Identification of Neutral $B$ Mesons using Correlated Hadrons

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ABSTRACT

The identification of the flavor of a neutral $B$ meson can make use of hadrons produced nearby in phase space. Examples include the decay of “$B^{**}$” resonances or the production of hadrons as a result of the fragmentation process. Some aspects of this method are discussed, including time-dependent effects in neutral $B$ decays to flavor states, to eigenstates of CP and to other states, and the effects of possible coherence between $B^0$ and $\bar{B}^0$ in the initial state. We study the behavior of the leading hadrons in $b$-quark jets and the expected properties of $B^{**}$ resonances. These are extrapolated from the corresponding $D^{**}$ resonances, of whose properties we suggest further studies.
I. INTRODUCTION

For thirty years, the only system in which CP violation has been observed is that of neutral kaons, for which several possible explanations exist [1]. The most popular, that of phases in the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2], can be tested using systems of $B$ mesons [3].

Asymmetries in rates of decays of neutral $B$ mesons to CP eigenstates such as $J/\psi K_S$ and $\pi^+\pi^-$ are particularly easy to interpret in terms of fundamental phases in the CKM matrix. However, the flavor of the decaying meson [$B^0(=\bar{b}d)$ or $\overline{B}^0(=bd)$] must be identified at some reference time which is to be compared with the time of decay.

One suggestion for tagging the flavor of a produced neutral $B$ meson [4] is to study its correlation with pions produced nearby in phase space. This method has been used to identify neutral $D$ mesons though the decays of charged $D^*$ resonances [5]. The $D^{*\pm}$ resonance, with spin-parity $J^P = 1^-$, lies just above threshold for this decay, giving rise to a characteristically soft pion. The corresponding $1^- B^*$ lies only about 46 MeV above the $B$, so $B^* \rightarrow \pi B$ is forbidden, but there are positive-parity resonances with $J^P = 0^+$, $1^+$, and $2^+$ and masses below about 5.8 GeV/c$^2$ that are expected to couple to $\pi B$ and/or $2 B^*$. Even if these “$B^{**}$” resonances cannot all be identified, one expects some $\pi B^{(*)}$ correlations as a result of the fragmentation process, as illustrated in Fig. 1.

In the present paper we expand upon the method proposed in Ref. [4]. The number of $B$ mesons in modes such as $J/\psi K$ and $J/\psi K^*$ reconstructed by the CDF [6] and various LEP [7] collaborations is large enough that the correlations of these mesons with pions nearby in phase space are already under investigation [8]. Our purpose is to provide guidance for such studies. As a corollary, we have found a general description of initial “tagged” states of neutral $B$ mesons which may be of use in any study of CP-violating decay asymmetries.

In Section II we generalize the approach of Ref. [4] to time-dependent decays. In view of the considerable precision in $B$ lifetime measurements which has already been achieved by the CDF and LEP groups, the study of time-dependent effects may not be too far in the future. Indeed, the first results of such studies have already been presented by the ALEPH Collaboration [9].

The measurement of time dependences in decays can shed light on the question, addressed only briefly in Ref. [4], of whether interferences between
$B^0$ and $\bar{B}^0$ produced in conjunction with a pion of a given charge can ever occur. We assumed in Ref. [4], and shall assume in the present study, that in high energy $e^+e^-$ collisions and in a hadronic reaction $B^0$ and $\bar{B}^0$ are always incoherent with respect to one another. In Sec. III we show how to test this hypothesis by also allowing a coherent or partially coherent admixture of $B^0$ and $\bar{B}^0$. This method has applications to any study of neutral $B$ mesons, as we have pointed out in a shorter communication [10].

We mention several aspects of tagging $B$'s in Section IV, drawing attention to methods using hadrons other than pions [11, 12] and stressing the importance of corresponding studies using charmed mesons. We also discuss the question of whether explicit $B^{**}$ resonances are needed in order for the method to succeed.

Section V is devoted to some general remarks on resonances which can decay to a tagging hadron and a neutral $B$ or $B^*$. We refer the reader to Refs. [13] and [14] for recent more detailed discussions of properties of some of these resonances. As in Section IV, we stress the importance of corresponding studies using charmed mesons. We conclude in Section VI.

II. TIME DEPENDENCES

A. Identification of $B^0 - \bar{B}^0$ oscillations

The study of same-sign lepton production in the reaction $e^+ + e^- \rightarrow B^0 + \bar{B}^0$ led to the conclusion that the neutral $B$ meson underwent significant mixing with its antiparticle [15]. The current estimate of the mixing parameter, averaged over ARGUS and CLEO data, is [16] $\Delta m/\Gamma = 0.66 \pm 0.10$.

Explicit $B^0 - \bar{B}^0$ oscillations have been identified in high-energy $e^+e^-$ collisions at LEP by the ALEPH collaboration [9]. In the reaction $e^+e^- \rightarrow Z^0 \rightarrow b\bar{b}$, the flavor of a produced neutral $B$ meson is tagged by means of the semileptonic decay of the $b$ quark not incorporated into this meson.

In Ref. [4] we defined the relative rates of production of $B^0$ and $\bar{B}^0$ mesons in low-mass combinations with charged pions to be

\[ N(B^0\pi^-) \equiv P_1, \quad N(B^0\pi^+) \equiv P_2 \]

\[ N(B^0\pi^+) \equiv P_3, \quad N(B^0\pi^-) \equiv P_4. \]

(1)
For $e^+e^- \to Z^0 \to b\bar{b}$ and for $\bar{p}p \to B\pi + \ldots$, charge conjugation symmetry implies $P_3 = P_1$, $P_4 = P_2$. Let us imagine that a neutral $B$ decays to a state of identifiable flavor, e.g.,

$$B^0 \to J/\psi K^{*0}, \quad \bar{B}^0 \to J/\psi \bar{K}^{*0},$$

(2)

with the flavor of the neutral $K^*$ identified by the decay $K^{*0} \to K^+\pi^-$ or $\bar{K}^{*0} \to K^-\pi^+$. Let us denote a “right-sign” combination $R$ as $\bar{B}^0\pi^-$ or $B^0\pi^+$, and a “wrong-sign” combination $W$ as $\bar{B}^0\pi^+$ or $B^0\pi^-$. Then as a function of proper decay time, the relative numbers of right-sign and wrong sign combinations are:

$$R(t) = e^{-\Gamma t} \left[ P_1 \cos^2 \left( \frac{\Delta m t}{2} \right) + P_2 \sin^2 \left( \frac{\Delta m t}{2} \right) \right],$$

(3)

$$W(t) = e^{-\Gamma t} \left[ P_1 \sin^2 \left( \frac{\Delta m t}{2} \right) + P_2 \cos^2 \left( \frac{\Delta m t}{2} \right) \right],$$

(4)

so that the time-dependent asymmetry is given by

$$\frac{R(t) - W(t)}{R(t) + W(t)} = \frac{P_1 - P_2}{P_1 + P_2} \cos(\Delta m t).$$

(5)

(Here we have ignored very small CP-violating effects in the decays in question.) The corresponding time-integrated asymmetry is

$$\frac{\int [R(t) - W(t)] dt}{\int [R(t) + W(t)] dt} = \frac{P_1 - P_2}{P_1 + P_2} \frac{1}{1 + x_d^2},$$

(6)

where $x_d \equiv (\Delta m / \Gamma)_{B^0}$. The factor $[1 + x_d^2]^{-1}$ is about 2/3. Thus, since time-dependent information is available anyway in extraction of the $B$ signals from many experiments, Eq. (5) may provide information on $(P_1 - P_2)/(P_1 + P_2)$ which is at least as statistically compelling as the time-integrated asymmetry (6).

Of course, in the decays of charged $B$’s, e.g. to $J/\psi K^\pm$, no $B - \bar{B}$ oscillations will occur, and one will measure just the dilution factor $(P_1 - P_2)/(P_1 + P_2)$ when forming the corresponding right sign – wrong sign asymmetry. Here, with $r \equiv B^-\pi^+$ or $B^+\pi^-$ and $w \equiv B^-\pi^-$ or $B^+\pi^+$, one has

$$\frac{(r - w)}{(r + w)} = \frac{P_1 - P_2}{P_1 + P_2}.$$

(7)
The comparison of this result with (5) or (6) will form a useful test of isospin independence of the production process. Such independence is frequently but not universally expected to occur [4]. A specific case in which it could be violated would be if a meson is produced by fragmentation of a proton into a $b$-flavored baryon and a meson containing a $\bar{b}$, as shown in Fig. 2. Since the proton has more $u$ than $d$ valence quarks, one might expect more $B^+$ than $B^0$ mesons in such a process.

B. Time-dependent CP-violating asymmetries

The time-integrated asymmetry for decays of states of identified flavor at $t = 0$ into a CP eigenstate $f$ may be defined as

$$A(f) \equiv \frac{\Gamma(B^0_{t=0} \to f) - \Gamma(\overline{B}^0_{t=0} \to f)}{\Gamma(B^0_{t=0} \to f) + \Gamma(\overline{B}^0_{t=0} \to f)}.$$  \hfill (8)

For $f = J/\psi \ K_S$, one has

$$A(J/\psi \ K_S) = -\frac{x_d}{1 + x_d^2} \sin(2\beta),$$  \hfill (9)

where $\beta$ is an angle in the triangle expressing unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [3]. In cases in which $B^0$ and $\overline{B}^0$ production cross sections are equal, we derived in [4] the simple relation

$$A_{obs}(f, \pi) = \frac{P_1 - P_2}{P_1 + P_2} A(f),$$  \hfill (10)

where the observed asymmetry is defined in terms of the charged pion. The corresponding time-dependent rates for states which were initially $B^0$ or $\overline{B}^0$ at $t = 0$, aside from common overall factors, are

$$\Gamma(t) \equiv \frac{d\Gamma}{dt}(B^0_{t=0} \to f) = e^{-\Gamma t}(1 - \sin 2\beta \sin \Delta mt)$$  \hfill (11)

$$\Gamma(t) \equiv \frac{d\Gamma}{dt}(\overline{B}^0_{t=0} \to f) = e^{-\Gamma t}(1 + \sin 2\beta \sin \Delta mt)$$  \hfill (12)

so the time-dependent asymmetry is

$$A(t) = \frac{\Gamma(t) - \Gamma(t)}{\Gamma(t) + \Gamma(t)} = -\sin 2\beta \sin(\Delta mt)$$  \hfill (13)
The observed asymmetry associated with pions of opposite charge will be diluted by a common factor \((P_1 - P_2)/(P_1 + P_2)\), as in the time-integrated case.

It may be necessary to take into account the explicit time-dependences (11) and (12) when discussing efficiencies for detecting \(B\) mesons. Typically such efficiencies vary as a function of proper decay lifetime.

C. States which are not CP eigenstates

Angles of the unitarity triangle can also be determined from neutral \(B\) decays to states \(f\) which are not CP eigenstates. This is feasible when both a \(B^0\) and a \(\bar{B}^0\) can decay to a final state \(f\) which appears in only one partial wave, provided that a single weak CKM phase dominates each of the corresponding decay amplitudes. Two interesting examples are [17] \(B^0 \to \rho^- \pi^+\), for which one must neglect the penguin amplitude, and \(B^0 \to D_s^- K^+\), where a single amplitude is known to contribute in the standard model. In both these cases, in contrast to the form (13), one sees both sines and cosines of \(\Delta m t\).

Let us comment in passing on the specific signatures of the decays \(B_s\) or \(\bar{B}_s\) \(\to D_s^+ K^-\). The graphs contributing to this process are shown in Fig. 3. A good mode for detecting the \(D_s^+\) is via its \(\phi \pi^+\) decay, where \(\phi \to K^+ K^-\). The final state of the strange \(B\) then contains three charged kaons and one charged pion coming from the secondary vertex. As we shall discuss in more detail below, the initial flavor of a strange \(B\) is to be determined on a statistical basis by means of correlations with a charged kaon [11]. This kaon comes from the primary production vertex. It would be highly desirable to invent a trigger for hadronically produced events containing four charged kaons to increase the sensitivity for such events.

We wish to demonstrate how our tagging method can be applied to a general case of a non-CP eigenstate. The time-dependent rates for states which were \(B^0\) or \(\bar{B}^0\) at \(t = 0\) and decay at time \(t\) to the state \(f\) or its CP-conjugate \(\bar{f}\) are given by [18]:

\[
\Gamma_f(t) = e^{-\Gamma t} [\lambda] \cos^2\left(\frac{\Delta m t}{2}\right) + |\lambda|^2 \sin^2\left(\frac{\Delta m t}{2}\right)
+ |AA| \sin(\delta + \phi_M + \phi_D) \sin(\Delta m t)\]

\[
\Gamma_f(t) = e^{-\Gamma t} [\bar{\lambda}] \cos^2\left(\frac{\Delta m t}{2}\right) + |\lambda|^2 \sin^2\left(\frac{\Delta m t}{2}\right)
\]

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\[ - |A\bar{A}| \sin(\delta + \phi_M + \phi_D) \sin(\Delta mt) \] ,

\[ \Gamma_f(t) = e^{-\Gamma t} \left[ |A|^2 \cos^2 \left( \frac{\Delta mt}{2} \right) + |A| \sin^2 \left( \frac{\Delta mt}{2} \right) - |A\bar{A}| \sin(\delta - \phi_M - \phi_D) \sin(\Delta mt) \right] , \]

\[ \Gamma_{\bar{f}}(t) = e^{-\Gamma t} \left[ |A|^2 \cos^2 \left( \frac{\Delta mt}{2} \right) + |\bar{A}| \sin^2 \left( \frac{\Delta mt}{2} \right) - |A\bar{A}| \sin(\delta - \phi_M - \phi_D) \sin(\Delta mt) \right] . \] (14)

Here \(|A|\) and \(|\bar{A}|\) are the magnitudes of the decay amplitudes of \(B^0\) and \(\bar{B}^0\) to \(f\), \(\delta\) and \(\phi_D\) are the strong and weak phase-differences between these amplitudes, and \(\phi_M\) is the phase of \(B - \bar{B}\) mixing. The corresponding time-dependent rates for states \(f\) or \(\bar{f}\) in conjunction with pions of positive or negative charges are then:

\[ \Gamma_{f\pi^\pm}(t) = (1/2)e^{-\Gamma t} \left\{ |A|^2 + |\bar{A}|^2 \pm |P_1 - P_2|[|A|^2 - |\bar{A}|^2] \cos(\Delta mt) + 2|A\bar{A}| \sin(\delta + \phi_M + \phi_D) \sin(\Delta mt) \right\} , \]

\[ \Gamma_{\bar{f}\pi^\pm}(t) = (1/2)e^{-\Gamma t} \left\{ |A|^2 + |\bar{A}|^2 \mp |P_1 - P_2|[|A|^2 - |\bar{A}|^2] \cos(\Delta mt) + 2|A\bar{A}| \sin(\delta - \phi_M - \phi_D) \sin(\Delta mt) \right\} . \] (15)

where we have taken \(P_1 + P_2 = 1\).

These four rates depend on four unknown quantities, \(|A|\), \(|\bar{A}|\), \(\sin(\delta + \phi_M + \phi_D)\) and \(\sin(\delta - \phi_M - \phi_D)\). Measurement of the rates allows a determination of the weak CKM phase \(\phi_M + \phi_D\), apart from a two-fold ambiguity [17]. In the two cases \(B^0 \to \rho^-\pi^+\) and \(B^0 \to D_s^- K^+\) this phase obtains the values \(2\alpha\) and \(\gamma\), respectively.

III. THE QUESTION OF COHERENCE

The search for CP-violating asymmetries in decays of neutral \(B\) mesons produced at the \(\Upsilon(4S)\) resonance involves correlations between the particle whose decay is studied and the particle whose decay serves to “tag” the flavor of its partner. Here, coherence between a \(B^0\) and a \(\bar{B}^0\) is crucial. There have been several studies [3, 19] of such coherence, both at the \(\Upsilon(4S)\), where a \(B^0\bar{B}^0\) pair is in a state with \(C = -1\), and in configurations where an extra photon has been produced leading to \(C = +1\) for the \(B^0\bar{B}^0\) pair.

In a hadronic production environment or in high energy \(e^+e^-\) reactions, it is much less likely to find coherence between a \(B\) and \(\bar{B}\), since they are
usually separated in rapidity by many intermediate hadrons. The absence of coherence was an assumption which was made not only in Ref. [4] but which appears in many other treatments of hadronic production [20]. It seems prudent to test for such coherence directly. We have found that such a test is possible, and describe it briefly in the present section. We have reported these results in more detail in Ref. [10].

We denote particle and antiparticle basis states by spinors with spin up and spin down in an abstract “quasispin” space [19, 21]:

\[
|B^0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad |\overline{B}^0\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.
\]

A density matrix \( \rho \) allows one to discuss incoherent and coherent states in a unified manner. The most general such 2 \( \times \) 2 matrix has the form

\[
\rho = (1 + Q \cdot \sigma)/2,
\]

where \( Q \) is a vector describing polarization in quasispin space, satisfying \( Q^2 \leq 1 \), and \( \sigma_i \) \((i = 1, 2, 3)\) are the Pauli matrices.

A pure state corresponds to a linear combination of \( B^0 \) and \( \overline{B}^0 \) with arbitrary complex coefficients, whose sum of absolute squares equals unity. Such a state can be denoted by a density matrix with \( Q \equiv |Q| = 1 \). An arbitrary incoherent combination of \( B^0 \) and \( \overline{B}^0 \) with relative probabilities \( P_1 \) and \( P_2 = 1 - P_1 \) corresponds to a diagonal density matrix with \( Q_1 = Q_2 = 0 \), \( Q_3 = 2P_1 - 1 \). This is the case we considered to hold in Ref. [4]. As a special case of either example, one describes the density matrices for initial \( B^0 \) and \( \overline{B}^0 \) by \( \text{diag}(1,0) \) and \( \text{diag}(0,1) \), respectively.

The probability for a transition from an initial state denoted by the density matrix \( \rho_i \) to a final state denoted by \( \rho_f \) is then \( I(f) = \text{Tr} (\rho_i T^\dagger \rho_f T) \), where \( T \) is the operator which time-evolves the state from \( i \) to \( f \). Here \( \rho_f \) can denote an arbitrary coherent superposition of \( B^0 \) and \( \overline{B}^0 \) at time \( t \), or can also take account of the decay of this superposition.

To discuss mass eigenstates, which have simple time-evolution properties, we neglect differences between their lifetimes [3], and denote them by \( B_L \) (“light”) and \( B_H \) (“heavy”):

\[
|B_L\rangle = (|B^0\rangle + |\overline{B}^0\rangle)/\sqrt{2} \quad , \quad |B_H\rangle = (|B^0\rangle - |\overline{B}^0\rangle)/\sqrt{2}.
\]

We adopt a convention in which the \( B^0 - \overline{B}^0 \) mixing amplitude is real, and use the fact [3] that the mixing amplitudes are of approximate magnitude
$1/\sqrt{2}$. This differs from a more standard convention by a phase which we take into account when calculating amplitudes for decays of $b$ quarks. The transformation between flavor eigenstates and mass eigenstates is then implemented by the unitary matrix $U = (\sigma_1 + \sigma_3)/\sqrt{2}$. The matrix describing the time evolution in the mass eigenstate basis is

$$e^{-iM_{Bt}} \equiv e^{-\Gamma t/2} \text{diag}(e^{-im_L t}, e^{-im_R t}) = e^{-\Gamma t/2} e^{-imt} e^{i\alpha t} \Delta mt/2 ,$$

(18)

where $\Delta t \equiv (m_H + m_L)/2$, $\Delta m \equiv m_H - m_L$. Thus the time evolution operator $T$ in the $B^0, \bar{B}^0$ basis is just $T = U^\dagger e^{-iM_{Bt}} U = e^{-\Gamma t/2} e^{-imt} e^{i\alpha t} \Delta mt/2$.

The trace for the transition probability $I(f)$ can be computed by applying the matrices $U$ and $U^\dagger$ to the initial and final density matrices. Defining $\rho' = U \rho U^\dagger$, we find that the effect of $U$ is to rotate $Q$ into $Q'$, where

$$Q'_1 = Q_3 \quad , \quad Q'_2 = -Q_2 \quad , \quad Q'_3 = Q_1 .$$

(19)

The transition probability can now be written in terms of traces as

$$I(f) = Tr\left( \rho'_1 e^{iM_{Bt}} \rho'_f e^{-iM_{Bt}} \right) .$$

(20)

The states $B^0$ and $\bar{B}^0$ at the time of decay $t$ may be identified by their decays to states of identifiable flavor, e.g., $B^0 \rightarrow J/\psi K^{*0}$, with $K^{*0} \rightarrow K^+ \pi^-$. We assume that a single weak subprocess contributes to the decay, which is an excellent approximation for these final states [3].

With the convention in which the mixing amplitudes in the neutral $B$ mass eigenstates are real, the weak decay amplitudes for $B^0 \rightarrow J/\psi K^{*0}$ and $\bar{B}^0 \rightarrow J/\psi \bar{K}^{*0}$ may be denoted $Ae^{-i\beta}$ and $Ae^{i\beta}$, respectively, where $\beta = \arg (-V_{cb}^* V_{cd}/V_{ub}^* V_{ud})$, and $V_{ij}$ are elements of the Cabibbo-Kobayashi-Maskawa matrix specifying the charge-changing weak couplings of quarks. We find

$$I \left( \frac{B^0}{\bar{B}^0} \right) = \frac{1}{2} |A|^2 e^{-\Gamma t} [1 \pm Q_{\perp} \cos(\Delta mt + \delta)] ,$$

(21)

$$Q_{\perp}' \equiv Q_{\perp} \cos \delta \quad , \quad Q_{\perp}'' \equiv Q_{\perp} \sin \delta .$$

(22)

When the initial state is an incoherent mixture of $B^0$ and $\bar{B}^0$ with relative probabilities $P_1$ and $P_2 = 1 - P_1$, respectively, one sets $Q_{\perp}' = 2P_1 - 1$ and $\delta = 0$ in the above expression, recovering the results of Sec. II A.
We consider a charge-symmetric production process in which an arbitrary combination of neutral $B^0$ and $\bar{B}^0$ is produced, with an additional particle of specific charge (such as a charged lepton or pion) bearing some specific kinematic relation to it. We wish to determine the relation of this process to the one in which the charge of the tagging particle is the opposite.

Under the phase convention we have chosen for $B^0$ and $\bar{B}^0$, if we take $CP |B_L\rangle = |B_L\rangle$ and $CP |B_H\rangle = -|B_H\rangle$, the charge conjugation operation has the phase

$$C |B^0\rangle = -|\bar{B}^0\rangle \quad ; \quad C |\bar{B}^0\rangle = -|B^0\rangle \quad .$$

(23)

Under charge conjugation, the first and second rows and columns of the density matrix $\rho$ are interchanged, so that $Q_1 \rightarrow Q_1$, $Q_2 \rightarrow -Q_2$, $Q_3 \rightarrow -Q_3$, or $Q_1' \rightarrow -Q_1'$, $Q_2' \rightarrow -Q_2'$, $Q_3' \rightarrow Q_3'$. Therefore, the decay rates $I(f)$ for states tagged with antiparticles are given in terms of those $I(f)$ for states tagged with particles by

$$I(f; Q_1', Q_2', Q_3') = I(f; -Q_1', -Q_2', Q_3') \quad .$$

(24)

For a final state identified as a $B^0$ by its decay to $J/\psi K^{*0}$, the time-dependent asymmetry is

$$A(J/\psi K^{*0}) = \frac{I(J/\psi K^{*0}) - \bar{I}(J/\psi K^{*0})}{I(J/\psi K^{*0}) + \bar{I}(J/\psi K^{*0})} = Q'_\perp \cos(\Delta m t + \delta) \quad .$$

(25)

As a consequence of the assumed charge symmetry of the production process, one has $\bar{I}(J/\psi K^{*0}) = I(J/\psi K^{*0})$ and $\bar{I}(J/\psi K^{*0}) = I(J/\psi K^{*0})$.

We can then measure the components $Q'_\perp$ and $\delta$ using decays to flavor eigenstates. (The ALEPH Collaboration [9] has measured a time-dependent asymmetry of the above form, fitting it under the assumption $\delta = 0$.)

A measurement of $Q'_\perp$ for neutral nonstrange $B$ mesons can be performed by utilizing their decays to the specific CP eigenstate $J/\psi K_S$. In our phase convention, the amplitudes for $B^0$ and $\bar{B}^0$ to decay into $J/\psi K_S$ are $A' e^{-i\beta} / \sqrt{2}$ and $-A' e^{i\beta} / \sqrt{2}$. Here we have taken into account the intrinsic negative CP of the $J/\psi K_S$ state, and neglected the small CP violation in the kaon system. The density matrix for the final state is then

$$\rho_{J/\psi K_S} = \frac{1}{2} |A'|^2 \begin{bmatrix} e^{-i\beta} & -e^{i\beta} \\ -e^{i\beta} & e^{-i\beta} \end{bmatrix} = \frac{1}{2} |A'|^2 \begin{bmatrix} 1 & -e^{2i\beta} \\ -e^{-2i\beta} & 1 \end{bmatrix} ,$$

(26)
with off-diagonal terms in $\rho$ changed in sign for $J/\psi K_L$. The expressions for the decay rates for states prepared with particle and antiparticle tags are

$$I \left( \frac{J/\psi K_L}{K_S} \right) = \frac{1}{2} |A|^2 e^{-\Gamma t} \{ 1 \pm [Q'_3 \cos 2\beta + Q'_1 \sin 2\beta \sin(\Delta mt + \delta)] \},$$

$$\bar{I} \left( \frac{J/\psi K_L}{K_S} \right) = \frac{1}{2} |A|^2 e^{-\Gamma t} \{ 1 \pm [Q'_3 \cos 2\beta - Q'_1 \sin 2\beta \sin(\Delta mt + \delta)] \}.$$  

(27)

As in the case of decays to the flavor eigenstates, the time-dependent term has a phase shift $\delta$ and a modulation amplitude $Q'_1$. The decay asymmetry for the $J/\psi K_S$ final state is

$$A(J/\psi K_S) = \frac{I(J/\psi K_S) - \bar{I}(J/\psi K_S)}{I(J/\psi K_S) + \bar{I}(J/\psi K_S)} = \frac{-Q'_1 \sin 2\beta \sin(\Delta mt + \delta)}{1 - Q'_3 \cos 2\beta}.$$  

(28)

The component $Q'_3$ (which appears even in the absence of CP violation) is a necessary ingredient in the discussion of possible coherence. It is this component that leads to correlations between $K_S$ and $K_L$ produced in $\phi$ decay, as discussed in Ref. [22]. In order to learn its value, we measure the rate for $J/\psi K_S$ production (summing over particle and antiparticle tags, so that the time-dependent terms cancel). We compare this with the corresponding sum of rates (which also has no time-dependence, and is independent of $Q'_3$) for production of a flavor eigenstate $K^0$.

To measure the $K^0$ production rate, there are a couple of possibilities. If the rate of production of charged and neutral $B$'s is equal, as is expected for high-energy $\epsilon^+\epsilon^-$ collisions [4], one can use the observed rate for $J/\psi K^+$ production, making use of the fact that the decays of $B$ mesons to $J/\psi K$ involve the quark subprocess $b \to c\bar{c}$, which conserves isospin [23]. Or, one can measure the $B^0/B^+$ production ratio via the observed $K^{*0}/K^{*+}$ ratio, and infer the $K^0$ rate from the observed $K^+$ rate. In short, the ratio $|A'/A|^2$ is measurable.

Once we learn the relative normalization of rates for decays to flavor eigenstates and CP eigenstates, we can determine the magnitude of the term $Q'_3 \cos 2\beta$, and then use the asymmetry in $J/\psi K_S$ decay to measure $\sin 2\beta$. With the possibility of a discrete ambiguity (unlikely for known ranges of CKM parameters), we then obtain $\cos 2\beta$, thereby finding $Q'_3$ itself.
The corresponding decay asymmetry for the $\pi^+\pi^-$ final state is easily calculated. We neglect penguin effects [24], which can be dealt with by studying the $2\pi^0$ final state. With our phase convention for $b$ quarks, the result can be obtained by the substitution $\beta \to -\alpha$ in the corresponding result for the $J/\psi K_L$ final state, where $\alpha = \text{Arg} \left(-V_{tb}^* V_{td}/V_{ub}^* V_{ud}\right)$. We find

$$A(\pi^+\pi^-) \equiv \frac{I(\pi^+\pi^-) - I(\pi^+\pi^-)}{I(\pi^+\pi^-) + I(\pi^+\pi^-)} = \frac{-Q'_\perp \sin 2\alpha \sin(\Delta m t + \delta)}{1 + Q'_3 \cos 2\alpha} .$$

(29)

Since we have already measured all components of $Q'$ and the phase $\delta$, this result can be used to extract $\alpha$.

**IV. FURTHER REMARKS ON TAGGING**

**A. Resonances vs. more general correlations**

The fragmentation diagrams shown in Fig. 1 indicate that a correlation between the charge of the leading pion and the flavor of the neutral $B$ is possible whether or not that pion resonates with the $B$ (or its parent $B^*$, decaying to $B\gamma$). Very recently this correlation was calculated for LEP energies [25] using a soft fragmentation version of JETSET 7.3. It was found that the correlation factor $[N(B^0\pi^+) - N(B^0\pi^-)]/[N(B^0\pi^+) + N(B^0\pi^-)]$, for pions with the lowest $M(B\pi)$ value in each event, increases from a value of 0.17 at $M(B\pi) = 5.5$ GeV/$c^2$ to the value of 0.27 at 5.8 GeV/$c^2$, and stays constant up to 6.2 GeV/$c^2$, where very small rates are expected. This estimate, which does not include resonance effects, is quite encouraging, and should be tested experimentally.

It is an observed feature of hadron physics, however, that whenever a quark $q$ is contained in a meson $M_1$ and the same antiquark $\bar{q}$ is contained in another meson $M_2$, the mesons $M_1$ and $M_2$ form their first resonance no higher than several hundred MeV above threshold [26]. Moreover, meson-pseudoscalar scattering in such “non-exotic” channels (such as $\pi^+\pi^-$ or $K^+\pi^-$) is substantially stronger than in “exotic” channels like $\pi^0\pi^0$ or $K^0\pi^0$, even at nonresonant energies.

The advantage of explicit $\pi B$ or $\pi B^*$ resonances, as stressed in Ref. [4], may be particularly great in eliminating combinatorial backgrounds rather than in obtaining a correlation. This advantage is likely to be most pronounced for narrow resonances. As calculated in [13] and mentioned in [4]
and below, we expect one of the $J^P = 1^+$ resonances and the $J^P = 2^+$ resonance to be narrow, but the other $J^P = 1^+$ resonance and the $J^P = 0^+$ resonance are likely to be considerably broader.

B. Tagging using hadrons other than pions

For completeness, we wish to mention methods for tagging neutral non-strange and strange $B$ mesons which rely upon their correlations with kaons [11] and protons [12]. The corresponding fragmentation diagrams are shown in Fig. 4 for correlations of kaons with nonstrange $B$'s, Fig. 5 for correlations of kaons with strange $B$'s, and Fig. 6 for correlations of protons or antiprotons with nonstrange $B$'s.

The common feature of all these methods is that the neutral $B$ meson contains the same quark or antiquark as the corresponding antiquark or quark in the tagging particle, and that this should serve to uniquely specify the tagging particle. Thus, in Fig. 4, it would not be suitable to use a $K^0$ or $\bar{K}^0$ as a tagging particle, since one would actually observe $K^0 \rightarrow \pi^+ \pi^-$. Generalizations of the diagrams in Figs. 4-6 are easily made.

C. Lessons from correlations of hadrons with $D$ and $D^*$ mesons

The presence of positive-parity "$D^{**}$" resonances has already been established. We shall discuss their properties more explicitly in Sec. V, since they can provide valuable information about the corresponding $B^{**}$ resonances. Here we wish to note that the same sorts of correlations can be studied for charmed mesons as for $B$ mesons. This type of study is not essential for tagging neutral $D$ mesons themselves, since the $D$ decays $D^{*+} \rightarrow \pi^+ D^0$ and $D^{*-} \rightarrow \pi^- D^0$ [5] are ideal for that. The purpose of studying hadron-$D$ or hadron-$D^*$ correlations in which the effective mass lies above the $D^*$ is to calibrate what sorts of correlations one might expect for systems containing $b$ quarks.

As one example, consider the correlations of a charged kaon and a $D_s^{(*)}$ or $\bar{D}_s^{(*)}$, as shown in Fig. 7. These diagrams are identical to those in Fig. 5 with the substitution of a charmed quark for a $b$ quark. Moreover, both $B_s^{*0}$ and $D_s^{*+}$ decay via photon emission to $B_s^0$ and $D_s^+$, respectively. Thus, aside from the fact that we expect [27] $M(B_s^{*0}) - M(B_s^0) = M(B^{*0}) - M(B^0) \simeq 46$ MeV while we have $M(D_s^{*+}) - M(D_s^+) \simeq 141$ MeV, the two systems should

13
be very similar. Differences could arise as a result of these different hyperfine splittings if there are resonances very close to threshold; this possibility is assessed in Sec. V.

V. RESONANCE LORE

A. Positive-parity D mesons

The bound states of a charmed quark c with a light anti-quark \( \bar{q} \) in an \( L = 1 \) system have been discussed in many places, including Refs. [28], [29], and [30]. The understanding of such resonances will help in anticipating the properties of the corresponding mesons involving \( b \) quarks.

The fine structure of the \( L = 1 \) \( c\bar{q} \) system is dominated by whether the sum \( L + S_q \equiv j \) corresponds to \( j = 1/2 \) or \( 3/2 \). The states with \( j = 1/2 \) and their expected decay modes are:

\[
J_{2j}^P = 0_1^+ : \rightarrow (D\pi)_{\ell=0} ,
\]

\[
J_{2j}^P = 1_1^+ : \rightarrow (D^*\pi)_{\ell=0} ,
\]

Neither of these states has been observed yet. The states with \( j = 3/2 \) are expected to be:

\[
J_{2j}^P = 1_3^+ : \rightarrow (D^*\pi)_{\ell=2} ,
\]

\[
J_{2j}^P = 2_3^+ : \rightarrow (D\pi)_{\ell=2}, (D^*\pi)_{\ell=2} .
\]

The states in these two pairs of equations are expected to be split by an interaction whose strength depends on one inverse power of the heavy quark mass.

Candidates for the \( 1_3^+ \) and \( 2_3^+ \) states exist [31, 32, 33, 34]:

\[
D^*(2420) \rightarrow D^*\pi ,
\]

\[
D^*(2460) \rightarrow D\pi, D^*\pi .
\]

The identification of the \( 2_3^+ \) state is unique just on the basis of decay modes. The identification of the \( 1_3^+ \) state is supported by the small mass splitting between the states and by the Dalitz plot distribution in the \( D\pi\pi \) final state. This distribution is consistent with the production of an \( \ell = 2 \) \( D^*\pi \) final state [32, 33].
Adjusting the predictions of Ref. [28] to make the $1^+_3$ and $2^+_3$ states correspond to the observed ones, one then expects the $0^+_1$ and $1^+_1$ states and to show up around 2.34 and 2.35 GeV/c², respectively. Other predictions for these states have been summarized in Ref. [29]. A recent interesting suggestion [35] is that these particles could be the parity doublets of the $0^-D$ and $1^-D^*$ mesons, split from them by chiral symmetry breaking.

The failure to observe the $0^+_1$ and $1^+_1$ states up to now has usually been ascribed to their ability to decay via $S$-waves, and thus to be extremely broad. It is important, nonetheless, to see if such states can be identified, perhaps by comparison with exotic channels. Thus, for instance, to search for the $0^+_1$ state one might compare $\pi^+D^0$ (non-exotic) and $\pi^-D^0$ (exotic) channels, while to search for the $1^+_1$ state one might compare $\pi^-D^{**}$ (non-exotic) and $\pi^+D^{**}$ (exotic) channels.

One also expects $0^+_1, 1^+_1, 1^+_3$ and $2^+_3$ strange charmed mesons, about 100 MeV above the corresponding nonstrange ones. (This is about the observed splitting between the $D^+_s$ and the $D^+$, and between the $D^{*+}_s$ and the $D^{*+}$.) A candidate for the $1^+_3$ strange state has been seen [36]:

$$D^{*+}_s(2536) \rightarrow D^+K^-,$$

(36)

The absence of a $D\overline{K}$ mode suggests that this is not the $2^+_3$ state.

B. Extrapolation to positive-parity B mesons

A detailed study of the spectroscopy of $L = 1$ $b\bar{q}$ mesons has recently been performed in Ref. [13]. Some earlier treatments are contained in Ref. [37]. Here we comment on those features which can be obtained primarily from extrapolating the known or expected properties of the $L = 1$ $c\bar{q}$ mesons.

The fine-structure splitting between the states $1^+_3$ and $2^+_3$ scales as $1/m_Q$, where $Q$ is the heavy quark. Thus, we expect the corresponding $b\bar{q}$ states to be split by $m_c/m_b \simeq 1/3$ times the splitting in the charm system, or about 13 MeV. Now, the spin-weighted average of the charmed $1^+_3$ and $2^+_3$ masses is about 2445 $MeV/c^2$, which lies about 470 $MeV/c^2$ above the spin-weighted average of the $D$ and $D^*$ masses. Thus, if the dynamics of the $c\bar{q}$ and $b\bar{q}$ systems are similar, we expect the spin-weighted average of nonstrange $1^+_3$ and $2^+_3 b\bar{q}$ states to lie about 470 $MeV/c^2$ above $[3M(B^*) + M(B)]/4 \simeq 5313$ $MeV/c^2$, or at 5783 $MeV/c^2$. (Taking account of the slightly greater binding
energy of the $b\bar{q}$ system, the authors of Ref. [13] find this value to be 20 MeV/$c^2$ lower.)

The $(1^+_1, 2^+_1)$ states should then lie at $(5775, 5788)$ MeV/$c^2$ (or $(5755, 5767)$ MeV/$c^2$ in the estimate of Ref. [13]). The $(0^+_1, 1^+_1)$ states should lie about 100 MeV lower. For the corresponding strange states, one should add about 100 MeV. (This appears to be true in comparing the $B^0$ with the recently observed $B^0_s$ [38], and in comparing nonstrange and strange $J^P = 1^+$ charmed mesons.) We summarize these expectations in Table I.

Table 1. Expected properties of $L = 1$ $b\bar{q}$ states.

<table>
<thead>
<tr>
<th>$J^P_{2J}$</th>
<th>$\bar{q} = \bar{u}$ or $\bar{d}$</th>
<th>$\bar{q} = \bar{s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^+_1$</td>
<td>$5.68 \ (\bar{B}\pi)_{\ell=0}$</td>
<td>$5.78 \ (\bar{B}K)_{\ell=0, \bar{B}_s^0\gamma}$</td>
</tr>
<tr>
<td>$1^+_1$</td>
<td>$5.68 \ (\bar{B}\pi)_{\ell=0}$</td>
<td>$5.78 \ \bar{B}_s\gamma, \ \bar{B}_s^0\gamma$</td>
</tr>
<tr>
<td>$1^+_3$</td>
<td>$5.78 \ (\bar{B}^*\pi)_{\ell=2}$</td>
<td>$5.88 \ (\bar{B}^*K)_{\ell=2}$</td>
</tr>
<tr>
<td>$2^+_3$</td>
<td>$5.79 \ (\bar{B}\pi)<em>{\ell=2, (\bar{B}^*\pi)</em>{\ell=2}}$</td>
<td>$5.89 \ (\bar{B}K)<em>{\ell=2, (\bar{B}^*K)</em>{\ell=2}}$</td>
</tr>
</tbody>
</table>

The $\ell = 0$ decays [except for $b\bar{s}(0^+_1) \rightarrow \bar{B}K$, which has very little energy release] should correspond to very broad resonances, while the $\ell = 2$ decay widths should be tens of MeV or less (as in the $D^*(2420)$ and $D^*(2460)$ cases). Detailed estimates have been made in Ref. [13].

C. The 2S states

In order to make use of methods for tagging $D^+_s = c\bar{s}$ or $\bar{B}_s^0 = b\bar{s}$ using an associated kaon, one must study $K^-D^+_s$ or $K^-\bar{B}_s^0$ combinations above threshold: 2.46 or 5.87 GeV/$c^2$, respectively. The $2^+_3$ $c\bar{u}$ state, $D^*(2460)$, should be just barely able to decay to $K^-D^+_s$. The $K^-\bar{B}_s^0$ threshold is above any of the nonstrange resonances in Table I. If a resonance is to be responsible for $K^-\bar{B}_s^0$ or $K^-\bar{B}_s^{0*}$ correlations, the lowest candidate will be a $2S$ state.

The spin-weighted averages of $2S$ $c\bar{c}$ and $b\bar{b}$ states probably lie about 0.6 GeV/$c^2$ above the corresponding $1S$ states. The spacing between $1S$
and $2S$ states of one light quark and one heavy quark is probably slightly greater than this [39, 40]. In Ref. [13] the $2S - 1S$ spacings are found in a QCD-motivated potential of the Buchm"uller-Tye [41] type to be about (740, 720, 680, 660) MeV/$c^2$ for ($D$, $B$, $D_s$, $B_s$) states. At any rate, the decay modes $B^0_s K^-$ and $\bar{B}^0_s K^-$ appear to be allowed for the $J^P = 1^- 2S b\bar{u}$ state.

Making use of the estimates of Ref. [13] for nonstrange states but just adding 100 MeV/$c^2$ for strange states, we expect the $2S$ $c\bar{q}$ and $b\bar{q}$ levels to have the approximate masses shown in Table II. If the strange states really have smaller $2S - 1S$ spacings than the nonstrange ones, as predicted in Ref. [13], one should subtract about 60 MeV/$c^2$ from the estimates in the second column of Table II.

<table>
<thead>
<tr>
<th>$J^P$</th>
<th>$q = s$ or $\bar{d}$</th>
<th>$q = \bar{s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mass $(GeV/c^2)$</td>
<td>Decay mode(s)</td>
</tr>
<tr>
<td>$c\bar{q}$ (0$^-$)</td>
<td>2.68</td>
<td>$D^<em>\pi$, $D^</em>_s K$</td>
</tr>
<tr>
<td>$c\bar{q}$ (1$^-$)</td>
<td>2.82</td>
<td>$D^{(<em>)}\pi$, $D^{(</em>)}_s K$</td>
</tr>
<tr>
<td>$b\bar{q}$ (0$^-$)</td>
<td>6.00</td>
<td>$\bar{B}^<em>\pi$, $\bar{B}^</em>_s K$</td>
</tr>
<tr>
<td>$b\bar{q}$ (1$^-$)</td>
<td>6.05</td>
<td>$\bar{B}^{(<em>)}\pi$, $\bar{B}^{(</em>)}_s K$</td>
</tr>
</tbody>
</table>

Here we have assumed the same hyperfine splittings as in the $1S$ cases. The hyperfine splitting in a nonrelativistic model should be proportional to $|\Psi(0)|^2$, where $\Psi(r)$ is the Schrödinger wave function. For a system of reduced mass $\mu$ bound in a linearly rising potential $V(r) = ar$, $|\Psi(0)|^2 = (\mu/4\pi) \langle dV/dr \rangle = (\mu a/4\pi)$ independently of principal quantum numbers. There is some reason to suspect that this is an appropriate limit for a light quark bound to a heavy one.

The results of Tables I and II suggest that $\bar{B}^{(*)}_s K$ correlations may be similar to $D^{(*)}_s K$ correlations, which should be easier to study. One possible
exception is that the $D(2460)$ should be just barely to decay to $D_s^+K^-$, while the corresponding $J^P = 2^+$ resonance in the $B$ system is expected to be too light to decay to $B_sK^-$.  

D. Angular distributions and kinematics

1. Effect of loss of photon in $B^* \rightarrow B\gamma$. The $D^*$ can decay to $D\pi$ or $D\gamma$, but the $B^*$ is only able to decay to $B\gamma$. The energy of this photon is so low (about 46 MeV) that its detection is unlikely in most experiments. (See, however, Ref. [42].) Even if the photon is missed in the decay $B^{**} \rightarrow B^*\pi \rightarrow B\gamma\pi$, the effective mass of the $B\pi$ system is shifted down from the true $B^{**}$ mass, but not broadened appreciably.

To see this, let $p_\pi$, $p_B$ and $p_\gamma$ be the momenta of the pion, $B$, and photon in the $B^*$ rest frame (Fig. 8). Let $\theta_\gamma$ be the angle between the photon and the pion in this frame. We have $|\vec{p}_\gamma| = E_\gamma = 46$ MeV = $|\vec{p}_B|$, while for $M(B^{**}) = 5.79$ GeV (the value we predict for the $2^+_3$ state), one has $p_\pi = 464$ MeV. A bit of arithmetic leads to

$$M_{B\pi} \simeq M_{B^{**}} - E_\gamma + \frac{E_\gamma p_\pi}{M_{B^{**}}} \cos \theta_\gamma,$$

where $M_{B\pi}$ is the physical mass, while $M_{B^{**}}$ is the mass of the $B^{**}$ without the photon. The predicted mass differences between the $B\pi$ system and the $B$ are then:

$$M(B\pi) - M(B) = \begin{cases} 
(448 + 4 \cos \theta_\gamma) & \text{MeV}/c^2 (1^+_3) \\
(461 + 4 \cos \theta_\gamma) & \text{MeV}/c^2 (2^+_3) 
\end{cases}$$

or, for $M(B^{**}) = 5.79$ GeV, $M(B\pi) \simeq M(B^{**}) - [46 - 3.8(\cos \theta_\gamma)] \text{ MeV}/c^2$. The predicted mass differences between the $B\pi$ system and the $B$ are then:

$$M(B\pi) - M(B) \simeq 500 \text{ MeV}/c^2.$$

The relative strengths of the peaks in $1^+_3$ and $2^+_3$ decay are 3:2 as shown in Refs. [29] and [30].

2. Dalitz plot analysis of $D'^{**} \rightarrow D^*\pi \rightarrow D\pi\pi$. Let us define kinematic variables for the decays $D'^{**} \rightarrow D^*\pi_1$, $D^* \rightarrow D\pi_2$, as shown in Fig. 9. We
recall some results already quoted in Ref. [29] for the distribution in $\theta$ (equivalent to a Dalitz plot variable). When a spin-2 $D^{**}$ decays to $D^*\pi$, it does so via a $D$-wave, and the decay probability $W(\theta)$, normalized in such a way that

$$\frac{1}{2} \int_{-1}^{1} d(\cos \theta) W(\theta) = 1,$$

is $W(\theta) = (3/2) \sin^2 \theta$. When a spin-1 $D^{**}$ decays to $D^*\pi$, it can do so either by an $S$-wave (as expected for the $1^+_2$ state) or a $D$-wave (as expected for the $1^+_3$ state). The corresponding distributions are

$$W(\theta) = \begin{cases} 
1 & (S \text{ wave}), \\
(1 + 3 \cos^2 \theta)/2 & (D \text{ wave}).
\end{cases}$$

(41)

It appears that the decay $D(2420) \to D^*\pi$ is compatible with the distribution for $D$ wave [32, 33]. This supports the identification of the $D(2420)$ as the $1^+_3$ state. The $D(2460)$ indeed appears to have $J^P = 2^+$ [31, 32].

When and if another resonance decaying to $D^*\pi$ is discovered, we predict that the distribution will be isotropic in $\theta$ as expected for the $1^+_1$ state.

3. Dalitz plot analysis of $B^{**} \to B^*\pi \to B\gamma\pi$. The Dalitz plot distribution associated with the configuration noted in Fig. 8 can be measured if one can detect the photon [42]. Normalizing distributions $W(\theta,\gamma)$ as above, we find for a spin-2 $B^{**}$ decaying to $B^*\pi$, with subsequent decay of the $B^*$ to $\gamma B$, that $W(\theta,\gamma) = \frac{3(1 + \cos^2 \theta,\gamma)}{4}$. This function is peaked at $\theta,\gamma = 0$ and $\pi$. The corresponding distributions for a spin-1 $B^{**}$ decaying to $B^*\pi$ in a state of angular momentum $\ell$ are $W(\theta,\gamma) = 1$ for $\ell = 0$ and $W(\theta,\gamma) = \frac{2 + 3 \sin^2 \theta,\gamma}{4}$ for $\ell = 2$. This last function is peaked at $\theta,\gamma = \pi/2$.

4. Distributions for polarized $D^{**}$ and $B^{**}$. The Dalitz plot distributions corresponding to Fig. 9 cannot be measured for $B^{**}$ decays since the decay $B^* \to B\pi$ is kinematically forbidden. However, if $D^{**}$ or $B^{**}$ resonances are produced with any polarization, their decays to $D(\pi)$ or $B(\pi)$ may produce pions with a non-isotropic distribution with regard to the polarization axis. This point has recently been emphasized in Ref. [14].

Let us imagine that a spin-J resonance $R$ (standing for $D^{**}$ or $B^{**}$) is produced along some axis $\hat{n}$. By parity invariance one expects the same probability for helicity $\lambda$ and $-\lambda$ with respect to $\hat{n}$, but, aside from this, populations associated with different helicities can differ. This, in turn, can
lead to non-trivial distributions in the angle $\theta_1$ between the momentum of the pion $\pi_1$ to which the resonance $R$ decays and the direction $\hat{n}$. Labelling these relative decay probabilities by $W_{|\lambda|}(\theta_1)$, where

$$\frac{1}{2} \int_{-1}^{1} d(\cos \theta_1) W_{|\lambda|}(\theta_1) = 1 \ , \quad (42)$$

$$W_0(\theta_1) + 2 \sum_{|\lambda|>0} W_{|\lambda|}(\theta_1) = 2J + 1 \ , \quad (43)$$

we have (for $P \equiv D$ or $B$, $V \equiv D^*$ or $B^*$):

\[ R(2^+) \rightarrow P\pi \]

\[ W_0(\theta_1) = (5/4)(3\cos^2 \theta_1 - 1)^2 \quad (44) \]

\[ W_1(\theta_1) = (15/2)\sin^2 \theta_1 \cos \theta_1 \quad (45) \]

\[ W_2(\theta_1) = (15/8)\sin^4 \theta_1 \quad (46) \]

\[ R(1^+) \rightarrow (V\pi)_{\ell=0} \]

\[ W_0(\theta_1) = W_1(\theta_1) = 1 \quad (47) \]

\[ R(1^+) \rightarrow (V\pi)_{\ell=2} \]

\[ W_0(\theta_1) = (3/4)(1 + 3\cos^2 \theta_1) \quad (48) \]

\[ W_1(\theta_1) = (3/4)(1 + [3/2]\sin^2 \theta_1) \quad (49) \]

\[ R(2^+) \rightarrow V\pi \]

\[ W_0(\theta_1) = (15/2)\sin^2 \theta_1 \cos^2 \theta_1 \quad (50) \]

\[ W_1(\theta_1) = (5/4)(1 - 3\cos^2 \theta_1 + 4 \cos^4 \theta_1) \quad (51) \]

\[ W_2(\theta_1) = (5/4)(1 - \cos^4 \theta_1) \quad (52) \]

Of course, for $R(0^+) \rightarrow P\pi$ there is no $\theta_1$ dependence.

The above distributions are relevant to any attempt to select pion-$D$ or pion-$B$ correlations by means of angular rather than effective-mass cuts. If different values of $|\lambda|$ are populated differently, such angular cuts can either
enhance or degrade a signal which was due originally to a specific resonance
or band of resonances.

VI. CONCLUSIONS

We have discussed the possibility of identifying neutral $B$ mesons using
hadrons produced nearby in phase space. The simplest example is the ex-
pected correlation between a $B^0$ and a $\pi^+$, which we expect to be stronger
(with relative probability $P_1$) than that between a $B^0$ and a $\pi^-$ (with relative
probability $P_2 < P_1$). The correlation is expected to be most pronounced for
low effective masses or small rapidity differences. It can exist as a result of
resonances in the $B\pi$ system, but can also be due simply to the fragmentation
of a $b$ quark. All statements are of course valid also for the charge-conjugate
systems.

A number of issues have been treated in this article, which serves as a
sequel to Ref. [4].

(1) We have noted some simple time-dependences in decays which are
"tagged" by means of an associated hadron. In general a dilution of the
observed asymmetry with a very simple form $(P_1 - P_2)/(P_1 + P_2)$ occurs.

(2) Although we assume no coherence between $B^0$ and $\overline{B}^0$ in the initial
state, we have shown how to test for this coherence experimentally.

(3) We have stressed that explicit $B\pi$ resonances are not required for
"tagging," although the presence of such resonances may help to reduce com-
binatorial backgrounds.

(4) We have mentioned the use of correlations with hadrons other than
pions. Quark diagrams describing fragmentation are particularly helpful in
visualizing which correlations are likely to prove fruitful.

(5) We have stressed the need for detailed studies of the corresponding
correlations involving $D$ mesons, aside from the prominent production of
very soft pions in the decays $D^* \rightarrow D\pi$ which have no counterpart in the $B$
system.

(6) We have treated several issues regarding resonances, discussing some
properties of the positive-parity charmed mesons and their extrapolation to
$B$ mesons, expected masses of $2S$ states, and angular distributions in decays.

In the study of CP-violating decays of neutral $B$ mesons, the identifica-
tion of their initial flavor is a topic of keen interest. The use of correlated hadrons
in this context is a promising possibility. Whether it will be realized in
practice depends on a number of experimental questions, some of which we have raised in the present work.

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[16] D. Besson, invited talk presented at XVI International Symposium on Lepton and Photon Interactions (see Ref. [7]).


[25] G. Gustafson and J. Hakkinen, work in preparation. We thank G. Gustafson for a private communication and for allowing us to quote these unpublished results.


[37] See, in particular, Refs. [12]–[14] of Ref. [29].


[42] L3 Collaboration, paper No. 227 contributed to the XVI International Symposium on Lepton-Photon Interactions (see Ref. [7]).
FIGURE CAPTIONS

FIG. 1. (a) Fragmentation of a $b$ quark into a $B^0$ or $B^{*0}$ with production of a $\pi^-$; (b) charge-conjugate process.

FIG. 2. (a) Fragmentation of a proton into a $b$-flavored baryon and (a) a $B^+$ or (b) a $B^0$.

FIG. 3. Diagrams describing decays of a $B_s$ or $\bar{B}_s$ into $D_s^+K^-$. 

FIG. 4. Correlations of neutral nonstrange $B$ mesons with neutral $K^*$ resonances. (a) $\bar{B}^0$ or $\bar{B}^{*0}$ with $K^{*0}$; (b) charge-conjugate process.

FIG. 5. Correlations of strange $B$ mesons with charged kaons. (a) $\bar{B}_s^0$ or $\bar{B}_s^{*0}$ with $K^-$; (b) charge-conjugate process.

FIG. 6. Correlations of neutral nonstrange $B$ mesons with protons or antiprotons.

FIG. 7. Correlations of charged kaons and charmed-strange mesons.

FIG. 8. Momenta of particles in the decay $B^{**} \rightarrow B^*\pi$, $B^* \rightarrow B\gamma$, as expressed in the $B^*$ rest frame.

FIG. 9. Momenta of particles in the decay $D^{**} \rightarrow D^*\pi_1$, $D^* \rightarrow D\pi_2$, as expressed in the $D^*$ rest frame.