.4d cannot be parallel and still produce nonzero $p_t$ at the lower vertex.

In the case of $W$ boson production in $\gamma p$ collisions (15) the cancellation mechanism starts earlier than in the preceding subcase. The lowest order diagrams corresponding to (15) are, for direct as well as resolved $\gamma$ components, sketched in Figs.5a,c. Both of these contributions are of the order $O(\alpha_\gamma^2 W)$. As the produced $W$ is massive, the $q\bar{q}$ pair originating in Fig.5c from the incoming photon can, contrary to diagram in Fig.4d, be parallel. Consequently, the parallel singularity of the $O(\alpha_\gamma W)$ direct $\gamma$ term has to be subtracted, leading to the $M$-dependent finite part. The subtracted, $M$-dependent, part is included in the $M$-dependent photon distribution function $D_{q/\gamma}(x, M)$, entering the LO contribution to the resolved $\gamma$ component of Fig.5a. Higher order QCD corrections (Fig.5b) work as before. In this case the mechanism of factorization thus starts to operate already at the order $O(\alpha_\gamma W)$ and provides another evidence against the claim that $D_{q/\gamma}(x, M) = O(\alpha/\alpha_s)$. Used in this process the latter behaviour would imply for the resolved component $\sigma^{\text{res}}(\gamma + p \rightarrow W) = O(\alpha_{\text{W}}/\alpha_s)$. Consequently, the LO direct and resolved contributions would be of different orders in $\alpha_s$ and, worst of all, $\sigma^{\text{res}}$ would diverge for $\alpha_s \rightarrow 0$.

So far I have discussed only the real photon interactions. In ep collisions the photon exchanged in NC interactions ($Z^0$ exchange can be neglected in the region of $Q^2$ considered) is virtual and one has to be careful to treat properly the dependence of photon distribution functions $D_{i/\gamma}(x, M, Q^2)$, $i=q, \bar{q}, g$ on the virtuality $Q^2$ of the exchanged photon [11]. In this note I restrict my attention to the region of low $Q^2$, where the virtual photon behaves to a very good approximation like the real one. The spectrum of these photons inside the
The incoming electron is given by the Weizsäcker-Williams approximation

\[
D_{\gamma/e}(x, Q_0) = \frac{\alpha}{2\pi} \left( \frac{1 + (1 - x)^2}{x} \right) \ln \frac{Q_0^2(1 - x)}{m_e^2 x^2},
\]

where \( Q_0 \) denotes the maximum virtuality of the exchanged \( \gamma \) taken into account. The “optimal” choice of \( Q_0 \) (in the sense that the cross-section of the 2\( \rightarrow \)3 subprocess \( e^+q/g\rightarrow e^+q+g/\bar{q} \) behaves as \( 1/Q^2 \) up to \( Q^2 = Q_0^2 \)) depends on the transverse momentum of produced partons and is roughly linear function thereof. My considerations concern this region.

Up to now all theoretical analyses of data on jet production in ep collisions have included, along with the resolved \( \gamma \) contribution of Fig.4a, also (and only) the LO direct \( \gamma \) one of Fig.4d. These analyses show that around \( p_t \approx 25 \) GeV, \( d\sigma^{\text{res}}/dp_t \approx d\sigma^{\text{dir}}/dp_t \). As emphasized above, the \( \mathcal{O}(\alpha^2\alpha_s^2) \) resolved \( \gamma \) component should, however, always be considered simultaneously with NLO direct \( \gamma \), which is of the same order. Although there exists [14] a comprehensive analysis of direct \( \gamma \) contributions to jet production in ep collisions up to the NLO, there is so far no theoretically consistent treatment of full jet production to the order \( \mathcal{O}(\alpha^2\alpha_s^2) \), including LO resolved (Fig.4a) together with LO+NLO direct \( \gamma \) contributions (Fig.4d,c). The results of [15] show that the ratio \( r \equiv (d\sigma^{\text{NLO}}/dp_t)/(d\sigma^{\text{LO}}/dp_t) \) is of the order of 2 in the region of \( p_t \) around 20 GeV and that \( r \) grows as \( p_t \) decreases so that the inclusion of NLO direct \( \gamma \) contribution shifts the crossing point \( p_t^c \) to smaller values. It would be very useful to have the calculations of [14, 15] available down to \( p_t \) around 10 GeV, where most of the available HERA data are, to see how much the situation will actually change by including the NLO direct \( \gamma \) term.

Moreover, in these circumstances the final state in the NLO direct \( \gamma \) channel will often lead to kinematically very close final state configurations as the resolved one. This is due to the fact that the on-mass shell quark/antiquark, originating from the incoming photon and not participating in further interaction with the constituents of the proton, may fly close to the direction of the parent photon and thus is essentially distinguishable from the “remnant” jets of the resolved \( \gamma \), Fig.4a. For small \( p_t \), the factorization mass \( M \) to be used in \( D_{q/\gamma}(x, M) \) should be roughly proportional to \( p_t \), which implies small opening angle between \( q/\bar{q} \) in the the direct \( \gamma \) contribution of Fig.4c. For high \( p_t \) the “remnant” jet from NLO direct component is expected to have bigger angle with respect to the beam (\( \gamma \)) direction and thus to be distinguishable from the resolved \( \gamma \) one.

Let us now return to the interpretation of recent HERA data on two jet events. In [1, 2] only the LO direct and LO resolved \( \gamma \) contributions were included. As theoretically consistent \( \mathcal{O}(\alpha^2\alpha_s^2) \) order analysis requires the inclusion of the NLO direct \( \gamma \) term, the interpretation of the mentioned observation as an evidence for resolved \( \gamma \) component is premature. What can be safely said is that the data require the presence of the \( \mathcal{O}(\alpha^2\alpha_s^2) \) effects. Even this, however, is an important conclusion.

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References

Figure captions

Fig.1: Basic QED vertex (a) and example of lowest order QCD correction to it (b). In this and all following figures the big solid circles stand for parton distribution functions of the colliding hadrons (labelled by double lines) or the photon.

Fig.2: Examples of Feynman diagrams contributing to the resolved (a) and direct (b) $\gamma$ photoproduction of jets.

Fig.3: Factorization for the two gluon final state in proton-proton collisions: the $M$-dependence of $D_{q/p}(x, M)$ used in $\mathcal{O}(\alpha_s^2)$ Feynman diagram of (a) is compensated by the explicit $M$-dependence of the $\mathcal{O}(\alpha_s^3)$ hard scattering cross-section (b).

Fig.4: Factorization for the two gluon final state in $\gamma p$ collisions: the $M$-dependence of $D_{q/\gamma}(x, M)$ used in $\mathcal{O}(\alpha_s^2)$ Feynman diagram for resolved $\gamma$ (a) is compensated in part by the explicit $M$-dependence of the $\mathcal{O}(\alpha_s^3)$ resolved $\gamma$ hard scattering cross-sections (b) and in part by the $M$-dependence of the $\mathcal{O}(\alpha_s^2)$ direct $\gamma$ hard scattering cross-section (c). In (d) the LO, $\mathcal{O}(\alpha_s)$, direct $\gamma$ contribution is plotted for comparison.

Fig.5: Factorization in photoproduction of $W$ boson: the $M$-dependence of $D_{q/\gamma}(x, M)$ used in $\mathcal{O}(\alpha_s\alpha_W)$ resolved $\gamma$ contribution (a) is cancelled in part by the explicit $M$-dependence of $\mathcal{O}(\alpha_s\alpha_W\alpha_s)$ hard scattering cross-sections (b) and in part by the explicit $M$-dependence of $\mathcal{O}(\alpha_s\alpha_W)$ hard scattering cross-section (c). Dashed lines denote $W$ boson.