Rare $B$-decays and Heavy to Light Semileptonic Transitions in the Isgur and Wise Limit

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Abstract

From the experimental branching ratios for $B^- \to \rho^0 \ l^- \bar{\nu}_l$ and $D^+ \to \bar{K}^{*0}(K^0) \ e^+ \nu_e$ one finds, in the Heavy Quark Limit of $HQET$, $|V_{ub}| = (8.1 \pm 1.7) \times 10^{-3}$, larger but consistent with the actual quoted range $(2 - 7) \times 10^{-3}$. In the same framework one predicts for $R(B \to K^*\gamma) = (2 \pm 2) \times 10^{-2}$.

The study of the Cabibbo-Kobayashi-Maskawa [1] suppressed decays $B^- \to \rho^0 e^- \bar{\nu}_e$, interesting in itself for the determination of $|V_{ub}|$ [2], has been recently related by the spin-flavour symmetries of the HQET [3] (in the Heavy Quark Limit) to the rare $B$-decays [4][5]. In such a way the predictions for the branching ratio of the decay $B \to K^*\gamma$, which provide a test of the Standard Model [6], depend strongly on the value of $|V_{ub}|$, for which the experimental data about the $b \to u e \bar{\nu}_e$ (inclusive exclusive) decays give an information depending also from the theoretical approach followed to evaluate the corresponding amplitudes.

Here we shall reach rather firm conclusions by following the suggestion of N. Isgur and M.B. Wise of relating the involved form factors by flavour symmetry [7].

The $\bar{B} \to \rho$ semileptonic decays are chosen rather than the decays with $\pi$ in the final state, because in this last case the upper limit of the invariant mass of the final leptons is very near to the $B^*$ resonance (which is expected equal to $m_B$ in the HQL); as a consequence the pole of the $B^*$ dominates the spectrum in that region so that the prediction of HQET fails near to the no-recoil point [8].

In the first section we review the spectrum $B \to V l \nu_l$ and give the involved form factors. In the second we get $|V_{ub}|$ by comparing experiment with the theoretical predictions. In the third are discussed the rare $B$ decays in the HQL.

1. The spectrum in the invariant mass $q^2$ of the lepton pair for the semileptonic decays of a heavy meson with one vector meson in the final state ($H_j \to V_k l \nu_l$) is given by e.g. in [9] (neglecting lepton masses):

$$\frac{d\Gamma(H_j \to V_k e \nu_e)}{dq^2} = \frac{G_F^2 |V_{jk}|^2}{192\pi^3 m_H^2} \lambda(m_H^2, m_V^2, q^2)$$

$$\cdot \left[ \left| A_1^{(jk)}(q^2) \right|^2 2(m_H + m_V)^2 q^2 + \left( m_H^2 + m_V^2 \right)^2 \left( m_H - m_V - q^2 \right)^2 \right]$$

$$+ \left| A_2^{(jk)}(q^2) \right|^2 \frac{\lambda^2(m_H^2, m_V^2, q^2)}{4m_V^2(m_H + m_V)^2}$$

$$- A_1^{(jk)}(q^2) A_2^{(jk)}(q^2) \lambda(m_H^2, m_V^2, q^2) \frac{(m_H^2 - m_V^2 - q^2)}{2m_V^2}$$

$$+ \left| V^{(jk)}(q^2) \right|^2 \frac{2q^2}{(m_H + m_V)^2} \lambda(m_H^2, m_V^2, q^2) \right] ; \quad (1)$$

where

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz) , \quad (2)$$

$$q^2 = (p_H - p_V)^2 . \quad (3)$$
The form factors of the weak currents for an initial \( \bar{B} \) meson are:

\[
\langle V_j(\bar{\epsilon}, p_j) | A_\mu | \bar{B}(p) \rangle = (m_B + m_V) A_1^{(b)}(q^2) \left( \bar{\epsilon}_\mu - \frac{\bar{\epsilon} \cdot q}{q^2} q_\mu \right) - A_2^{(b)}(q^2) \frac{\bar{\epsilon} \cdot q}{m_V + m_B} \left( p^\mu_j + p^\mu - \frac{m_B^2 - m_V^2}{q^2} q^\mu \right) + 2m_V A_0^{(b)}(q^2) \frac{\bar{\epsilon} \cdot q}{q^2} q_\mu ,
\]

and are related, by HQET (leading order), to the corresponding ones for the process \( D \to V l \nu_l \) if \( V \) is a light vector meson.

N. Isgur and M.B. Wise [7] found as a consequence of flavour symmetry the relations between the weak form factors for \( \bar{B} \to K^* \) and \( D \to K^* \), which, for the our parameterization, imply:

\[
A_1^{(b)}(q_B^2) = C_{bc} \left( \frac{m_D + m_V}{m_B + m_V} \right) \sqrt{\frac{m_B}{m_D}} A_1^{(c)}(q_D^2) ,
\]

\[
A_2^{(b)}(q_B^2) = \frac{C_{bc}}{2} \sqrt{\left( \frac{m_D}{m_B} \right)^3 \left( \frac{m_B + m_V}{q_D^2} \right)} \left\{ y_2(m_D + m_V) A_1^{(c)}(q_D^2) + A_2^{(c)}(q_D^2) \left[ y_1 \frac{q_D^2}{m_D + m_V} - y_2(m_D - m_V) \right] - 2m_V y_2 A_0^{(c)}(q_D^2) \right\} ,
\]

\[
A_0^{(b)}(q_B^2) = \frac{C_{bc}}{4m_V} \sqrt{\left( \frac{m_D}{m_B} \right)^3} \left\{ \frac{2m_V x_2}{q_D^2} A_0^{(c)}(q_D^2) + \left[ 2 \left( \frac{m_B}{m_D} \right)^2 - \frac{x_2}{q_D^2} \right] (m_D + m_V) A_1^{(c)}(q_D^2) + \left[ m_D - m_V \right] \frac{x_1}{q_D^2} A_2^{(c)}(q_D^2) \right\} ,
\]

\[
V^{(b)}(q_B^2) = C_{bc} \frac{m_B + m_V}{m_D + m_V} \sqrt{\frac{m_D}{m_B}} V^{(c)}(q_D^2) ,
\]

where

\[
q_B^2 = (m_B v - p_V)^2 \quad q_D^2 = (m_D v - p_V)^2 \quad v^2 = 1 ,
\]

\[
y_1 = \frac{(m_B + m_D)}{m_D} \quad y_2 = \frac{(m_D - m_B)}{m_D} ,
\]

\[
x_1 = y_2 q_B^2 + y_1 (m_B^2 - m_V^2) \quad x_2 = y_1 q_B^2 + y_2 (m_B^2 - m_V^2) ,
\]

\[
C_{bc} \cong \left( \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right)^{-6/25} \cong 1.1 .
\]

It is worth noticing that the \( q_D^2 \) values corresponding to the physical region for \( q_B^2 \) are not located in the \( q^2 \) allowed range for \( D \to K^* \) semileptonic decays.
The symmetry breaking corrections to equations (6)-(9) are proportional in the full kinematical range to \(\alpha_s m_L / m_H\), where \(\alpha_s\) is the coupling constant of QCD and \(m_L\) is the mass of light quark coupled to \(b\) by the weak current; for the case considered here \(V \equiv \rho\), \(m_L = m_c\) and therefore we may safely neglect these corrections [10].

2. From the experimental data on the semileptonic \(D^+ \to K^0 (\bar{K}^0) \ell^+ \nu_\ell\) decays, within the pole approximation for the form factors, the E691 Collaboration [11] found the following values for their residua:

\[
A_0^{(cs)}(0) = 0.71 \pm 0.16, \quad A_2^{(cs)}(0) = 0.00 \pm 0.22, \quad V^{(cs)}(0) = 0.90 \pm 0.32,
\]

\[
A_1^{(cs)}(0) = \frac{1}{m_D + m_K^*} \left\{ 2m_K^* A_0^{(cs)}(0) + (m_D - m_K^*) A_2^{(cs)}(0) \right\}.
\]  

(11)

From \(SU(3)\) invariance we may identify the residua for \(D^0 \to K^{*-}\) with ones for \(D^0 \to \rho^-\) weak form factors and from equations (6)-(9) and the values given in equation (11) one may predict the form of the spectrum for

\[
\frac{d\Gamma(B \to \rho \ell^- \bar{\nu}_\ell)}{dq^2},
\]

and the rate

\[
\frac{1}{|V_{ub}|^2} \frac{\Gamma(B^- \to \rho^0 l^- \bar{\nu}_l)}{\Gamma(B^- \to \rho^0 l^- \bar{\nu}_l)} = 0.80 \times 10^{-11} \text{ GeV}.
\]  

(12)

From the measured branching ratio \(Br(B^- \to \rho^0 l^- \bar{\nu}_l) = (10.3 \pm 3.6 \pm 2.5) \times 10^{-4}\) [12] one obtains (neglecting the errors on values of the form factors in equation (11))

\[
|V_{ub}| = \sqrt{\frac{Br(B^- \to \rho^0 l^- \bar{\nu}_l)|_{\text{exp}}}{\Gamma(B^- \to \rho^0 l^- \bar{\nu}_l)/\tau_B}} = (8.1 \pm 1.7) \times 10^{-3}
\]

(13)

larger but still consistent, within the experimental uncertainties, with the value found from the experimental information on \(|V_{ub}/V_{cs}| = 0.10 \pm 0.03\) [2][13] and for the value \(|V_{cs}| = 0.043 \pm 0.003\) obtained from the study of semileptonic \(B\)-decays in the Heavy Quark Limit [14]:

\[
|V_{ub}| = (4.3 \pm 1.3) \times 10^{-3}.
\]  

(14)

The spectrum predicted is described in figure 1.

3. A high precision prediction about the rate of the rare decay \(B \to K^*\gamma\) [16], which is induced at one loop in the Standard Model, is very important, since the discrepancy between theory and experiment would be indirect evidence of new physics.

\[\text{If we ignore the location of } B^* \text{ resonance and calculate the spectrum and the rate of } B \to \pi \text{ semileptonic decay we obtain a linear dependance of } d\Gamma/dq^2 \text{ by the } \bar{q}^2. \text{ The result is very similar to the one obtained by J.G. Körner and G.A. Schuler [15].}\]
By relating $B \to K^*\gamma$ to the semileptonic $\bar{B} \to \rho$ decay, P.J. O’Donnell and H.K.K. Tung [5] obtained for the ratio

$$R(B \to K^*\gamma) = \frac{\Gamma(B \to K^*\gamma)}{\Gamma(b \to s\gamma)} = \frac{m_B^3}{m_s^3} \left(\frac{m_B^2 - m_{K^*}^2}{m_B^2 - m_{K^*}^2}\right)^3 \frac{1}{2} \left\{|F_1(0)|^2 + 4|F_2(0)|^2\right\}$$  \hspace{1cm} (15)

de the prediction, which differs from the one deduced by us \footnote{I am indebted to Patrick O’Donnell and Humphrey Tung for a clarifying communication.}

$$R(B \to K^*\gamma) \left(\frac{d\Gamma(\bar{B} \to \rho [\bar{p} \bar{\nu}_e])}{dq^2}\right)_{q^2=0}^{-1} = \frac{192\pi^3}{G_F^2} \frac{1}{|V_{ub}|^2} \left(\frac{m_B^2 - m_{K^*}^2}{m_B^2 - m_{\rho}^2}\right)^3 \left(\frac{m_B^2 - m_{K^*}^2}{m_B^2 - m_{\rho}^2}\right)^3 |I|^2,$$  \hspace{1cm} (16)

because we invoke $SU(3)_{u,d,s}$ symmetry for $V^{(u)}(0)$ rather than for $T_1^{B-V}(0) = V^{(s)}(0)/(m_B + m_{\nu})$ as in [5].

Note that $|I| = 1$ in the HQL and the corrections to the prediction of HQET in relating the form factor in equations (4)-(5) (with $j = u$ or $s$) to the one of the matrix element $\langle V_j|\sigma_{\mu\nu} q^\nu b_R|\bar{B}\rangle$ are expected to be small (cfr. [5]) and we neglect them.

In effect the conclusions of the paper of P.J. O’Donnell and H.K.K. Tung are that the corrections to the assumption that $K^*$ and $\rho$ are heavy mesons are negligible (at least to relate the form factors of the matrix elements of the weak currents and the $\bar{s}\sigma_{\mu\nu} b_R$ and $\bar{u}\sigma_{\mu\nu} b_R$ operators respectively at $q^2 = 0$). The universality of the Isgur-Wise function, in principle, allow us to give, without relating the ratio $R$ to the spectrum of $B \to \rho l \nu_l$, the $R(B \to K^*\gamma)$ in terms of the Isgur-Wise function $\xi(w^2)$ extracted, for example, by the experimental data on charmed semileptonic $B$-decays (cfr. for example [14]). In such case the following relations hold \footnote{The first relation is more general. For example it is necessary to avoid an unphysical pole in $q^2 = 0$ for the $h(q^2)$ and $g_-(q^2)$ form factors introduced in [7].}

$$F_1(0) = 2F_2(0),$$  \hspace{1cm} (17)

$$F_1(q^2) = \frac{(m_B + m_{K^*})}{2\sqrt{m_B m_{K^*}}} \xi(w^2(q^2)).$$

But the value

$$w^2(q^2 = 0) \equiv \left(\frac{p_B}{m_B} - \frac{p_{K^*}}{m_{K^*}}\right)^2_{q^2=0} = -\left(\frac{m_B - m_{K^*}}{m_B m_{K^*}}\right)^2$$  \hspace{1cm} (18)

is too far from the physical range of $\bar{B} \to D^{(*)}$ semileptonic decays and the predictions depend strongly on the behaviour assumed for $\xi(w^2)$ \footnote{From the data on $D \to K(K^*)\ell^+\nu_\ell$ decays A. Ali and T. Mannel [17] extracted the values $w_3 = 1.8$ and $\beta = 0.25$ respectively for the pole and exponential parameterization for $\xi(w^2)$.}

A more reliable prediction for the ratio $R$ from equation (16) is obtained by relating the spectra of $B \to \rho l \nu_l$ and $D \to K^*$ semileptonic decays.
Following the hypothesis of section 2 about the residua of the involved form factors, \( SU(2) \) flavour symmetry and (cfr. (1))

\[
\frac{dI(B \to \rho \Gamma)}{dq^2} \bigg|_{q^2=0} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^2} \left( \frac{m_B^2 - m_{\rho^*}^2}{4 m_{\rho^*}^3} \right)^3 \left| (m_B + m_{\rho}) A_1^{(b)}(0) - (m_B - m_{\rho}) A_2^{(b)}(0) \right|^2
\]

\[
= \frac{G_F^2 |V_{ub}|^2}{192\pi^3} \frac{(m_B^2 - m_{\rho^*}^2)^3}{m_B^3} \cdot \left| A_0^{(b)}(0) \right|^2,
\]

from the equations (16), (8) (or (6) and (7)) and (11) we derive

\[
R(B \to K^\ast \gamma) = \frac{m_b^3}{(m_b^2 - m_{\rho^*}^2)^3} \frac{(m_B^2 - m_{\rho^*}^2)^3}{m_B^3} \cdot \left| A_0^{(b)}(0) \right|^2
\]

(20)

giving \((m_b = 5 \text{ GeV} \text{ and } m_s = 0.55 \text{ GeV})\)

\[
R(B \to K^\ast \gamma) = (35 \pm 28) \times 10^{-2}.
\]

(21)

The central value is very near to the one given in [17] in the polar approximation for \( \xi(w^2) \). The error quoted in (21) depends on the large error in the determination of \( A_2^{(c)}(0) \).

Obviously one expects the assumption that \( K^\ast \) and \( \rho \) are heavy less reliable than \( HQL \) for \( b \) and \( c \) quarks; thus the previous result can be modified by taking \( I \) from ref. [5]:

\[
R(B \to K^\ast \gamma) = (35 \pm 28) \times 10^{-2} \cdot |I|^2 = \begin{cases} 
(42 \pm 33) \times 10^{-2} & \text{for } I = 1.09 \\
(49 \pm 39) \times 10^{-2} & \text{for } I = 1.18 
\end{cases}
\]

(22)

It is worth recalling that the static limit for \( b \) (\( \gamma_0 b = b \)) and \( SU(2)_{bc} \) heavy flavour symmetry imply

\[
R(B \to K^\ast \gamma) = \frac{G_{b,c}^2 m_b^3}{(m_b^2 - m_{\rho^*}^2)^3} \frac{(m_B^2 - m_{K^*}^2)^3}{4 m_B^3 m_D} \left| (m_D + m_{K^*}) A_1^{(c)}(q_D^2) - \left( \frac{m_B^2 - m_{K^*}^2}{m_D + m_{K^*}} \right) \frac{m_D}{m_B} V^{(c)}(q_D^2) \right|^2
\]

\[
= (2 \pm 2) \times 10^{-2}.
\]

(23)

The result is substantially equivalent to the one dictated by \( HQL \) for \( b, c \) and \( s \) and a wave function model for the Isgur-Wise function [18].

We derived, in the \( HQL \) for \( b \) and \( c \), the spectrum predicted for \( B \to \rho \nu \overline{\nu} \) and, comparing theory and experiment, we give \( |V_{ub}| \).

Also relating \( R(B \to K^\ast \gamma) \) to the \( \frac{dI(B \to \rho \nu \overline{\nu})}{dq^2} \) at \( q^2 = 0 \) we obtained the ratio \( R \) in terms of \( A_1^{(b)}(0) \) and \( A_2^{(b)}(0) \) (or \( A_0^{(b)}(0) \)), estimated by extrapolation from the corresponding form factors for \( D \to K^* \) semi-leptonic decay.
We are more confident on the prediction for $R$ coming from the static limit for $b$ and $SU(2)_c$ heavy flavour symmetry.

A more precise determination of $c \to s$ weak form factors is needed for a more precise evaluation for $R(B \to K^{*-\gamma})$.

Acknowledgement

I am indebted to Professor Franco Buccella for many enlightening discussions.

References


and *ibidem*, **65**,1990,2630;


Erratum ibidem, B 274, 1992, 526;

Figure Caption

**Figure 1.** We report the spectrum predicted for the decay $B^{-} \to \rho^{0} l^{-}\bar{\nu}_{l}$. 