On Deusons or Deuteronlike Meson-Meson Bound States

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Abstract. The systematics of deuteronlike two-meson bound states, deusons, is discussed. Previous arguments that many of the present non-$qar{q}$ states are such states are elaborated including, in particular, the tensor potential. For pseudoscalar states the important observation is made that the centrifugal barrier from the P-wave can be overcome by the $1/r^2$ and $1/r^3$ terms of the tensor potential. In the heavy meson sector one-pion exchange alone is strong enough to form at least deuteron-like $Bar{B}^*$ and $B^*ar{B}$ composites bound by approximately 50 MeV, while $Dar{D}^*$ and $D^*ar{D}^*$ states are expected near the threshold.

Recently I suggested [1] that many of the best established light non-$qar{q}$ candidates [2] are in fact deuteronlike meson-meson bound states or deusons. The idea, that deuteronlike bound states of two mesons might exist is certainly not new, but has been discussed generally only in passing within general phenomenological models for meson-meson bound states (See [3]–[13]), where pion exchange is not given special attention. After my first letter, Ericson and Karl [14] has also studied the strength of pion exchange with similar conclusions, and Manohar and Weise [15] have studied flavour exotic two $B^*$-meson bound states. The heavy meson systems are an interesting testing ground for these ideas, since the predictions are less ambiguous than for light states.

One can write the one-pion exchange potential in an universal way by collecting all the constants into an overall number $\gamma$, the ”relative coupling number” which is a measure of the overall potential strength. Thus for $NN$ one has $\gamma_{NN}^S = -\frac{25}{9}(\tau_1 \cdot \tau_2)(\sigma_1 \cdot \sigma_2)$, for $(D^*\bar{D})_\pm$, $\gamma_{**}^P = \mp \tau_1 \cdot \tau_2$ and for $D^*\bar{D}^*$ $\gamma_{**}^V = \mp (\tau_1 \cdot \tau_2)(\Sigma_1 \cdot \Sigma_2)$. This number $\gamma$ measures the relative strength of the potential compared to the contribution from one pair of quarks in a spin triplet and isospin triplet state for which $\gamma_{**}^V = -1$. For example for the deuteron $\gamma_{**}^{NN} = 25/3$ and for $D^*\bar{D}^*$ in $I=0$, $S=0$, $\gamma_{**}^{VV} = 6$. The larger $\gamma$ is, the stronger is the attraction, and if it is negative there is repulsion. The universal one-pion exchange potential in $r$-space can then be written compactly:

$$V_\pi(r) = -\gamma V_0 \left[ D \cdot C(r) + S_{12}(\hat{r}) \cdot T(r) \right], \quad (1)$$

where $D$ is a diagonal matrix, $\hat{r}$ is the unit vector, and the $r$ dependence is given by the functions

$$C(r) = \frac{\mu^2 e^{-\mu r}}{m_\pi^2 m_\pi r}, \quad (2)$$

$$T(r) = C(r)[1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2}], \quad (3)$$

and $S_{12}(r)$ is the tensor operator in $r$ space, which, in general, connects different partial waves. In Eq. (1) we introduced the constant $V_0 = m_\pi^2 g^2/(12\pi f^2) \approx 1.3$ MeV, the numerical value of which is fixed by the $\pi N$ coupling constant.

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Because of the singular behaviour of the tensor potential it must be regularized at small distances. The perhaps most natural method is to introduce a form factor at each $\pi N$ vertex, such as $(\Lambda^2 \mu^2)/(\Lambda^2 + t)$, which in $r$-space can be looked upon as a spherical pion source with rms radius $R = \sqrt{10}/\Lambda$.

The deuteron. Our prime reference state is of course the deuteron, the existence of which nobody doubts. It has been studied in great detail over the years (See Ref. [16] and the recent reviews [17], [18]). There one knows that the dominant binding energy comes from pion exchange between two colourless $qqq$ clusters - a proton and a neutron.

One defines conventional basis vectors $|\beta S_1>$ and $|\beta D_1>$ such that the wave function is in general $u(r)|\beta S_1> + w(r)|\beta D_1>$. The deuteron potential $V_d(r)$ can then be written in matrix form as:

$$V_d(r) = -\frac{25}{3} V_0 \left[ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} C(r) + \begin{pmatrix} 0 & \sqrt{8} \\ \sqrt{8} & -2 \end{pmatrix} T(r) \right],$$

(4)

The overall strength of the potential is given by $\gamma V_0 = 11.4$ MeV. There are a few important points to be learned from the deuteron and the $NN$ system, which are essential also for the present application to meson-meson states:

- The central part of the one pion potential $\gamma_{10}^{NN} V_0 C(r)$ is insufficient to bind the deuteron. The overall strength is too small by a large factor of 3.
- The tensor potential is very important in providing the binding [18]. It is much larger in magnitude than the central part at small distances.
- The potential must be regularized at small $r$, by some cut off procedure. The binding energy is very sensitive to this procedure, as well as to any added short range potential from $2\pi, \rho, \omega$ etc. exchange. But, once the binding energy is right all the other static deuteron properties follow correctly.

Unfortunately because of this last fact one will not, in general, be able to predict reliably the binding energies for deusons. Certainly, if the potential is strong enough one can be quite confident that such bound states must exist, but their exact binding energy will always depend on details of regularization, and on the short range potential from heavier exchanges. Again, in borderline cases where the expected binding is small, like for the deuteron, one similarly cannot be sure that such bound states actually exist.

For the numerical solutions we use a method (See Refs. [19],[20]) where one discretizes the $r$ dependence to a finite dimensional vector, whereby the Hamiltonian becomes a finite matrix, which can be solved by using efficient standard matrix routines for finding eigenvalues and eigenvectors. The method is particularly accurate for finding the ground state in problems with coupled channels, which is precisely what is needed here. The method is comprehensively discussed for the one-channel problem in Ref. [20].

$PV$ deusons. Since parity forbids two pseudoscalars to be bound by one-pion exchange, the lightest deusons are pseudoscalar-vector states. Again in such $PV$ systems the pion is too light to be a constituent, because the small reduced mass of a $\pi V$ system would give a too large kinetic term, which cannot be overcome by the potential. Thus the deuson spectrum can start at the earliest with $K \bar{K}$ or at $\approx 1400$ MeV. In general,
the relative coupling number, \( \gamma \), is 3 times larger for \( I=0 \) states than for \( I=1 \), and the \( J^{PC} \) quantum numbers of interest for \( PV \) systems are \( 0^{-} \) and \( 1^{++} \). For these \( \gamma = 3 \) and the potentials are:

\[
V_{0^{-}} = -3V_0 [C(r) + 2T(r)] = -3V_0 \frac{\mu^2}{m^2} e^{-\mu r} \left[ 3 + \frac{6}{\mu r} + \frac{6}{(\mu r)^2} \right], \tag{5}
\]

\[
V_{1^{++}} = -3V_0 \left[ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} C(r) + \begin{pmatrix} 0 & \sqrt{2} \\ -\sqrt{2} & 1 \end{pmatrix} T(r) \right]. \tag{6}
\]

Here \( \mu \) is not exactly the pion mass, since the vector and pseudoscalar do not have the same mass. This leads to a recoil effect through which one has instead: \( \mu^2 = m^2 - (M_V - M_P)^2 \). For the axial vector states \( 1^{++} \) (6) there are two channels like for the deuteron, and we define basis vectors such that in general \( |1^+> = u(r)|^3S_1 > + w(r)|^3D_1 > \).

The pseudoscalar channel is a single channel case, Eq. (5), to which the tensor part contributes with same sign as the central part since \( -3P_0|S_{12}|^3P_0 > = +2 \). Therefore these add giving a remarkably strong potential (5), which is attractive for \( C=+ \) i.e., \( J^{PC} = 0^{-} \). Thus an especially interesting new situation appears: The tensor part with its \( r^{-2} \) and \( r^{-3} \) terms contributes, with much stronger attraction than the central part, directly to the \( 3P_0 \) wave, and not through a second order coupling of S and D waves as for the deuteron. This means that the \( 1/r^2 \) and the \( 1/r^3 \) terms of the tensor potential can compensate, at least partially, the centrifugal barrier \( 2/(Mr^2) \).

Numerically by solving the Schrödinger equation one finds using the one-pion potential (5-6) alone, regularized with \( \Lambda = 1.2 \text{ GeV} \) that

- that there certainly must exist an \( \eta_b (\approx 10545) \), that there very likely exists an \( \eta_c (\approx 3870) \), and that possibly in the iota peak, \( \eta(1440) \), there is a \( K\bar{K}^* \) deuson.

- that certainly there must exist a \( \chi_{1b} (\approx 10562) \), that rather likely there exists a \( \chi_{1c} (\approx 3870) \), and that possibly the \( f_1(1420) \) could be a \( K\bar{K}^* \) deuson.

**VV deusons.** For composites of two vector mesons the strongest attraction, as measured by the relative coupling number \( \gamma^{VV}_{SI} \), is in the spin and isospin singlet channels \( (\gamma^{VV}_{00} = 6) \), followed by the spin triplet, isosinglet channels \( (\gamma^{VV}_{10} = 3) \). For \( I=1 \) one has either repulsion or very weak attraction. The S-waves appear for \( J^{PC} = 0^{++}, 1^{+-} \) and \( 2^{++} \), but these can mix with D-waves and the potential for the two first mentioned channels are:

\[
V_{0^{++}} = -6V_0 \left[ \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} C(r) + \begin{pmatrix} 0 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix} T(r) \right], \tag{7}
\]

\[
V_{1^{+-}} = -3V_0 \left[ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} C(r) + \begin{pmatrix} 0 & \sqrt{2} \\ -\sqrt{2} & 1 \end{pmatrix} T(r) \right]. \tag{8}
\]

For the single channel cases the most interesting is the pseudoscalar \( 3P_0 \):

\[
V_{0^{--}} = -3V_0 [C(r) + 2T(r)] = -3V_0 \frac{e^{-m \pi r}}{f_\pi} [3 + \frac{6}{m \pi r} + \frac{6}{(m \pi r)^2}], \tag{9}
\]

which is in fact very similar to that for pseudoscalar \( (P\bar{V})_+ \) states, (Eq. 5), and is remarkably strong, as was already discussed for the \( PV \) case. There remains two spin
parities which are of interest, 1−− and 2++, which are discussed in Ref. [21], and in
which the tensor potential connects 3 respectively 4 partial waves. In summary by
solving the Schrödinger equation one finds \( VV \) deusons of the following kinds:

- In the pseudoscalar sector an \( \eta_b(\approx 10590) \) should exist and \( \eta_c(\approx 4015) \) should very
  likely also exist and an \( \eta(\approx 1790) \) is possibly bound. These states should all mix
to some extent with their \( PV \) counterparts, making the lighter (mostly \( PV \)) more
  bound.

- In the scalar sector there should exist a \( \chi_{b0}(\approx 10582) \) \( B^*B^* \) bound state and a
  \( \chi_{c0}(\approx 4015) \) also very likely exists. In the light sector the \( f_0(1720) \) (the ”theta”)
could be a \( K^*\bar{K}^* \), while the \( f_0(1520) \) could be a \((\rho\rho - \omega\omega)/\sqrt{2} \) deuson.

- In the axial sector \( h_b(\approx 10608) \) should exist, possibly also a \( h_c(\approx 4015) \), while \( h_1(\approx
  1790) \) is less likely.

- In the tensor sector the numerical calculations show, perhaps surprisingly, that
  there should exist a \( \chi_{b2}(\approx 10602) \) \( B^*B^* \) bound state, and that a \( \chi_{c2}(\approx 4015) \) at
  threshold is also very likely. With some extra attraction the \( f_2(1720) \) [22] could
  be a \( K^*\bar{K}^* \), and the \( f_2(1520) \) could be a \((\rho\rho + \omega\omega)/\sqrt{2} \) deuson.

The widths are expected to be quite narrow. This is especially the case for the four
\( D\bar{D}^* \) and \( B\bar{B}^* \) states, which because of parity cannot decay to \( D\bar{D} \), respectively \( B\bar{B} \). Of
course through annihilation of the heavy quarks, decays to light mesons are possible.
However, such annihilation should be suppressed by form factors, since these states
are much larger in size than normal \( Q\bar{Q} \) states (assuming the states are weakly bound).
The \( D^*\bar{D}^* \) and \( B^*\bar{B}^* \) deusons can generally decay into \( D\bar{D} \) and \( B\bar{B} \), which should be
their main decay mode giving widths of a few tens of MeV.

In channels with exotic flavour or CP quantum numbers pion exchange is generally
repulsive or quite weak. Therefore one does not expect that such exotic deusons exist,
although \( B^*B^* \) may be an exception. Neither does the deuson model predict new non-
\( q\bar{q} \) states which should have been seen. E.g., for \( I=1 \) channels one pion exchange is
generally a factor 3 weaker than for \( I=0 \), and one certainly does not expect such states
within the light meson sector.

Where could the predicted heavy deusons of Table 8 be produced and seen experi-
mentally? Unfortunately, this will not be easy, but at least two good places are: \( \pi N \)
in flight at the Fermilab antiproton accumulator, and \( \Upsilon \) decay looking at final states
including e.g. \( J/\psi \omega \) (for the \( D\bar{D}^* \) deusons) or \( D\bar{D}, D\bar{D}^* \) (for the \( D^*\bar{D}^* \) deusons).

To find these states would be important, not only because they would confirm the
expectations from pion exchange and constrain the parameters of the model presented
here. More importantly, if these deusons are found, they at the same time would give
strong support for the interpretation that many, perhaps all, of the present light non-
\( q\bar{q} \) candidates really are deuteronlike states. This would then imply that experimental
evidence for baglike multiquark states and glueballs would have to be looked for at
higher energies.

More details about the results presented here are given in Ref. [21].
References


