The Effective Potential of Electroweak Theory
with Two Massless Higgs Doublets
at Finite Temperature

Koichi Funakubo*, Akira Kakuto† and Kazunori Takenaga‡

Department of Physics, Saga University, Saga, 840 JAPAN

1Department of Liberal Arts, Kinki University in Kyushu, Iizuka, 820 JAPAN

2Department of Physics, Kyushu University, Fukuoka, 812 JAPAN

HEP-PH 9310267

The effective potential of electroweak theory with two massless Higgs doublets at finite
temperature is studied. We investigate phase structure and critical temperature in this
model by numerical analysis without high-temperature expansion. The phase transition is
found to be of first order. The critical temperature is shown to be relatively low for typical
scalar masses. The free energy of the critical bubble is calculated with some approximations
and we find that the bubble nucleation can occur at a temperature a little below the critical
temperature. We also discuss the possibility of the electroweak baryogenesis.

* funakubo@sagagw.cc.saga-u.ac.jp
† kakuto@fuk.kindai.ac.jp
‡ f77498a@kyu-cc.cc.kyushu-u.ac.jp
1. Introduction

Recently, much attention has been paid to baryogenesis at the electroweak phase transition (EWPT). In addition to the great success of low-energy phenomenology in the standard model, it is well known that the standard model may satisfy the famous Zakharov’s three conditions[1] to generate baryon number: (i) There are processes which violate baryon number. (ii) Underlying dynamics involves CP violating processes. (iii) There exist non-equilibrium processes which are caused by first-order phase transition. There has been a lot of studies to try to generate baryon number at the EWPT to explain the baryon asymmetry of the Universe observed today. Whether the condition (iii) is satisfied by underlying dynamics or not is usually determined by analyzing the effective potential at finite temperature. In the standard model, the negative tree-level mass term of the Higgs field causes a serious infrared problem that the effective potential becomes complex at one-loop level. Thus it is very difficult to obtain a reliable result by perturbation theory. One usually resorts to high-temperature expansion and takes terms including $O(M^3T)$ term in bosonic contributions to the effective potential\(^1\). The $O(M^3T)$ term is very crucial for the first-order phase transition. However this term is complex when the order parameter takes small values. Many works have been performed to improve the potential at finite temperature[2][3]. It is suggested that the phase transition is weakly first order for relatively light Higgs boson in some recent works[4]. In order to preserve baryon number after finishing EWPT, the rate of anomalous process induced by the sphaleron must be smaller than that of the expansion of the Universe. This, in turn, is converted into a strong constraint on Higgs boson mass. It leads us to an upper bound of the Higgs boson mass, $M_H = 35 \sim 50$ GeV. On the other hand, the lower bound of the Higgs boson mass found by experiments at LEP is about 60 GeV. Moreover, it is believed that the CP violation in Kobayashi-Maskawa scheme of the standard model is too feeble to generate baryon asymmetry of the Universe observed today.

Two-Higgs-doublet model has recently been investigated because of possibilities to avoid these difficulties[2]. In two-Higgs-doublet model, there could appear a new CP violating phase in the Higgs sector, and it is expected that this new CP violation could make baryon density large enough to be comparable with the observed value. Moreover it is argued that the constraint on the lightest Higgs boson mass in order not to wash out baryon density after EWPT is not so stringent and the Higgs boson could be as heavy as

\(^1\) For fermions there is no $O(M^3T)$ term.
about 100 GeV \[2\], which is consistent with experiments. In general there are three order parameters\(^2\) in this model, and these parameters are relevant for both baryogenesis and new CP violation in weak interactions. It is complicated to analyze the effective potential including one-loop effects at finite temperature, and to find the minimum of the potential to study whether the new CP violation is realized or not. Difficulties are mainly due to mixing mass terms of the two Higgs doublets if one needs to include a possibility of the new CP violation. Furthermore, as in the usual standard model, there appears a serious infrared problem associated with negative tree-level mass terms of Higgs fields. We believe that many analyses should be worked out to understand the two-Higgs-doublet model with negative mass terms.

In this paper we shall investigate phase structure of the electroweak theory with two massless Higgs doublets. This model is much simpler than the two-doublet model with tree-level mass terms. There is no infrared difficulty and it is easy to analyze the effective potential at finite temperature. We shall study critical temperature and the order of EWPT by analyzing the one-loop effective potential at finite temperature without high-temperature expansion. In fact it turns out that high-temperature expansion cannot be applied at EWPT period in this model. The order of phase transition is found to be first order for wide ranges of scalar masses. At very high temperatures, loop expansion of the effective potential might be problematic [5]. In our model, the critical temperatures are found to be significantly low compared with those in the standard model for wide ranges of scalar masses. This fact suggests that our calculations may be reliable at this range of temperature. We will find that the rate of the anomalous process after EWPT is sufficiently slow in order not to wash out baryon density. We obtain a critical temperature \(T_C \simeq 100\) GeV for relatively heavy scalar masses\(^3\). It is expected these features of the phase transition in this model are also shared in the model with small tree-level masses of Higgs fields. The phase transition in the supersymmetric standard model may also have the same features if the tree-level Higgs masses are small. We believe that our model can provide many attractive features for generation of baryon number at the EWPT, though new CP violating phase in the Higgs sector cannot be introduced.

In the next section we give the one-loop effective potential at finite temperature in the present model. In section 3 we study the effective potential for various sets of scalar

\(^2\) Assuming the \(U(1)_{\text{em}}\) invariance.

\(^3\) This \(T_C\) corresponds to the scalar mass set II defined in section 2.
mass parameters and find the critical temperature for each case by numerical analysis. The EWPT is found to be of first order. We also study briefly the bubble nucleation below the critical temperature. Section 4 is devoted to conclusions and discussions.

2. The Effective Potential at Finite Temperature

In imaginary time formalism, the effective potential is automatically separated into zero-temperature and finite-temperature parts [5]. In the case of one-Higgs-doublet model, we are afflicted with a serious infrared problem in temperature-dependent part, which is due to negative tree-level mass term of the Higgs field as mentioned before. In the present model, however, there is no such kind of problem, because tree-level Higgs mass terms are absent by assumption. It is not necessary to use high-temperature expansion in order to study the structure of phase transition. As we will see later, high-temperature expansion should not be adopted in this model, because it turns out that the critical temperature is low compared with the mass of the heaviest scalar. The zero-temperature part of the effective potential was analyzed in detail by Inoue, Nakano and one of the authors (A.K.) [6]. We will start with a review on the zero-temperature effective potential in the massless two-Higgs-doublet model in the next subsection.

2.1. Model and one-loop potential at zero temperature

We work in the usual $SU(2) \times U(1)$ gauge theory of the electroweak model with two massless Higgs doublets and with $N_g$ generations of quarks and leptons. The effect of $SU(3)_c$ color strong interactions appears only through quark loops in the effective potential. The number of colors is denoted by $N_c (= 3$ for $SU(3)_c$). Quantum number assignment of $SU(2) \times U(1)$ for quarks and leptons are as follows:

$$Q_{LA} \equiv \left( \frac{u_A}{d_A} \right)_L; \quad Y(Q_{LA}) = \frac{1}{6}, \quad l_{LA} \equiv \left( \frac{\nu_A}{\epsilon_A} \right)_L; \quad Y(l_{LA}) = -\frac{1}{2},$$

$$u_{RA}; \quad Y(u_{RA}) = \frac{2}{3}, \quad d_{RA}; \quad Y(d_{RA}) = -\frac{1}{3}, \quad \epsilon_{RA}; \quad Y(\epsilon_{RA}) = -1,$$

where $Y$ stands for the $U(1)$ hypercharge and $A$ distinguishes generations. We introduce two massless Higgs doublets $\Phi_1$ and $\Phi_2$ with $Y(\Phi_1) = Y(\Phi_2) = 1/2$. In order to avoid dangerous tree-level flavor-changing neutral interactions mediated by scalar exchange, we
restrict allowed Yukawa couplings by imposing the following discrete symmetry[7], by which only one kind of Higgs fields interacts with $u_R$ or $d_R$:

$$\Phi_2 \rightarrow -\Phi_2, \quad u_{RA} \rightarrow -u_{RA}.$$  \hfill (2.1)

Then we have the following Yukawa couplings:

$$\mathcal{L}_Y = -\sum_{A,B}^{N_2} \left[ \bar{Q}_{LA} f^{(u)}_{AB} \Phi_2 u_{RB} + \bar{Q}_{LA} f^{(d)}_{AB} \Phi_1 d_{RB} + \bar{t}_{LA} f^{(l)}_{AB} \Phi_1 \epsilon_{RB} + \text{h.c.} \right],$$  \hfill (2.2)

where $\Phi_2 = i\tau^2 \Phi_2^*$, with $\tau^a (a = 1, 2, 3)$ being Pauli matrices. The most general renormalizable and $SU(2) \times U(1)$ invariant Higgs quartic interactions contain seven types of independent couplings. The discrete symmetry (2.1) must also be imposed on the Higgs self-interactions, otherwise the Yukawa couplings (2.2) lose their meaning\(^4\). Then the Higgs potential is written as

$$V_H = \frac{\lambda_1}{2} (\Phi_1 \dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2 \dagger \Phi_2)^2 + \lambda_3 (\Phi_1 \dagger \Phi_1)(\Phi_2 \dagger \Phi_2)$$

$$+ \lambda_4 (\Phi_1 \dagger \Phi_2)(\Phi_2 \dagger \Phi_1) + \frac{\lambda_5}{2} [(\Phi_1 \dagger \Phi_2)^2 + \text{h.c.}] \quad (\lambda_{1,2} > 0).$$  \hfill (2.3)

The vacuum expectation values (VEV) of the Higgs fields are determined by minimizing the effective potential and their norm is fixed by $v^2 = 1/\sqrt{2}G_F$ to describe low-energy phenomena. Here $G_F = 1.16639 \times 10^{-5} (\text{GeV})^{-2}$ is the Fermi coupling constant. In our model the Higgs potential is a homogeneous polynomial of Higgs fields. Thus spontaneous symmetry breaking does not occur at the tree level. We must consider at least the one-loop potential.

Calculating the contribution from one-loop order to a potential in a model with more than one Higgs doublets is not so straightforward as it appears. This is because in multi-doublet case one cannot choose a renormalization scale such that all the quartic couplings are small\(^8\). A comprehensive study of the effective potential in multi-doublet model was carried out by Gildner and Weinberg\(^5\) [9]. We follow their arguments here. First, by use of $SU(2) \times U(1)$ gauge degrees of freedom, we write VEV of the Higgs fields as

$$\langle \Phi_1 \rangle = \frac{\rho}{\sqrt{2}} \binom{0}{n_1}, \quad \langle \Phi_2 \rangle = \frac{\rho}{\sqrt{2}} \binom{n_4}{n_2 + in_3},$$  \hfill (2.4)

\(^4\) Discussion is given in ref. [6].

\(^5\) Simple discussion is also given in ref. [8].
where $\rho > 0$, $n_i \ (i = 1 \sim 4)$ are real and $\sum_{i=1}^{4} n_i^2 = 1$.

Then VEV of the tree-level potential becomes

$$V_H^{Tree} = \frac{\rho^4}{8} \left[ \lambda_1 n_1^4 + \lambda_2 (n_2^2 + n_3^2 + n_4^2)^2 + 2 \lambda_3 n_1^2 (n_2^2 + n_3^2 + n_4^2) \\
+ 2 \lambda_4 n_1^2 (n_2^2 + n_3^2) + 2 \lambda_5 n_1^2 (n_2^2 - n_3^2) \right].$$  \hspace{1cm} (2.5)

We set a condition that the tree-level potential has minimum value of zero on some ray $\rho n = \rho n_0$ for arbitrary positive $\rho$. The condition implies that

$$\frac{\partial}{\partial n_i} \left( \frac{V_H^{Tree}}{\rho^4} \right) = 0 \quad \text{for} \quad i = 1 \sim 4. \hspace{1cm} (2.6)$$

A solution of (2.6) specifies a direction $n \equiv (n_1, n_2, n_3, n_4)$ of VEV, where the tree-level potential takes minimum value of zero. The solution that breaks the $SU(2) \times U(1)$ symmetry and keeps the $U(1)_{em}$ invariance is uniquely determined as \hspace{1cm} 6

$$\langle \Phi_1 \rangle = \frac{\rho}{\sqrt{2}} \left( \begin{array}{c} 0 \\
0_{n_01} \end{array} \right), \quad \langle \Phi_2 \rangle = \frac{\rho}{\sqrt{2}} \left( \begin{array}{c} 0 \\
0_{n_02} \end{array} \right), \hspace{1cm} (2.7)$$

with $\lambda \equiv \sqrt{\lambda_1 \lambda_2 \lambda_3 \lambda_4 \lambda_5} = 0$, \hspace{1cm} (2.8)

where $n_{01}$ and $n_{02}$ are defined by

$$n_{01}^2 \equiv \frac{\sqrt{\lambda_2}}{\sqrt{\lambda_1} + \sqrt{\lambda_2}}, \quad n_{02}^2 \equiv \frac{\sqrt{\lambda_1}}{\sqrt{\lambda_1} + \sqrt{\lambda_2}}.$$

The condition (2.8) guarantees the minimum value of the tree-level potential to be zero. In this model there is no new CP violating phase in the Higgs sector. As Gildner and Weinberg pointed out, perturbative calculations are reliable along the ray $\rho n_0$ and the minimum of the one-loop potential is the deepest one compared with those along other directions. We assume that the tree-level relation, $\lambda = 0$ is still valid at one-loop order. This means that we choose a renormalization point $M_R$ at which $\lambda(M_R) = 0$.

Following the usual procedure, the one-loop potential, $V_1$ at zero temperature in $R_\xi$ gauge is given as

$$V_1 = \frac{1}{64\pi^2} \left[ \frac{3}{8} g^4 + \frac{3}{16} (g^2 + g'^2)^2 + \frac{1}{2} (\sqrt{\lambda_1 \lambda_2 + \lambda_3})^2 + \lambda_1 \lambda_2 + \lambda_5^2 \\
- \frac{\lambda_2}{(\sqrt{\lambda_1} + \sqrt{\lambda_2})^2} \sum_A (N_c f_A^{(d)4} + f_A^{(t)4}) \\
- \frac{\lambda_1}{(\sqrt{\lambda_1} + \sqrt{\lambda_2})^2} \sum_A N_c f_A^{(u)4} \right] \hspace{1cm} \times \rho^4 \left( \ln \frac{\rho^2}{M_R} - \frac{25}{6} \right), \hspace{1cm} (2.9)$$

---

6 We choose the sign of $\lambda_5$ to be negative. It can be arbitrary chosen by a phase convention of the Higgs field.
where the renormalization point is defined by
\[
\left[ \frac{d^4 V_1}{d \rho^4} \right]_{\rho = M_R} = 0.
\]

We denote \(SU(2), U(1)\) and diagonalized Yukawa couplings by \(g, g'\) and \(f_A^{(u,d)}\) respectively. Note that the gauge-dependent terms vanish in the Gilder-Weinberg method. As mentioned before, VEV is determined by the minimum\(^7\) of \(V_1\), at which it is given by
\[
\rho^2 = v^2 = M_R^{2/11}.
\]

Eliminating \(M_R\) in (2.9) and rescaling \(\rho\) as \(\rho/v \equiv \varphi\), we have
\[
V_1 = \frac{1}{64\pi^2} B \varphi^4 (\ln \varphi^2 - \frac{1}{2}),
\]  
(2.10)

where
\[
B \equiv 6M_W^4(v) + 3M_Z^4(v) + 2M_{H^\pm}(v) + M_h^4(v) + M_A^4(v) \\
- 4 \sum_A (N_c M_A^{(u)}(v) + N_c M_A^{(d)}(v) + M_A^{(l)}(v)).
\]  
(2.11)

Here various kinds of particle masses are defined as
\[
M_W^2(v) \equiv \frac{g^2}{4} v^2, \quad M_Z^2(v) \equiv \frac{g^2 + g'^2}{4} v^2, \\
M_{H^\pm}^2(v) \equiv \frac{1}{2} (\sqrt{\lambda_1 \lambda_2 + \lambda_3}) v^2, \quad M_h^2(v) \equiv \sqrt{\lambda_1 \lambda_2} v^2, \\
M_A^2(v) \equiv -\lambda_5 v^2, \quad M_A^{(u)}(v) \equiv \frac{\sqrt{\lambda_1}}{\sqrt{\lambda_1 + \lambda_2}} f_A^{(u)} v^2, \\
M_A^{(d)}(v) \equiv \frac{\sqrt{\lambda_2}}{\sqrt{\lambda_1 + \lambda_2}} f_A^{(d)} v^2, \quad M_A^{(l)}(v) \equiv \frac{\sqrt{\lambda_2}}{\sqrt{\lambda_1 + \lambda_2}} f_A^{(l)} v^2.
\]  
(2.12)

The scalon, which is a particle associated with scale invariance in this model, becomes massive after spontaneous symmetry breaking. The scalon mass, \(M_S\) is defined by the inverse propagator evaluated at zero momentum and is given by [9],
\[
M_S^2 = \left[ \frac{d^2 V_1}{d \rho^2} \right]_{\rho = v} = \frac{G_F}{4\sqrt{2} \pi^2} \left[ 6M_W^4(v) + 3M_Z^4(v) + 2M_{H^\pm}(v) + M_h^4(v) + M_A^4(v) - 4 \sum_A M_A^{(l)}(v) - 4N_c \sum_A M_A^{(u)}(v) - 4N_c \sum_A M_A^{(d)}(v) \right].
\]  
(2.13)

\(^7\) The coefficient of \(\rho^4 (\ln \frac{\rho^2}{M_R} - \frac{25}{6})\) must be positive for \(V_1\) to be lower bounded.
Here we give sample sets of particle masses, which are used in later numerical analysis. Recent precision electroweak measurements of various observables give us important information on the top quark mass $M_t$. It is known, by comparing precision measurements with higher-order corrections in the standard model, that $M_t \simeq 131^{+47}_{-28}$ GeV \cite{10} for the Higgs mass range 60GeV–1000GeV. We assume $M_t = 140$ GeV throughout our analysis. The other fermion masses are neglected in numerical calculations. On the other hand, masses of Higgs bosons are not so strictly constrained by experiments due to their feeble contributions to electroweak observables. In our model there are four kinds of scalar particles, charged Higgs $M_{H^\pm}$, CP odd Higgs $M_A$, neutral Higgs (CP even) $M_h$, and scalon $M_S$. All these masses are free parameters\textsuperscript{8}. These scalars have not been detected at laboratories but effects of these particles can enter into the electroweak observables, which are well measured now, through loop diagrams \textsuperscript{9}. The scalon can be regarded as the usual physical Higgs boson in the standard model. Thus it must be heavier than about 60 GeV. In this paper we take the following two sets of scalar masses for illustrative calculations:

set I $\ (M_{H^\pm}, M_A, M_h : M_S) = (350, 250, 200 : 80.95)$ GeV,
set II $\ (M_{H^\pm}, M_A, M_h : M_S) = (450, 350, 250 : 142.19)$ GeV.

For the above scalar masses all the quartic couplings are within perturbative ranges\textsuperscript{10}:

$$0.04 < \frac{|\lambda_i|}{4\pi} < 0.23 \quad (i = 1 \sim 5) \quad \text{for set I,}$$

$$0.08 < \frac{|\lambda_i|}{4\pi} < 0.45 \quad (i = 1 \sim 5) \quad \text{for set II.}$$

\textbf{2.2. One-loop effective potential at finite temperature}

The finite-temperature part of the effective potential is given by the following prescription\textsuperscript{5},

$$\int \frac{d^4 k}{(2\pi)^4} \rightarrow \int_k = i T \sum_n \int \frac{d^3 k}{(2\pi)^3} \ ,$$

$$k_0 \rightarrow \omega_n \equiv \begin{cases} 
\frac{2 \pi n i T}{\pi (2n + 1) i T} & \text{for boson} \\
\frac{2 \pi n i T}{\pi (2n + 1) i T} & \text{for fermion}
\end{cases}$$

\textsuperscript{8} $M_S$ is determined from (2.13) once we fix $M_{H^\pm}, M_A, M_h$ and $M_t$.

\textsuperscript{9} We are going to analyze effects of these new particles on electroweak observables in forthcoming paper.

\textsuperscript{10} We assume that $\lambda_1$ and $\lambda_2$ are the same order.
where \( n \) runs over \( 0, \pm 1, \pm 2 \cdots \). Then one-loop contributions at finite temperature can be written as
\[
I_B(a) = \frac{T^4}{2\pi^2} \int_0^\infty dx \ x^2 \ln(1 - e^{-(x^2 + a^2)^{1/2}}),
\]
\[
I_F(a) = \frac{-4T^4}{2\pi^2} \int_0^\infty dx \ x^2 \ln(1 + e^{-(x^2 + a^2)^{1/2}}).
\] (2.14)

Here \( a^2 \equiv \varphi^2 M^2(\nu)/T^2 \) with \( M(\nu) \) being a mass of the relevant particle, and \( B(F) \) stands for boson (fermion). Thus the finite-temperature part of the effective potential is
\[
\tilde{U}^T(\varphi, T) = 6I_B(\varphi M_W/T) + 3I_B(\varphi M_Z/T) \\
+ 2I_B(\varphi M_{H^\pm}/T) + I_B(\varphi M_A/T) \\
+ I_B(\varphi M_h/T) + I_B(0) + 2I_B(0) \\
+ \sum_A [3I_F(\varphi M_A^{(u)}/T) + 3I_F(\varphi M_A^{(d)}/T) + I_F(\varphi M_A^{(l)}/T)] \\
+ 16I_B(0) + \frac{3}{2}I_F(0),
\] (2.15)

where we have set \( N_g = N_c = 3 \). We have added \( \varphi \)-independent contributions for completeness: \( 16I_B(0) \) for gluons, \( 2I_B(0) \) for photon, \( I_B(0) \) for scalar and \( (3/2)I_F(0) \) for neutrinos. Gauge-dependent terms vanish in \( \tilde{U}^T \) as in \( V_1 \). For numerical analysis, we ignore masses of fermions except for the top quark mass, and normalize the potential \( \tilde{U}^T(\varphi, T) \) as
\[
\tilde{V}^T(\varphi, T) \equiv \tilde{U}^T(\varphi, T) - \tilde{U}^T(\varphi = 0, T).
\] (2.16)

Eventually the effective potential at finite temperature in one-loop approximation is written as
\[
V(\varphi, T) = V_1(\varphi) + \tilde{V}^T(\varphi, T),
\] (2.17)

where \( V_1 \) is given by (2.10) and (2.11), and \( \tilde{V}^T \) by (2.15) and (2.16).

3. The Phase Transition

3.1. The critical temperature

In this section we investigate phase structure of our model, and determine critical temperature \( T_C \) by numerical analysis for various sets of scalar masses including set I and set II. In the case of the second-order phase transition, \( T_C \) is defined as a point at which the second derivative of the finite-temperature effective potential at the origin vanishes. On
the other hand $T_C$ in the first-order transition is a temperature at which there appear two degenerate minima in the potential. It is obvious that the potential (2.17) is a smooth real function of $\varphi$ at any $T$. We can, therefore, find nature of phase transition by calculating (2.17) directly without further approximations. Curves of the effective potential for various temperatures are depicted in fig. 1 (mass parameter set I), and fig. 2 (mass parameter set II). These figures show clearly that the phase transition is of first order. We find $T_C \simeq 67.5$ GeV for set I and $T_C \simeq 97.6$ GeV for set II. The critical temperatures for various sets of scalar masses are given in table 1. We see that $\varphi M_{\text{max}}(v)/T_C > 1$ for $\varphi \sim \varphi_+$, where $M_{\text{max}}(v)$ is the heaviest scalar mass and $\varphi_+ (\neq 0)$ the order parameter at a local minimum. That is, high-temperature expansion cannot be adopted in a parameter region of interest.

In the standard model, it is known that the condition not to wash out baryon density after EWPT is roughly expressed as [11]

$$\phi_+(T_C)/T_C > 1.4. \quad (3.1)$$

Here $\phi_+(T_C)$ is written as $v\varphi_+(T_C)$ in our notation. One can see from fig. 1 and fig. 2 that $\phi_+(T_C)/T_C$ is large due to low $T_C$ and large $\varphi_+(T_C) \sim 1$. The inequality (3.1) is satisfied for almost all cases in our model. Thus, if the sphaleron mass is the same order as in the single-doublet case, generated or primordial baryon number is preserved after EWPT. Of course the sphaleron solution does not exist at the classical level in the massless-doublet model. It is conceivable in principle, however, that the solution exists if one takes into account the one-loop potential $V_1(\varphi)$, because a mass scale appears as a renormalization scale or as the VEV $v$.

3.2. The critical bubble

It is well known that the bubble is nucleated in symmetric phase when the phase transition is of first order. The rate of bubble nucleation per unit volume is given by a formula[11]:

$$\Gamma_b = T^4 \left(\frac{F}{2\pi T}\right)^{3/2} e^{-F/T}, \quad (3.2)$$

where $F$ is the three-dimensional $O(3)$-invariant bubble action given by

$$F = \int d^3x \left[ \frac{1}{2}(\nabla \phi)^2 + V(\phi, T) \right]. \quad (3.3)$$
In order to convert the Universe into the broken phase, the bubble nucleation rate $\Gamma_b$ must at least be larger than that of the expansion of the Universe, $\Gamma_H \sim T/m_p$. The dominant contribution to $\Gamma_b$ is given by minimizing $F$. The critical bubble is the one which minimizes $F$. So we must compute the critical bubble action. To compute the critical bubble, we must solve the Euler-Lagrange equation given by

$$\frac{d^2 \phi}{dr^2} - \frac{2}{r} \frac{d\phi}{dr} + \frac{\partial V}{\partial \phi} = 0. \quad (3.4)$$

Substituting the solution of (3.4) back into (3.3), we obtain the free energy $F_C$ of the critical bubble. But in general it is very difficult to solve (3.4) analytically. In particular, in our case $V$ is given by the integral form so that even a numerical analysis is complicated. Instead of solving (3.4), we make some approximations to obtain the critical bubble. Given a potential as in fig. 3, which shows a potential below the critical temperature, we first minimize $F$ with respect to shape[11]. It is expected that the solution takes a spherically symmetric shape. Then $F$ can be approximately written as

$$F \simeq 4\pi R^2 \delta \left( \frac{1}{2} \left( \frac{\phi_+}{\delta} \right)^2 + V_{\text{max}} \right) - \frac{4\pi}{3} R^3 \epsilon, \quad (3.5)$$

where $R$ is the radius of the bubble and $\delta$ the thickness of the bubble wall. The depth $\epsilon (>0)$ at the absolute minimum, $V_{\text{max}}$ and $\phi_+$ are defined in fig. 3. Next we extremize $F$ with respect to $\delta$ and $R$ to obtain

$$\delta^2 = \frac{\phi_+^2}{2V_{\text{max}}}, \quad R = \frac{2\phi_+}{\epsilon} \sqrt{2V_{\text{max}}}. \quad (3.6)$$

Thus the free energy of the critical bubble $F_C$ is given by

$$F_C = \frac{16\pi}{3\epsilon^2} \left( \phi_+ \sqrt{2V_{\text{max}}} \right)^3. \quad (3.7)$$

In fig. 4 and fig. 5 we plot $F_C/T$ against temperature for scalar-mass sets I and II, respectively. The criterion, $\Gamma_C > \Gamma_H$ implies roughly $F_C/T < 145$, where $\Gamma_C \equiv \Gamma_b(F = F_C)$. At temperatures below the dotted lines drawn in fig. 4 and fig. 5, the bubble can nucleate in the symmetric phase and the Universe can be converted into the broken phase. The “critical” temperatures at which the bubble can nucleate are about 52.5 GeV and 80.5 GeV for sets I and II, respectively. These temperatures are close to the critical temperature for each case.
4. Conclusions and Discussions

We have studied the one-loop effective potential of electroweak theory with two massless Higgs doublets at finite temperature. We obtained the effective potential for various temperatures for two sets of scalar masses without high-temperature expansion. We have found that the phase transition in this model is of first order. We also calculated the free energy of the critical bubble with some approximations and found that the bubble nucleation can occur a little below the critical temperature.

In order to generate baryon number at the EWPT, it is necessary for the model to satisfy the Zakharov’s three conditions, (i), (ii) and (iii). In our model the condition (iii) is satisfied as shown in section 3. The condition (i) is obviously satisfied in our model because the gauge structure of this model is the same as that of one-Higgs-doublet model. Since there is no new CP violating phase in the Higgs sector in our model, the original Kobayashi-Maskawa phase is the only source of CP violation, though the condition (ii) is satisfied. It seems that our model is very attractive to generate baryon number at the EWPT. But there are some questions to be answered. We discuss these points below. Detailed analysis will be given in our forthcoming paper.

The first question is how we understand the sphaleron induced anomalous process in the broken phase in this model. The sphaleron is a solution of classical equations of motion in electroweak theory. Its mass (sphaleron energy) gives the height of energy barrier between topologically inequivalent vacua. If there are no tree-level mass terms of the Higgs fields, which corresponds to our model, no such kinds of solution exist. That is, no sphaleron solution exists in our model at the classical level. We expect, however, the solution may exist if we consider $V_1(\varphi)$, as mentioned in the previous section. The sphaleron solution in two-Higgs-doublet model with tree-level mass terms was studied in ref.[12] by Kastning et al. From their analysis we see that the dependence of the sphaleron energy on the tree-level mass terms is not so significant and that the energy mainly depends on the VEV of the Higgs fields and on the gauge coupling. So it is quite possible that the rate of the anomalous process in this model in the broken phase may be estimated by using the result of the sphaleron solution in the model with tree-level mass terms. The second question is whether observed baryon asymmetry can be realized or not. In our model rather strong first-order phase transition occurs and the latent heat is released after the phase transition. This latent heat is converted into the entropy factor which has much effect on baryon asymmetry. If the entropy is very large, there is a possibility to wash out produced
baryon asymmetry. Fortunately, we found in section 3 that the “critical” temperatures at which the bubble begins to nucleate are very close to the critical temperatures. Therefore we expect that the entropy generation is not so significant that baryon density may not be washed out, because the period of supercooling is short. It is also an important problem to clarify how the magnitude of the CP violation in the standard model affects on the baryon number in our model. In addition to above two questions, we must analyze many problems to confirm scenarios of electroweak baryogenesis. Specifically, detailed understanding of the dynamics of the EWPT must be developed. Understanding the nucleation of the bubbles of the broken-symmetry phase is one of the most important subjects. At this stage the bubble nucleation and its growth are not so well-understood even in the standard model.

There are many open problems to be studied in two massless Higgs-doublet model, but we believe that this model can provide many attractive features for electroweak baryogenesis.

**Acknowledgements**

The authors would like to express their cordial gratitude to S. Otsuki, F. Toyoda and other colleagues at Saga, Kyushu and Kinki Universities for discussions and encouragement. One of the authors (K.F.) is partially supported by the Grant-in-Aid for Encouragement of Young Scientist of the Ministry of Education, Science and Culture (No. 05740182).

**References**

Table Caption

Table 1. The critical temperatures for various sets of scalar masses. We assume $M_t = 140$ GeV. GeV unit is used.

Figure Captions

Fig. 1. The effective potential for scalar mass set I. The critical temperature is 67.5 GeV.

Fig. 2. The effective potential for scalar mass set II. The critical temperature is 97.6 GeV.

Fig. 3. The effective potential below the critical temperature.

Fig. 4. The free energy of the critical bubble divided by $T$ for set I vs. temperature in GeV unit.

Fig. 5. The free energy of the critical bubble divided by $T$ for set II vs. temperature in GeV unit.
<table>
<thead>
<tr>
<th>$M_{H^\pm}$</th>
<th>$M_A$</th>
<th>$M_h$</th>
<th>$M_S$</th>
<th>$T_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>350</td>
<td>350</td>
<td>73.57</td>
<td>65.7</td>
</tr>
<tr>
<td>150</td>
<td>350</td>
<td>350</td>
<td>74.93</td>
<td>65.0</td>
</tr>
<tr>
<td>150</td>
<td>400</td>
<td>250</td>
<td>74.22</td>
<td>64.8</td>
</tr>
<tr>
<td>250</td>
<td>350</td>
<td>70</td>
<td>62.49</td>
<td>57.9</td>
</tr>
<tr>
<td>250</td>
<td>350</td>
<td>150</td>
<td>63.29</td>
<td>57.8</td>
</tr>
<tr>
<td>250</td>
<td>400</td>
<td>400</td>
<td>107.06</td>
<td>81.2</td>
</tr>
<tr>
<td>350</td>
<td>70</td>
<td>350</td>
<td>92.41</td>
<td>74.8</td>
</tr>
<tr>
<td>350</td>
<td>250</td>
<td>200</td>
<td>80.95</td>
<td>67.5</td>
</tr>
<tr>
<td>350</td>
<td>400</td>
<td>400</td>
<td>126.88</td>
<td>89.9</td>
</tr>
<tr>
<td>400</td>
<td>250</td>
<td>250</td>
<td>107.06</td>
<td>81.1</td>
</tr>
<tr>
<td>400</td>
<td>350</td>
<td>350</td>
<td>126.88</td>
<td>89.9</td>
</tr>
<tr>
<td>450</td>
<td>350</td>
<td>250</td>
<td>142.19</td>
<td>97.6</td>
</tr>
</tbody>
</table>

Table 1
Fig. 2

$V(\times 10^6 \text{GeV}^4)$

- $T=0$
- $T=70.00 \text{GeV}$
- $T=97.58 \text{GeV}$
- $T=120.0 \text{GeV}$

$V$ is plotted against $\varphi$ for different temperatures $T$. The y-axis represents $V$ scaled by $10^6 \text{GeV}^4$. The lines correspond to different temperature values as indicated in the legend.
Fig. 3
\log_{10}(F_C/T)