Testing the Higgs system at a photon-photon collider

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Abstract

The level of sensitivity of the processes $\gamma \gamma \rightarrow ZZ$, $\gamma \gamma \rightarrow W^+W^-$ and $\gamma \gamma \rightarrow t\bar{t}$ to the Higgs sector of the Standard Model Lagrangian in the energy region between 200 GeV and 1 TeV is examined. The elementary Higgs boson is taken to have a mass less than 1 TeV. Sizeable effects are found in the $ZZ$ and $t\bar{t}$ channels if the incoming photons have the same helicity. Also the possibility that the elementary Higgs boson does not exist is examined. Assuming new physics to show up in the TeV energy region the cross sections are evaluated according to the heavy Higgs model. For center of mass energy values close to 1 TeV interesting effects are found in the $t\bar{t}$ channel if the photons have the same helicity. The limit of large Higgs mass is not unique. The parametrization of this arbitrariness may be interpreted as a representation of the new physics. The effects for the processes $\gamma \gamma \rightarrow ZZ$ and $\gamma \gamma \rightarrow t\bar{t}$ are investigated. These effects may be correlated to a possible resonance in $WW$ scattering in the TeV region.
1. Introduction

Lately photon-photon ($\gamma \gamma$) physics has received considerable attention. This is due to the work by Ginzburg et. al. [1] and others [2], who have shown that it is possible to convert a high energy $e^+e^-$ collider into a high energy $\gamma \gamma$ collider. This is done through backscattering of low energy photons off the initial $e^+(e^-)$ beam, where the backscattered photon receives up to 80% of the electron beam energy. It is expected that the $\gamma \gamma$ collider, just like the $e^+e^-$ collider, will be a facility that is able to do precision measurements. An interesting feature is that the energy distribution of the resulting $\gamma$ beam is increasingly monochromatic with increasing energy. Accuracy may thus be better at higher energies. At this stage, however, it is of course not exactly known what accuracy can be achieved.

In the study of the Higgs sector of the Standard Model $\gamma \gamma$ physics may prove to be a helpful tool, as the elementary Higgs boson may be produced indirectly through a vector boson loop or a top quark loop, see fig.1. The Higgs subsequently decays into a pair of charged $W$ or neutral $Z$ bosons, or a top quark pair (assuming the Higgs mass is sufficiently large). The reaction of fig.1 with initial electrons instead of photons gives a cross section proportional to the ratio of electron mass and vector boson mass, $m_e/M$, and is thus negligible. Another example where $\gamma \gamma$ physics may be complementary to $e^+e^-$ physics is the study of anomalous tree level couplings. For instance the $\gamma \gamma W^+W^-$ coupling may be tested through the process $\gamma \gamma \rightarrow W^+W^-$ [3]. In this paper we do not consider anomalous $\gamma$, $W^\pm$ and $Z$ couplings.

The Higgs sector of the Standard Model Lagrangian is needed to ensure the renormalizability of the theory. As is well known there are difficulties with respect to the cosmological constant. The model requires the existence of an elementary Higgs boson, of which there is so far no experimental evidence. The Higgs sector may be considered suspect, and perhaps the Higgs boson as predicted by the Standard Model Lagrangian does not exist as such. If this is indeed the case then as yet unknown new physics must exist that takes the place of the Higgs system. According to studies of longitudinally polarized vector boson scattering new physics must show up in the TeV region [4-6] if the mass of the elementary Higgs boson is not below 1 TeV. For example QCD-like models predict for longitudinal vector boson scattering the occurrence of a resonance in the isospin $I = 1$ channel at around 2 TeV [7,8]. Analysis [9,10] of longitudinal vector boson scattering shows that the occurrence of such a resonance is model dependent. It may be parametrized [9] by means of the Lehmann $\beta$ parameter [11]. If $\beta$ could be measured in $\gamma \gamma$ processes then that could be used to guess probable behaviour of the $WW$ scattering cross section in the $I = 1$ channel. Behaviour in other channels is dependent on other parameters.

It is perhaps useful to explain the question of parametrization in some detail. In the method of large Higgs mass the first parameter occurring in the results is $\ln m^2$, where $m$ is the Higgs mass. This logarithm occurs not only in $WW$ scattering but also in the $\rho$-parameter and in $WW$ production by photons. It is of prime interest if this parameter takes the same value in all those cases. In the heavy Higgs model differences would have to come from yet higher order terms, which in one study [12] have been shown to be small if
the Higgs mass is below 3 TeV. Use of an additional particle, the $U$-particle [13], introduces a new parameter that may be identified, within this model, with Lehmann's $\beta$ parameter. Then possible resonance effects in $WW$ scattering may be related to entirely different effects in $\gamma\gamma$ processes. Such effects appear in a significant way for $t\bar{t}$ and longitudinal $ZZ$ final state processes.

The experiments that explore the energy region between 100 GeV and 1 TeV must identify one of the following possibilities:
- there is an elementary Higgs with a mass below 1 TeV;
- there is new physics.

The first possibility might by identified by direct Higgs search. In the second case some model is required. Here we use as a model the Standard Model Lagrangian in the limit of a heavy Higgs with an additional Higgs interaction.

This paper is organized as follows. In section 2 we define the kinematics and write the equation for the cross-section of the process $\gamma\gamma \to XX$, where $XX$ represents a $ZZ$, a $W^+W^-$ or a $t\bar{t}$ pair. The numerical values used for the various parameters are listed. In section 3 the Higgs system is discussed in some detail. In particular the heavy Higgs model and the introduction of an additional interaction is described. In section 4 we present a calculation of the cross section for the process $\gamma\gamma \to ZZ$ to lowest non-zero order in perturbation theory. We examine the cross section as a function of the Higgs mass and compare the effects for the case of a low mass Higgs with those for the case of our heavy Higgs model. In section 5 we discuss the process $\gamma\gamma \to W^+W^-$ and in section 6 the process $\gamma\gamma \to t\bar{t}$. Section 7 contains a summary and a discussion of the results.

Our metric is such that $p^2 = -m^2$ for an on mass-shell particle with mass $m$ and momentum $p$.

2. Kinematics and definitions

The process $\gamma(k_1)\gamma(k_2) \to X(p_1)\bar{X}(p_2)$ is displayed in fig.2. Here $XX$ represents the final $ZZ$, $W^+W^-$ or $t\bar{t}$ pair. The momenta of the incoming photons $k_1$ and $k_2$ are defined to be aligned along the $z$-axis, thus

$$k_1 = E(0,0,1,i), \quad k_2 = E(0,0,-1,i), \quad (2.1)$$

with $E$ the photon beam energy. The corresponding transverse circular polarization vectors are defined by

$$\epsilon^+(k_1) = \epsilon^-(k_2) = \frac{1}{\sqrt{2}}(1,i,0,0), \quad \epsilon^-(k_1) = \epsilon^+(k_2) = \frac{1}{\sqrt{2}}(1,-i,0,0). \quad (2.2)$$

The polarization vectors are orthogonal to the momentum vectors, thus

$$k_1\epsilon(k_1) = k_2\epsilon(k_2) = 0. \quad (2.3)$$
In addition we have
\[ k_1 \epsilon(k_2) = k_2 \epsilon(k_1) = 0. \] (2.4)

The \( J = 0 \) state is for incoming \( \epsilon(k_1)^\pm \epsilon(k_2)^\pm \) polarization states and the \( J = 2 \) state is for incoming \( \epsilon(k_1)^\pm \epsilon(k_2)^\mp \) polarization states. The momentum vectors \( p_1 \) and \( p_2 \) of the outgoing vector boson or top quark pair are a function of the scattering angle \( \theta \). We may write
\[ p_1 = E(\beta_x \sin \theta, 0, \beta_x \cos \theta, i), \quad p_2 = E(-\beta_x \sin \theta, 0, -\beta_x \cos \theta, i), \] (2.5)

where the subscript \( x \) denotes the particle considered. For the neutral vector boson, the charged vector boson and the top quark we have, respectively,
\[ \beta_z = \sqrt{1 - \frac{4M_0^2}{s}}, \quad \beta_w = \sqrt{1 - \frac{4M^2}{s}}, \quad \beta_t = \sqrt{1 - \frac{4m_t^2}{s}}, \] (2.6)

where as usual \( s = 4E^2 \) is the center of mass energy squared. If the final state is a pair of vector bosons we need to define in addition to the two transverse polarization vectors \( \epsilon^+ \) and \( \epsilon^- \), the longitudinal polarization vector \( \epsilon^0 \):
\[ \epsilon^+(p_1) = \epsilon^-(p_2) = \frac{1}{\sqrt{2}}(\cos \theta, -i, -\sin \theta, 0), \]
\[ \epsilon^-(p_1) = \epsilon^+(p_2) = \frac{1}{\sqrt{2}}(\cos \theta, i, -\sin \theta, 0), \]
\[ \epsilon^0(p_1) = \frac{E}{M_v}(\sin \theta, 0, \cos \theta, i\beta_v), \]
\[ \epsilon^0(p_2) = \frac{E}{M_v}(-\sin \theta, 0, -\cos \theta, i\beta_v). \] (2.7)

The index \( v \) represents the charged or neutral vector boson, with \( M_z = M_0 \) and \( M_w = M \). The polarization vectors are orthogonal to the corresponding momentum vectors, thus \( p_i \epsilon(p_i) = 0 \) for each \( i = 1, 2 \) with \( \epsilon = \epsilon^\pm, \epsilon^0 \). For a specific helicity configuration of the photons and vector bosons the amplitude for \( \gamma \gamma \rightarrow VV \), with \( VV \) representing the \( ZZ \) or the \( W^+W^- \) pair, may be written as
\[ A^v (\lambda_1, \lambda_2, \lambda_3, \lambda_4) = \epsilon^\lambda_1(k_1)\epsilon^\lambda_2(k_2)\epsilon^\lambda_3(p_1)\epsilon^\lambda_4(p_2) \cdot A^{v\mu\nu}_{\alpha\beta\mu\nu}. \] (2.8)

There are altogether 36 different configurations, but not all of them are independent. Due to the relations
\[ A^v (\lambda_1, \lambda_2, \lambda_3, \lambda_4) = A^v (-\lambda_1, -\lambda_2, -\lambda_3, -\lambda_4), \] (2.9)

and
\[ |A^v (\lambda_1, \lambda_2, \lambda_3, \lambda_4)| = |A^v (\lambda_2, \lambda_1, \lambda_4, \lambda_3)|, \] (2.10)

12 independent amplitudes remain (in fact the number of independent amplitudes may be reduced to 8 by considering the replacement \( p \rightarrow -p \)). The differential cross-section for each helicity configuration is
\[ \frac{d\sigma^v (\lambda_1 \lambda_2 \lambda_3 \lambda_4)}{d\cos \theta} = \frac{\beta_v}{32\pi} \cdot |A^v (\lambda_1, \lambda_2, \lambda_3, \lambda_4)|^2 \cdot (389.352) \text{ pb} \quad \text{(energy in TeV).} \] (2.11)
When the total cross-section is calculated a factor 1/2 must be included for the process \( \gamma \gamma \rightarrow ZZ \) because of Bose statistics. Next we list the differential cross sections for the initial \( J = 0 \), 2 final \( TT \), \( TL \), \( LL \) helicity configuration states \( (T = \text{transverse}, \ L = \text{longitudinal}) \):

1. \( J = 0 \) (initial \( ++ = --- \) state only)

\[
\frac{d\sigma_{TT}^{+\nu}}{d\cos \theta} = \frac{d}{d\cos \theta} \left\{ \sigma^{+\nu}(++++) + 2\sigma^{+\nu}(+++-) + \sigma^{+\nu}(++--) \right\}
\]

\[
\frac{d\sigma_{TL}^{+\nu}}{d\cos \theta} = \frac{d}{d\cos \theta} \left\{ 2\sigma^{+\nu}(+++0) + 2\sigma^{+\nu}(+-0+) \right\}
\]

\[
\frac{d\sigma_{LL}^{+\nu}}{d\cos \theta} = \frac{d\sigma^{+\nu}(++00)}{d\cos \theta}
\]

(2.12)

2. \( J = 2 \) (initial \( +-- = --+ \) state only)

\[
\frac{d\sigma_{TT}^{+\nu}}{d\cos \theta} = \frac{d}{d\cos \theta} \left\{ \sigma^{+\nu}(+-+-) + 2\sigma^{+\nu}(+-++) + \sigma^{+\nu}(-+--) \right\}
\]

\[
\frac{d\sigma_{TL}^{+\nu}}{d\cos \theta} = \frac{d}{d\cos \theta} \left\{ 2\sigma^{+\nu}(+-+0) + 2\sigma^{+\nu}(+-0+) \right\}
\]

\[
\frac{d\sigma_{LL}^{+\nu}}{d\cos \theta} = \frac{d\sigma^{+\nu}(+-00)}{d\cos \theta}
\]

(2.13)

Note that we have used eqs. (2.9) and (2.10).

For the process \( \gamma \gamma \rightarrow t \bar{t} \), we consider final state unpolarized top quarks only and the amplitude is of the form:

\[
A^{t \bar{t}}(\lambda_1, \lambda_2) = \epsilon^t_{\alpha}(k_1)\epsilon^t_{\beta}(k_2) \cdot A^{t \bar{t}}_{\alpha\beta},
\]

(2.14)

with \( A^{t \bar{t}}(+++) = A^{t \bar{t}}(-,-) \) and \( A^{t \bar{t}}(+,-) = A^{t \bar{t}}(-,+). \) The corresponding differential cross-section for each helicity configuration is given by

\[
\frac{d\sigma^{t \bar{t}}(\lambda_1, \lambda_2)}{d\cos \theta} = \frac{N_c \beta_t}{32\pi s} \cdot |A^{t \bar{t}}(\lambda_1, \lambda_2)|^2 \cdot (389.352) \text{ pb \ (energy in TeV)}.
\]

(2.15)

\( N_c = 3 \) is the colour factor.

In the numerical evaluations presented in this paper, we used the following values for the relevant parameters:

\[
\alpha = \frac{e^2}{4\pi} = \frac{1}{128}; \quad \alpha_w = \frac{g^2}{4\pi} = \frac{1}{30};
\]

\( M = 80.22 \text{ GeV}, \) charged vector boson mass ;

\( s^2_w = 0.2326, \) sine squared of the weak mixing angle;

\( M_0 = 91.173 \text{ GeV}, \) neutral vector boson mass .

(2.16)
3. Vector bosons and the Higgs

3.a The conspiracy

So far, up to an energy of 100 GeV, still nothing is known about the Higgs sector in spite of the fact that the LEP experiments perform their measurements with very high precision. This is a consequence of the screening theorem [6]: when for a process the one loop correction is calculated due to a heavy Higgs (but still with a mass less than, say, 1 TeV), the dependence on the Higgs mass is only logarithmic. Quadratic dependence shows up at the two-loop level. For example the $\rho$ parameter including the heavy Higgs mass correction is given by [12]

$$
\rho = 1 - \frac{3\alpha}{16\pi e_w^2} \cdot \ln \left( \frac{m^2}{M^2} \right) + 9.49 \cdot 10^{-4} \cdot \frac{\alpha^2}{s_w^2 e_w^2 M^2} \cdot \frac{m^2}{M^2},
$$

where $m$ is the Higgs mass. The experimental value is

$$
\rho = 1 \pm 0.5\%. \quad (3.2)
$$

The error can still be reduced once more is known about the mass of the top quark.

Experiments performed at a center of mass energy greater than 170 GeV will be able to observe vector boson pair production. This opens up new possibilities in the study of the Higgs system, since the Higgs particle plays an important role concerning longitudinal vector boson interaction. This is quite easy to see. In the high energy limit the longitudinal polarization vector of eq.(2.7) is proportional to $E/M_v$,

$$
\epsilon^0(p) = \frac{p}{M_v} + O \left( \frac{M_v}{E} \right), \quad M_v = M \text{ or } M_0, \quad (3.3)
$$

and for example the amplitude $A^{\nu \nu}(\lambda_1, \lambda_2, 0, 0)$ of eq.(2.8) will seem to be proportional to $E^2/M_v^2$:

$$
A^{\nu \nu}(\lambda_1, \lambda_2, 0, 0) = \epsilon^\lambda_1(k_1) \epsilon^{\lambda_2}_\mu(k_2) \cdot \left\{ \frac{p_{1\mu} p_{2\nu}}{M_v^2} \right\} \cdot A^{\nu \nu}_{\alpha \beta \mu \nu}. \quad (3.4)
$$

If physics is described according to the renormalizable Standard Model Lagrangian containing a Higgs with a mass less than 1 TeV then the amplitude for any process, like the one above, behaves at most as a constant in the large energy limit. Thus leading energy cancellations must take place, which is indeed precisely what happens, first of all due to the structure of the Yang-Mills vertices, but also due to the Higgs interaction. Therefore if the Higgs does not exist and if we consider the Standard Model Lagrangian in the limit of a large Higgs mass, some leading energy terms may survive. Thus studying the Higgs system may be done by studying longitudinally polarized vector bosons. When we evaluate the
heavy Higgs mass correction for a typical process such as \( e^+e^- \rightarrow W_L^+W_L^- \), the screening theorem is still valid, but the \( \ln m^2 \) term is enhanced by a factor \( E^2/M_H^2 \) [13-16]. We note here that the \( \ln m^2 \) term should in fact be interpreted as an unknown parameter (this issue will be elaborated in section 3.c). It is expected that similarly enhanced corrections occur for \( \gamma \gamma \rightarrow V_LV_L \).

At high energies such corrections may thus become very large, which may seem very promising, were it not for the existence of the transverse polarized vector boson \( V_T \). The transverse vector boson production may prove to be a severe background to the longitudinal vector boson production for the following reasons:

1. with respect to the Higgs system the \( V_T \) behaves much like the photon, i.e. it is rather insensitive to the Higgs system;
2. although the decay products of the \( V_L \) have a different angular distribution than those of the \( V_T \), still the \( V_L \) can never be fully isolated;
3. \( V_T \) production is always dominant over \( V_L \) production, the difference often being a factor 10 or more.

There seems to exist a conspiracy to obstruct testing the Higgs system. For energy values up to the vector boson mass the only heavy Higgs effects are proportional to \( \ln m^2 \). While the screening theorem is still valid, for energies well above the vector boson mass the limit of a heavy Higgs mass leads to survival of \( E^2/M_H^2 \) terms. Other circumstances effectively make detection of even these effects very difficult.

Sensitivity to the Higgs system may thus not improve when the center of mass energy is increased from the threshold value to 1 TeV. As will be derived in section 5, the leading \( \ln m^2 \) correction to the \( \gamma \gamma \rightarrow W^+W^- \) cross section near threshold is

\[
< 1\% \cdot \ln m^2,
\]

depending on the renormalization scheme considered.

We now turn to the description of the Higgs system for the special cases that are considered in this paper.

3.3. Standard Model Higgs boson

The existence of the Standard Model elementary Higgs boson with a mass below 1 TeV is assumed. An experimental lower bound of 60 GeV is provided by the LEP experiments. Here however we assume that the Higgs is at least as heavy as twice the vector boson mass. We assume therefore the following mass range

\[
2M < m < 1 \text{ TeV}.
\]  

Our aim is to examine the sensitivity of the processes \( \gamma \gamma \rightarrow ZZ \) and \( \gamma \gamma \rightarrow W^+W^- \) (the \( t\bar{t} \) channel will be discussed in section 6) to the Higgs exchange diagram of fig.1. Its contribution may possibly be significant, notably in the resonance region, where \(-s + m^2 = \mathcal{O}(g^2)\). As we will see in section 4, for \( \gamma \gamma \rightarrow ZZ \) this is indeed the case for a relatively
light Higgs. For this process the diagram of fig.1 gives the following contribution to the amplitude of eq.(2.8):

\[ A_H^{\pm}(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = \epsilon_{\alpha}^{\lambda_1}(k_1)\epsilon_{\beta}^{\lambda_2}(k_2)\epsilon_{\mu}^{\lambda_3}(p_1)\epsilon_{\nu}^{\lambda_4}(p_2) \cdot \delta_{\alpha\beta} \delta_{\mu\nu} \cdot P_H \cdot A(H) \cdot \frac{\alpha\alpha_w}{c_w^2}. \]  

(3.7)

For \( \gamma \gamma \rightarrow W^+W^- \) we have

\[ A_H^{w}(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = \epsilon_{\alpha}^{\lambda_1}(k_1)\epsilon_{\beta}^{\lambda_2}(k_2)\epsilon_{\mu}^{\lambda_3}(p_1)\epsilon_{\nu}^{\lambda_4}(p_2) \cdot \delta_{\alpha\beta} \delta_{\mu\nu} \cdot P_H \cdot A(H) \cdot \alpha\alpha_w, \]  

(3.8)

where

\[ A(H) = 6M^2 + m^2 - \frac{C_0^w}{i\pi^2} \cdot (M^2m^2 - 7M^2s + 12M^4) \]

\[ + Q^2 N_c m_t^2 \cdot \left(-4 + \frac{C_0^{w'}}{i\pi^2} \cdot (8m_t^2 - 2s)\right), \]  

(3.9)

with as usual \( N_c = 3 \) and \( Q = 2/3 \) is the fractional charge of the top quark. Note that, due to eqs.(2.3) and (2.4), the \( k_{1\alpha}k_{2\beta} \) and \( k_{1\beta}k_{2\alpha} \) pieces do not contribute and we therefore did not bother to write them down. Only the \( \delta_{\alpha\beta} \) piece survives, which implies that the Higgs exchange graph of fig.1 will only contribute when the incoming photons have the same helicity. For the process \( \gamma \gamma \rightarrow W^+W^- \), the effect of the diagram of fig.1 has been investigated in ref.[17] for Higgs mass values up to 300 GeV. The effect is small. For higher Higgs mass values the effect is completely negligible. The reason is that, since it is a next to leading order effect, it would only be significant if the size of the \( W_L^+W^+_L \) cross section is of comparable order of magnitude to the size of the \( W_T^+W_T^- \) cross section. Such is not the case and except near threshold, the longitudinal final state is highly suppressed relative to the transverse final state.

The scalar three-point function \( C_0^x \) appearing in eq.(3.9) is defined by

\[ \frac{C_0^x}{i\pi^2} = \int d\eta \frac{1}{\{q^2 + m^2\} \{(q + k_1)^2 + m^2\} \{(q + k_1 + k_2)^2 + m^2\}}. \]  

(3.10)

The Higgs propagator \( P_H \) is given by \( 1/(-s + m^2 - i\epsilon) \). Near the resonance we need to employ the Dyson summed propagator, i.e.

\[ P_H = \frac{1}{-s + m^2 - \Pi(s)}. \]  

(3.11)

where \( \Pi(s) \) is the collection of all irreducible self energy graphs to all orders of perturbation theory. To first approximation we may write

\[ P_H = \frac{1}{-s + m^2 - im \cdot \Gamma(g^2)}, \]  

(3.12)

8
where $\Gamma(g^2)$ is the decay width of the Higgs boson evaluated to lowest non-zero order in $g$. Such an expansion, however, is only useful if perturbation theory can be applied to the self energy $\Pi(s)$. For a heavy Higgs, matters become rather ambiguous and for example for $m = 1$ TeV the width is 0.5 TeV. In ref.[18] this issue has been investigated for Higgs mass values greater than 700 GeV and it has been suggested to employ $\Gamma(s, g^2)$ in the propagator instead of $\Gamma(g^2)$, with

$$ \Gamma(s, g^2) = \frac{s^2}{N^4} \cdot \Gamma(g^2). \quad (3.13) $$

3.3 Heavy Higgs model

If we consider the Standard Model in the limit of a large Higgs mass, and if we take this limit in the tree level Lagrangian we end up with an effective field theory where the Higgs sector is given by the non-linear $\sigma$-model. According to this effective theory, the tree amplitude for $W_L W_L$ scattering grows like the energy squared and the unitarity limit is reached at about 1 TeV. Thus for this theory, beyond 1 TeV, physics becomes non-perturbative and we do not know how to calculate things anymore. In the TeV region new physics will show up, but what kind of new physics is at this moment anybody’s guess. At the same time we do know that up to 100 GeV the Standard Model works extremely well, Higgs or no Higgs. In other words, given the fact that there exists new physics, the Standard Model Lagrangian in the heavy Higgs mass limit is a very good approximation. Here we assume that up to 1 TeV the one-loop approximation is still reasonable. If the non-linear $\sigma$-model is used for the Higgs sector, then after performing a typical one-loop calculation infinities remain, i.e. terms containing

$$ -\frac{2}{n-4} = \Delta, \quad (3.14) $$

if we use dimensional regularization. They need to be interpreted as arbitrary parameters which must be fixed by experiment. Sensitivity to these unknown parameters implies sensitivity to the, still unspecified, new physics.

If we consider the renormalizable Standard Model Lagrangian with the linear $\sigma$-model as the Higgs sector and take the heavy Higgs mass limit at the end of a one-loop calculation no infinities survive. Instead we are left with terms proportional to $\ln m^2$. There is a direct correspondence between these $\ln m^2$ terms and the $\Delta$ terms [19]:

$$ \Delta \leftrightarrow \ln \left( \frac{m^2}{M^2} \right), \quad (3.15) $$

and thus the $\ln m^2$ terms are to be interpreted as unknown parameters. Since the one loop dependence on the Higgs mass is given by the $\ln m^2$ term only, one would expect the heavy Higgs model to contain just one arbitrary parameter. However there is at least one
more. Take the amplitude for $W_L W_L$ scattering to one loop order accuracy. Evaluated in the energy region $M \ll p \ll m$, where $p$ is a typical momentum, it will be of the form

$$A(W_L W_L \rightarrow W_L W_L) = \alpha_w \cdot \frac{p^2}{M^2} \cdot c_1$$

$$+ \alpha_w \cdot \frac{p^4}{M^4} \cdot \left\{ c_2 \cdot \ln \frac{m^2}{p^2} + \beta + O \left( \frac{M^2}{p^2} \right) \right\} + O \left( \frac{p^2}{m^2} \right).$$

(3.16)

Here $c_1$, $c_2$ and $\beta$ are of the order one and are found by explicit calculation. Besides the $\ln m^2$ term, the subleading term $\beta$ also needs to be interpreted as an unknown parameter [9]. The reason is that the limit of a heavy Higgs mass is not unique. Taking the limit of a heavy Higgs mass in the Lagrangian (non-linear $\sigma$-model) gives $\beta = 1/3$, while taking that limit after computing the one loop graphs gives $\beta = -0.32$.

Evaluated in the same energy region, the amplitudes for the processes $\gamma \gamma \rightarrow W_L^+ W_L^-$ and $\gamma \gamma \rightarrow Z_L Z_L$ are of the form

$$A_{ww}^{\gamma\gamma}(+,-,0,0) = \alpha \cdot O \left( \frac{M^2}{p^2} \right)$$

$$+ \alpha \cdot \left\{ \alpha_w \cdot \frac{p^2}{M^2} \cdot c_2^w + \alpha_w \cdot \frac{p^4}{M^4} \cdot \left\{ c_2^w \cdot \ln \frac{m^2}{p^2} + c_3^w + \ldots \right\} \right\},$$

$$A_{zz}^{\gamma\gamma}(+,-,0,0) = \alpha \cdot \left\{ \alpha_w \cdot \frac{p^2}{M^2} \cdot c_1^z + \alpha_w \cdot \frac{p^4}{M^4} \cdot \left\{ c_2^z \cdot \ln \frac{m^2}{p^2} + c_3^z + \ldots \right\} \right\}. \quad (3.17)$$

The photons need to have the same helicity; the amplitudes for which the photons have the opposite helicity do not contain terms that lead to bad high energy behaviour in the heavy Higgs mass limit.

The terms $c_2^w$ and $c_2^z$ are found by explicit calculation. In the heavy Higgs model the parameters $\ln m^2$, $c_2^w$ and $c_2^z$ are interpreted as unknown parameters. Comparing the expansions of eqs.(3.16) and (3.17) we can relate the $\ln m^2$ terms, while the connection between $\beta$, $c_3^w$ and $c_3^z$ seems to be completely lost. This would suggest that for every process there is an extra arbitrary parameter. However, as we will demonstrate in the next section, these parameters may be uniquely related to each other. Thus assuming for $W_L W_L$ scattering the one loop approximation to be a reasonable one, the heavy Higgs model contains just two arbitrary parameters. We note that if this assumption is incorrect and if higher order corrections are important, the expansions of eqs.(3.16) and (3.17) cannot be related to each other; the set of parameters needed to describe one process may be different from the set of parameters for another process.

Consider first the process $\gamma\gamma \rightarrow ZZ$ described according to the heavy Higgs mass model. The Higgs exchange diagram of fig.1 contributes to the lowest non-zero order in perturbation theory and from eqs.(3.7), (3.9) and (3.11) we find in the limit of large $m$:

$$\lim_{m \rightarrow \infty} A_{HH}^{\gamma\gamma}(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = e_\alpha^\lambda_1(k_1) e_\beta^\lambda_2(k_2) e_\mu^\lambda_3(p_1) e_\nu^\lambda_4(p_2)$$

$$\cdot \delta_{\alpha \beta} \delta_{\mu \nu} \cdot \frac{\alpha \alpha_w}{c_w^2} \cdot \left\{ 1 - \frac{C_w^0}{i \pi^2} \cdot M^2 + O \left( \frac{s}{m^2} \cdot \frac{M^2}{m^2} \right) \right\}. \quad (3.18)$$
Just like in $W_L W_L$ scattering, the amplitude for the process $\gamma \gamma \rightarrow Z_L Z_L$ shows bad high energy behaviour already in lowest non-zero order. In the expansion of eq.(3.17) $p^2 = -s$ with $c_f^b = 1/2$. The $m^2$ terms will arise at the next to leading order, in this case thus at the two-loop level.

Unfortunately, unlike in $WW$ scattering, $\gamma \gamma \rightarrow ZZ$ is the perfect example of the conspiracy described in section 3.a: the $Z_T Z_T$ production is about a factor 100 larger, leaving the bad high energy behaviour of the $Z_L Z_L$ amplitude almost undetectable. The difference between a 1 TeV Higgs and an infinitely heavy Higgs is initially an enhancement of about a factor 2-4 (depending on the top quark mass) for the $Z_L Z_L$ cross-section at $\sqrt{s} = 1$ TeV. When transverse vector boson production is included, the enhancement is reduced to a few percent. Maybe such effects could be seen but now we are dealing with a theoretical uncertainty: first of all due to the broad Higgs width, but also if we need an accuracy of a few percent then next to leading order calculations must be done. This would entail a two-loop calculation.

For the process $\gamma \gamma \rightarrow W^+ W^-$ the background due to the transverse vector boson production is even more substantial. To order $\alpha$, the $W^+_T W^-_T$ amplitude is a constant in the large energy limit while the $W^+_L W^-_L$ amplitude vanishes like $M^2/s$. The quadratic energy term, that arises at order $\alpha \alpha_{\omega}$, is thus highly suppressed and heavy Higgs effects are not expected to be substantially different at threshold then at $\sqrt{s} = 1$ TeV. Effects only become interesting for center of mass energy values above 1.5 TeV.

It may be interesting to know what the leading Higgs mass correction will be near threshold. Making an expansion in the Higgs mass, with $M \simeq p \ll m$, the amplitude to one loop order accuracy is of the form

$$A(\gamma \gamma \rightarrow W^+ W^-) = \alpha \cdot c_{\omega}^W + \alpha \alpha_{\omega} \cdot \left( c_{\delta}^W \cdot \ln \frac{m^2}{M^2} + O(1) \right), \quad (3.19)$$

for any helicity configuration of the photons and vector bosons. The parameters $c_{\omega}^W$ and $c_{\delta}^W$ are found by explicit calculation. The one loop calculations that must be done in order to extract the $\ln m^2$ term, i.e. the parameter $c_{\delta}^W$, are given in full detail elsewhere [12,15]. In the end we only need to consider the one-loop corrected vertices, while the propagators remain unchanged (except for the Higgs propagator). After that only irreducible box diagrams must be calculated. In ref.[15] the leading $\ln m^2$ corrections are listed for a number of vertices.

Concerning finite renormalization, counterterms are fixed such that the masses are located at the pole of the propagators. This fixes $\delta_M$, $\delta_{\omega}$, $\delta_m$, and $\delta_M$ when making the shifts $M \rightarrow M(1 + \delta_M)$, etc. There is still one more parameter in the Standard Model tree level Lagrangian, namely the coupling constant $g$. Therefore one more measuring point is needed, thereby obtaining the corresponding value for $\delta_g$. For example when choosing muon decay, we find:

$$\delta_g(\text{muon decay}) = \frac{\alpha_{\omega}}{4\pi} \cdot \frac{1}{24} \cdot \ln m^2. \quad (3.20)$$

We also could have chosen the cross section for Coulomb scattering at zero momentum
transfer as our measuring point. We obtain a different value for \( \delta_g \):

\[
\delta_g (\text{Coulomb scattering}) = \frac{\alpha_w}{4\pi} \cdot \frac{-5}{12} \cdot \ln m^2. \tag{3.21}
\]

The correction due to \( \delta_g \) is always proportional to the amplitude in lowest non-zero order and will therefore not induce bad high energy behaviour.

3.6 The U particle

The heavy Higgs mass limit is arbitrary. Some time ago the U particle has been introduced to parametrize this, and to make this arbitrariness explicit [13]. To the U particle corresponds a scalar singlet field, added to the Lagrangian in a gauge invariant way. The U-particle, not coupled to the vector bosons, is coupled to the Higgs with a strength \( m^2 \) and its mass is taken to be equal to the Higgs mass:

\[
\mathcal{L}(U) = -\frac{1}{2} (\partial_\mu U)^2 - \frac{1}{2} m^2 U^2 - \frac{g g^*_U m^2}{4M} U^2 H - \frac{g^2 g^*_U m^2}{4M^2} U^2 (H^2 + \phi^2). \tag{3.22}
\]

The field \( \phi \) is the usual Higgs ghost. Since the \( U \) is coupled only to the Higgs, no dependence on \( g_U \) should remain in the limit \( m \to \infty \), if the heavy Higgs mass limit is unique. This is not the case, and the contribution of the U-particle to the amplitude of eq. (3.16) is given by

\[
\beta \to \beta + \beta^U, \tag{3.23}
\]

with

\[
\beta^U = g_U^2 \cdot \left( \frac{\pi}{\sqrt{3}} - 2 \right), \tag{3.24}
\]

thus demonstrating the arbitrariness of \( \beta \). It is very interesting to note that within the framework of partial wave analysis the location of a resonance in the isospin \( I = 1 \) channel for \( W_L W_L \) scattering precisely depends on this \( \beta \) term [9]. When \( \beta = \frac{1}{3} \) the resonance is located at 9 TeV, while if \( \beta \leq 0 \) no resonance will occur. For \( \beta = 5 \) the resonance occurs at 2 TeV, much like the scaled up version of the \( \rho \) resonance in \( \pi \pi \) scattering. Thus, within the framework of partial wave analysis, the experimental sensitivity below 1 TeV to the \( \beta \) parameter will enable us to predict at what energy in the TeV region a resonance will occur in the \( I = 1 \) channel, however not explaining as to what kind of new physics could cause such a resonance.

The \( U \) particle effect for the \( \rho \) parameter has been calculated in ref.[20]; there the effect was found to be too small to be of practical interest. It may be quite interesting to see what an effect the existence of such a resonance would have for the process \( \gamma \gamma \to ZZ \). The \( U \) particle contribution to the diagram of fig.1 is given by

\[
A^U_{1/2} (\lambda_1, \lambda_2, \lambda_3, \lambda_4) = \varepsilon^\lambda_1 (k_1) \varepsilon^\lambda_2 (k_2) \varepsilon^\lambda_3 (p_1) \varepsilon^\lambda_4 (p_2)
\]

\[
\cdot \delta_{\alpha \beta} \delta_{\mu \nu} \cdot \frac{\alpha \alpha_w}{4\pi c_w} \left( -\frac{\beta^U}{8M^2} \right) \cdot \left\{ 6M^2 + s - \frac{C_0^w}{i\pi^2} 6M^2 (2M^2 - s) \right\}.
\]

\[
+ Q^2 N_c m^2 l \cdot \left( -4m^2_l + \frac{C_0^l}{i\pi^2} (8m^2_l - 2s) \right). \tag{3.25}
\]
When the initial photons have the same helicity and the outgoing vector bosons are longitudinally polarized, the contribution to the parameter \( c_3^- \) of eq.(3.17) is given by
\[
c_3^- \rightarrow c_3^+ + c_3^z U, \tag{3.26}
\]
with
\[
c_3^z U = \frac{\beta U}{64 \pi}. \tag{3.27}
\]
For \( \beta = 5 \) this implies a \(-50\%\) effect in the cross section for the process \( \gamma \gamma \rightarrow Z_L Z_L \) at \( \sqrt{s} = 1 \) TeV. But as we remarked before, such an effect is unfortunately probably undetectable when transverse polarized vector bosons are included in the final state: it is then reduced by a factor \( 10^{-2} \) to \(-0.5\%\).

4. The process \( \gamma \gamma \rightarrow ZZ \)

The evaluation of the cross-section in lowest non-zero order in perturbation theory has been done by Jikia [21], who found that the transverse vector boson production is overwhelmingly large as compared to the longitudinal vector boson production. Since at first it appears that \( \gamma \gamma \rightarrow Z_L Z_L \) is an ideal place to test for the Higgs sector, this result is disappointing; perhaps not so surprising in view of the conspiracy described in section 3.a.

Nevertheless such a result needs to be verified, not in the least since the corresponding calculation is quite complicated. While preparing this work we became aware of two more independent calculations, performed by Berger [22] and Dicus and Kao [23]. Our results confirm all of these three independent calculations.

Box diagrams need to be calculated, for which there is also a fermionic contribution. This contribution has been calculated in ref.[24] for the gluon fusion process, i.e. for the process \( gg \rightarrow ZZ \). This has provided us with an additional check, and we found agreement here as well.

We performed our calculations in the Feynman-'t Hooft gauge.

4.a The amplitude

The diagrams that contribute to the amplitude for \( \gamma \gamma \rightarrow ZZ \) to lowest non-zero order in perturbation theory are of order \( g^4 \), and are shown in fig.3. For the amplitude of eq.(2.8),
\[
A^{\pm \pm}(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = \epsilon_{\alpha}^{\lambda_1}(k_1)\epsilon_{\beta}^{\lambda_2}(k_2)\epsilon_{\mu}^{\lambda_3}(p_1)\epsilon_{\nu}^{\lambda_4}(p_2) \cdot A_{\alpha \beta \mu \nu}^{\pm \pm}, \tag{4.1}
\]
the expression for \( A_{\alpha \beta \mu \nu}^{\pm \pm} \) takes the form
\[
A_{\alpha \beta \mu \nu}^{\pm \pm} = \{k_1 \mu k_1 \nu p_1 \alpha p_1 \beta + k_2 \mu k_2 \nu p_1 \alpha p_1 \beta\} \cdot f_1 + k_1 \mu k_2 \nu p_1 \alpha p_1 \beta \cdot f_2 + k_2 \mu k_1 \nu p_1 \alpha p_1 \beta \cdot f_3 + \delta_{\mu \mu}p_1 \alpha p_1 \beta \cdot f_4 + \{\delta_{\mu \alpha}k_1 \nu p_1 \beta - \delta_{\nu \beta}p_1 \alpha k_2 \mu\} \cdot f_5 + \{\delta_{\mu \alpha}k_2 \nu p_1 \beta - \delta_{\nu \beta}p_1 \alpha k_1 \mu\} \cdot f_6 + \{\delta_{\mu \beta}p_1 \alpha p_1 \nu - \delta_{\nu \mu}k_1 \nu k_1 \mu\} \cdot f_7 + \delta_{\mu \beta}p_1 \alpha k_2 \nu \cdot f_8 + \delta_{\mu \beta}k_2 \nu k_1 \mu \cdot f_9 + \delta_{\mu \beta}k_1 \nu k_2 \mu \cdot f_{10} + \delta_{\mu \beta}k_2 \nu k_2 \mu \cdot f_{11} + \delta_{\mu \beta}k_1 \nu k_2 \mu \cdot f_{12} + \delta_{\mu \beta}k_1 \nu k_2 \mu \cdot f_{13} + \delta_{\mu \beta}k_2 \nu k_2 \mu \cdot f_{14}. \tag{4.2}
\]
We used momentum conservation, and substituted $p_2 = k_1 + k_2 - p_1$. As $A^{zz}_{\alpha\beta\mu\nu}$ is to be multiplied with the polarization vectors, the terms proportional to $k_{1\alpha}$, $k_{1\beta}$, $k_{2\alpha}$, $k_{2\beta}$, $p_{1\mu}$ and $p_{2\nu}$ do not contribute here and we left them out. In a Ward identity one will encounter such amplitudes multiplied by some momentum instead of one or more of the polarization vectors. Then terms containing $k_{1\beta}$ etc. must be kept.

The algebraic expressions for the functions $f_1, \ldots, f_{14}$ were derived using Schoonschip [25], and subsequently numerically evaluated using Form [26].

Referring to section 3, it may be deduced easily that the diagram of fig.1 contributes to the $f_{12}$ term only. For the case that the Higgs exists with a mass less than 1 TeV it is given by

$$f_{12H} = \frac{\alpha \alpha_w}{e_w^2} \cdot \frac{1}{s + m^2 - i m \cdot \Gamma(s, g^2)} \cdot \left\{ 6M^2 + m^2 - \frac{C_0}{i \pi^2} \cdot (M^2 m^2 - 7M^2 s + 12M^4) + Q^2 N_c m_t \cdot \left( -4 + \frac{C_0}{i \pi^2} \cdot (8m_t^2 - 2s) \right) \right\}$$

(4.3)

This expression differs slightly from the one given in [21], but the numerical consequences are negligible. For a very heavy Higgs we have,

$$\lim_{m \to \infty} f_{12H} = \frac{\alpha \alpha_w}{e_w^2} \cdot \left\{ 1 - \frac{C_0}{i \pi^2} \cdot M^2 \right\} \to \frac{\alpha \alpha_w}{e_w^2} \cdot \left\{ 1 \right\} \text{ for large } s.$$  

(4.4)

The $U$-particle contribution is given by:

$$f_{12U} = \frac{\alpha \alpha_w^2}{4 \pi e_w^2} \cdot \left( -\frac{\beta^U}{8M^2} \right) \cdot \left\{ 6M^2 + m^2 - \frac{C_0}{i \pi^2} \cdot 6M^2 \cdot (2M^2 - s) + Q^2 N_c m_t^2 \cdot \left( -4 + \frac{C_0}{i \pi^2} \cdot (8m_t^2 - 2s) \right) \right\} \to \frac{\alpha \alpha_w}{e_w^2} \cdot \left\{ -\frac{\alpha_w \beta^U s}{32 \pi M^2} \right\} \text{ for large } s.$$  

(4.5)

To order $g^4$ all other functions $f_i$ are insensitive to the Higgs system. If $\beta^U = 5$ the $U$ particle contribution leads to a $-25\%$ effect to the term $f_{12H}$ of eq.(4.4) at $\sqrt{s} = 1$ TeV, which corresponds to a $-50\%$ effect for the $Z_L Z_L$ cross section. As we mentioned before, in the process $\gamma \gamma \to ZZ$ such an effect is unfortunately reduced by two orders of magnitude due to the enormous $Z_T Z_T$ background.

4.b Ward identities; the equivalence theorem

In order to verify the correctness of the functions $f_1, \ldots, f_{14}$ we must consider the relevant Ward identities of which there are six altogether, which may be written as

$$(ik_{1\alpha}) \cdot \epsilon_\alpha^{\lambda_3} (k_2) \epsilon_\mu^{\lambda_4} (p_1) \epsilon_\nu^{\lambda_4} (p_2) \cdot A_{\alpha\beta\mu\nu}^{zz} = 0,$$

(4.6)

$$(ik_{2\beta}) \cdot \epsilon_\alpha^{\lambda_4} (k_1) \epsilon_\alpha^{\lambda_4} (p_1) \epsilon_\mu^{\lambda_4} (p_2) \cdot A_{\alpha\beta\mu\nu}^{zz} = 0,$$

(4.7)
\[(i k_1 \alpha i k_2 \beta) \cdot \epsilon^\alpha_{\mu} (p_1) \epsilon^\beta_{\nu} (p_2) \cdot A^{\pm \pm}_{\alpha \beta \mu \nu} = 0, \quad (4.8)\]

\[-ip_1 \mu \cdot \epsilon^\alpha_{\alpha} (k_1) \epsilon^\beta_{\beta} (k_2) \epsilon^\nu_{\nu} (p_2) \cdot A^{\pm \pm}_{\alpha \beta \mu \nu} + M_0 \cdot \epsilon^\alpha_{\alpha} (k_1) \epsilon^\beta_{\beta} (k_2) \epsilon^\nu_{\nu} (p_2) \cdot A^{\pm \pm}_{\alpha \beta \mu \nu} = 0, \quad (4.9)\]

\[-ip_2 \nu \cdot \epsilon^\alpha_{\alpha} (k_1) \epsilon^\beta_{\beta} (k_2) \epsilon^\nu_{\nu} (p_2) \cdot A^{\pm \pm}_{\alpha \beta \mu \nu} + M_0 \cdot \epsilon^\alpha_{\alpha} (k_1) \epsilon^\beta_{\beta} (k_2) \epsilon^\nu_{\nu} (p_2) \cdot A^{\pm \pm}_{\alpha \beta \mu \nu} = 0, \quad (4.10)\]

\[(ip_{1 \mu} i p_{2 \nu}) \cdot \epsilon^\alpha_{\alpha} (k_1) \epsilon^\beta_{\beta} (k_2) \cdot A^{\pm \pm}_{\alpha \beta \mu \nu} + M_0 \cdot -ip_{2 \nu} \epsilon^\alpha_{\alpha} (k_1) \epsilon^\beta_{\beta} (k_2) \cdot A^{\pm \pm}_{\alpha \beta \mu \nu} + M_0 \cdot -ip_{1 \mu} \epsilon^\alpha_{\alpha} (k_1) \epsilon^\beta_{\beta} (k_2) \cdot A^{\pm \pm}_{\alpha \beta \mu \nu} + M_0^2 \cdot \epsilon^\alpha_{\alpha} (k_1) \epsilon^\beta_{\beta} (k_2) \cdot A^{\pm \pm}_{\alpha \beta \mu \nu} = 0. \quad (4.11)\]

The first three Ward identities are also known as transversality conditions, i.e., the amplitude equals zero when the photon polarization vector is replaced by the corresponding momentum vector. The other three Ward identities involve the amplitude for which the polarization vector of the massive vector boson is replaced by its momentum vector. \(A^{\pm \pm}_{\alpha \beta \mu \nu} (\lambda_1, \lambda_2, \lambda_3) = \epsilon^\alpha_{\alpha} (k_1) \epsilon^\beta_{\beta} (k_2) \epsilon^\nu_{\nu} (p_2) A^{\pm \pm}_{\alpha \beta \mu \nu}\) denotes the amplitude for \(\gamma \gamma \rightarrow \phi Z\), obtained from \(\gamma \gamma \rightarrow ZZ\) with the replacement of \(Z\) by the Higgs ghost \(\phi\). Similarly, \(A^{\pm \pm}_{\alpha \beta \mu \nu} (\lambda_1, \lambda_2, \lambda_3)\) and \(A^{\pm \pm}_{\alpha \beta \mu \nu} (\lambda_1, \lambda_2)\) correspond to, respectively, the \(\gamma \gamma \rightarrow Z \phi\) amplitude and the \(\gamma \gamma \rightarrow \phi \phi\) amplitude. In Ward identities momenta are defined to be flowing into the vertex, which leads to the extra minus signs appearing in eqs.\((4.9), \)\((4.10)\) and \((4.11)\).

As an illustration, consider the Ward identity of eq.\((4.6)\) for the \(k_{1 \mu} k_{1 \nu} p_{1 \beta}\) term. After multiplication of \(A^{\pm \pm}_{\alpha \beta \mu \nu}\) of eq.\((4.2)\) with \(k_{1 \alpha}\), we obtain for this term:

\[\{(p_1 k_1) \cdot f_1 + f_5 - f_9\} \cdot k_{1 \mu} k_{1 \nu} p_{1 \beta}. \quad (4.12)\]

There is one additional term, which we did not explicitly write down; before multiplication with the external polarization or momentum vectors it is of the form \(k_{1 \mu} k_{1 \nu} k_{2 \alpha} p_{1 \beta} \cdot g_1\). This term does not contribute to \(A^{\pm \pm} (\lambda_1, \lambda_2, \lambda_3, \lambda_4)\) because \(\epsilon_1 k_2 = 0\). However, it does contribute to the Ward identity of eq.\((4.6)\) since \(k_1 k_2 = k_2^2 / 2 = -s / 2\) is not zero. The Ward identity is then given by

\[(p_1 k_1) \cdot f_1 + f_5 - f_9 - \frac{1}{2} s \cdot g_1 = 0. \quad (4.13)\]

There are of course many more such identities.

From the three Ward identities of eqs.\((4.9), \)\((4.10),\) and \((4.11)\) we can derive the equivalence theorem \([8,27]\), valid in the limit \(M_0 \ll E\). The equivalence theorem states that the leading
energy term of the $Z_L Z_L$ amplitude is given by the amplitude where the longitudinal vector boson $Z_L$ is replaced by the corresponding Higgs ghosts $\phi$. Here follows a sketch of the derivation. In the large energy limit, the longitudinal polarization vector $\epsilon^0_\mu(p)$ may be written as

$$\epsilon^0_\mu(p) = \frac{p_\mu}{M_0} + v_\mu, \quad v_\mu = O \left( \frac{M_0}{E} \right).$$  \hfill (4.14)

Using this relation, the first step is to eliminate the momentum vectors appearing in the Ward identities. This leads to identities involving the longitudinal polarization vectors $\epsilon^0_\mu$ and the vectors $v$. Next, subtracting the identities (4.9) and (4.10) from (4.11) leads to the relation

$$A^{zz}(\lambda_1, \lambda_2, 0, 0) = i^2 \cdot A^{\phi\phi}(\lambda_1, \lambda_2)$$

$$- i \cdot v_{1\mu} \epsilon^{\lambda_1}_\alpha(k_1) \epsilon^{\lambda_2}_\beta(k_2) \cdot A^{\phi\phi}_{\alpha\beta\mu}$$

$$- i \cdot v_{2\mu} \epsilon^{\lambda_1}_\alpha(k_1) \epsilon^{\lambda_2}_\beta(k_2) \cdot A^{\phi\phi}_{\alpha\beta\mu}$$

$$+ v_{1\mu} v_{2\mu} \epsilon^{\lambda_1}_\alpha(k_1) \epsilon^{\lambda_2}_\beta(k_2) \cdot A^{\phi\phi}_{\alpha\beta\mu}. \quad (4.15)$$

In the limit that $v$ is small, we obtain the equivalence theorem:

$$A^{zz}(\lambda_1, \lambda_2, 0, 0) = -A^{\phi\phi}(\lambda_1, \lambda_2) + O \left( \frac{M_0}{E} \right)$$  \hfill (4.16)

There do exist cases where it is not justified to throw away the diagrams that are multiplied with one or more $v$'s. Using the equivalence theorem in the large Higgs mass limit, we find

$$A^{zz}(+, +, 0, 0) = -a_{\alpha\mu} \cdot \frac{s}{2M^2}, \quad M_0 \ll E \ll m, \quad (4.17)$$

which may be compared to the term $f_{12H}$ of eq.(4.4). The equivalence theorem is thus in fact a Ward identity for the leading energy term only and it serves as another check on the calculation. We remark that in this case calculating the $\phi\phi$-amplitude is by far simpler than calculating the $Z_L Z_L$-amplitude.

Using the theorem, the process $\gamma \gamma \rightarrow Z_L Z_L$ has been studied in refs. [28,29] for center of mass energy values in the TeV range.

4.c Results

In fig.4a we show the $J = 0$ differential cross-sections at $\sqrt{s} = 200$ GeV. When we take the limit $m \rightarrow \infty$, the cross section is less than 10 fb. However if the Higgs exists, and if $\sqrt{s}$ is the resonance energy value, i.e. $m = 200$ GeV, the cross section is enhanced by almost a factor $10^3$ and is 7 pb.

It is of course possible that the center of mass energy is not the resonance energy value. In that case a tail of a resonance could be observed for energy values not too far away from the Higgs mass value. In order to have an indication of the size of the effect
of such a tail we introduce the function $R(m, \theta, \sqrt{s})$. It is the ratio of the differential cross-section for a given Higgs mass $m$ and the differential cross-section in the limit of a large Higgs mass, at the center of mass energy $\sqrt{s}$ considered. In percentage value:

$$R(m, \theta, \sqrt{s}) = \left( \frac{d\sigma(\theta)_m}{d\sigma(\theta)_{m \to \infty}} - 1 \right) \cdot 100\%, \quad d\sigma(\theta) = \frac{d\sigma}{d\cos \theta}. \quad (4.18)$$

As usual the dependence of the differential cross section on $\sqrt{s}$ is understood and needs not be indicated. $R(m, \theta, \sqrt{s})$ measures a lowest non-zero order effect for the process $\gamma \gamma \to ZZ$. Hence the $U$ particle effect, to be discussed at the end of this section, is not included. $R(m, \theta, \sqrt{s})$ is positive for $m > \sqrt{s}$, negative for $m < \sqrt{s}$. In fig.4b we show $R(m, \theta, \sqrt{s} = 200 \text{ GeV})$ for 4 different Higgs mass values, namely $m = 300, 400, 500$, and $600 \text{ GeV}$.

In fig.4c we show the $J = 2$ differential cross sections which to $\mathcal{O}(g^4)$ are insensitive to the Higgs system.

In all cases, varying the top quark mass from 120 to 180 GeV leads to an insignificant effect.

Figs. 5a-5c show similar curves, but here at $\sqrt{s} = 300 \text{ GeV}$. For the case $m \to \infty$, dependence on the top quark mass $m_t$ is negligible. The differential cross sections are of the order of 0.1 pb and already much larger than at $\sqrt{s} = 200 \text{ GeV}$. This is because the cross section for transverse vector boson production increases rather steeply, and is in fact near its maximum value. The longitudinal cross-section remains in the fb range.

The dependence on $m_t$ is significant when $m = 300 \text{ GeV}$. The reason is that for $\sqrt{s} > 2m_t$, the imaginary part of the top quark loop in the $\gamma \gamma H$ vertex is non-zero and may lead to a large effect. In fig. 5a the $J = 0$ differential cross sections, with unpolarized vector bosons in the final state, are shown for top quark mass values of $120 \text{ GeV}$ and $180 \text{ GeV}$.

In fig.5b $R(m, \theta, \sqrt{s} = 300 \text{ GeV})$ is plotted for Higgs mass values $m = 400, 500$ and $600 \text{ GeV}$ and for top quark mass values $m_t = 120$ and $180 \text{ GeV}$.

Figs.6a-6c display the curves at $\sqrt{s} = 500 \text{ GeV}$. At this energy, given a Higgs mass of $500 \text{ GeV}$, the resonance effect has disappeared. Nevertheless, for small values of $|\cos \theta|$, there is a 10-15% effect with respect to the cross-section for $m \to \infty$ for $m_t = 120 \text{ GeV}$. The effect is 5-8% for $m_t = 180 \text{ GeV}$.

Finally, figs.7a-7d give the cross-sections at $\sqrt{s} = 1 \text{ TeV}$. Effects of a Higgs with a mass of 1 TeV are at most 5% for $m_t = 120 \text{ GeV}$ and at most 1.6% for $m_t = 180 \text{ GeV}$.

We have also calculated the $U$-particle effect in the limit of a large Higgs mass, assuming a 2 TeV resonance in the $I = 1$ channel for $W_L W_L$ scattering. In fig.8 we show the $U$ particle effect to the $J = 0$ differential cross-section in percentage value. For the $LL$ final state the effect is always increasing with increasing energy: $-0.7\%$ at $\sqrt{s} = 200 \text{ GeV}$ and $-50\%$ at $\sqrt{s} = 1 \text{ TeV}$. However, when the $TT$ final state is included the effect is
always of the order \(-(0.1 - 0.4)\%\) and thus extremely difficult to observe. We note that effects only become substantial for $\sqrt{s} > 1.5$ TeV.

5. The process $\gamma \gamma \to W^+ W^-$

Regarding the anomalous tree level $\gamma WW$ and $\gamma \gamma WW$ couplings the process $\gamma \gamma \to WW$ has been discussed extensively in the literature [3]. Here we do not consider such effects, and we assume the tree level couplings to be the Yang-Mills couplings as predicted by the Standard Model Lagrangian.

If the initial photons have the same helicity the $W_L^+ W_L^-$ channel is highly supressed compared to the $W_T^+ W_T^-$ channel. Therefore if we consider unpolarized vector bosons in the final state the Higgs exchange diagram of fig.1 plays little or no role, even for center of mass energy values near the Higgs resonance [17]. In the case that the Higgs does not exist and if we consider the heavy Higgs model, leading energy terms survive. For the process $\gamma \gamma \to W_L^+ W_L^-$ this happens only when the incoming photons have the same helicity. This leading energy term, corresponding to the term $c_\gamma^W$ of eq.(3.17), has been calculated in refs. [28,29] with the help of the equivalence theorem. After transverse polarized vector bosons are included in the final state, this term leads to a very small effect for center of mass energy values up to 1 TeV. Here we neglect this term and instead we calculate the leading ln $m^2$ contribution due to a heavy Higgs for the unpolarized $\gamma \gamma \to W^+ W^-$ cross section. According to the heavy Higgs model the ln $m^2$ term must be interpreted as an unknown parameter.

5.5 The differential cross section in lowest non-zero order

In lowest non-zero order in perturbation theory, the contributing Feynman diagrams to the amplitude of the process $\gamma \gamma \to W^+ W^-$ are shown in fig.9. For the corresponding expression of eq.(2.8), i.e.

$$A^w_w(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = \epsilon^\lambda_1(k_1)\epsilon^{\lambda_2}(k_2)\epsilon^\lambda_3(p_1)\epsilon^\lambda_4(p_2) \cdot A^w_{\alpha\beta\mu\nu},$$

we have

$$A^w_{\alpha\beta\mu\nu} = 4\pi \alpha \cdot \left\{ -2 \delta_{\mu\nu} \delta_{\alpha\beta} - \delta_{\mu\alpha} \delta_{\nu\beta} \cdot \frac{s}{p_1 k_1} - \delta_{\nu\alpha} \delta_{\mu\beta} \cdot \frac{s}{p_1 k_2} 
+ \left( \delta_{\mu\alpha} k_{1\nu} p_{1\beta} + \delta_{\mu\beta} p_{1\alpha} k_{2\nu} - \delta_{\nu\alpha} k_{1\mu} p_{1\beta} - \delta_{\nu\beta} p_{1\alpha} k_{2\mu} \right) \cdot \left( \frac{2}{p_1 k_1} + \frac{2}{p_1 k_2} \right) 
+ \delta_{\alpha\beta} k_{1\mu} k_{2\nu} \cdot \frac{2}{p_1 k_1} + \delta_{\alpha\beta} k_{2\mu} k_{1\nu} \cdot \frac{2}{p_1 k_2} \right\}$$

(5.2)

The dotproducts are given by

$$p_1 k_1 = s/4 \cdot (-1 + \beta_w \cos \theta), \quad p_1 k_2 = s/4 \cdot (-1 - \beta_w \cos \theta).$$

(5.3)
When both outgoing vector bosons are longitudinally polarized we have

\[
A^{ww}(+,+,0,0) = \frac{4\pi\alpha}{1 - \beta_w^2 \cos^2 \theta} \cdot \frac{-8M^2}{s} \rightarrow \mathcal{O}\left(\frac{M^2}{s}\right) \text{ for large } s
\]

\[
A^{ww}(+,-,0,0) = \frac{4\pi\alpha}{1 - \beta_w^2 \cos^2 \theta} \cdot (1 - \cos^2 \theta) \cdot \left(-2 - \frac{4M^2}{s}\right) \rightarrow \mathcal{O}(1) \text{ for large } s. \tag{5.4}
\]

In the limit \( M \ll E \), \( A^{ww}_{\alpha\beta\mu\nu} \) is of the order one and when multiplied with the longitudinal polarization vectors, we expect \( A(+,+,0,0) \) and \( A(+,-,0,0) \) to contain \( \mathcal{O}(E^2/M^2) \) terms. Due to the Yang-Mills type cancellations these terms do not survive. The unpolarized differential cross section is given by

\[
\frac{d\sigma^{ww}}{d\cos \theta} = \frac{\beta_w}{32\pi s} \cdot (4\pi\alpha)^2 \cdot \left\{ 6 - \frac{32 + 48M^2/s}{1 - \beta^2 \cos \theta^2} + \frac{64 + 192M^4/s^2}{(1 - \beta^2 \cos \theta^2)^2} \right\}, \tag{5.6}
\]

for which the longitudinal contribution of eq.(5.4) may be neglected for energies already slightly above threshold. In fig.10 we show the unpolarized differential cross section as a function of \( \cos \theta \) for center of mass energy values \( \sqrt{s} = 200 \text{ GeV}, 400 \text{ GeV} \text{ and } 1 \text{ TeV} \).

5.6 The one loop correction due to a heavy Higgs

Up to a center of mass energy of 1 TeV, the only heavy Higgs effects that may possibly be observable are those that also exist in the \( W^+_r W^-_r \) channel. In this channel the leading heavy Higgs effect is the \( \ln m^2 \) correction, while there is no bad high energy behaviour. In accordance with the screening theorem the quadratic Higgs mass dependence shows up at the two loop level. Consider the amplitude for the process \( \gamma \gamma \rightarrow W^+ W^- \) near threshold. The leading heavy Higgs effect is given by eq.(3.19):

\[
A^{ww} = \alpha \cdot c_w^w + \alpha \alpha_w \cdot \left\{ c_w^w \cdot \ln \frac{m^2}{M^2} + \mathcal{O}(1) \right\}. \tag{5.7}
\]

We also could have considered the amplitude at some other energy up to 1 TeV. The leading heavy Higgs effect would still be as given above, while the energy dependence enters in the form of some \( \ln (p^2/M^2) \) term \( p \) is a typical momentum). The one loop calculation that must be done to extract the \( \ln m^2 \) term is a standard one. As we mentioned in section 3.c we do not give the details and we only give a sketch of how the \( \ln m^2 \) terms are obtained.

The receipe is as follows:

(1) leave the propagators and the external lines unchanged
(2) replace the occurring tree vertices in the diagrams of figs. 9a-9c, by the one loop corrected vertices
(3) evaluate the irreducible one loop \( \gamma \gamma W^+ W^- \) diagrams

Expressions for various one loop corrected vertices may be found in appendix E of ref.[15]. Note that there are no \( U \) particle effects, as they enter at the two loop level. We start with the vector boson exchange diagrams of fig.9a, which at the tree level may be written as

\[
A_{\alpha\beta\mu\nu}(a) = \{ \Gamma_{\gamma w w}^0 \cdot P_w \cdot \Gamma_{\gamma w w}^0 \} \alpha\beta\mu\nu. \tag{5.8}
\]
The notation should be clear: $\Gamma^0$ is the tree level vertex, and $P$ is the propagator. The heavy Higgs mass correction is found by replacing $\Gamma^0_{\gamma^0 \gamma^0}$ by $\Gamma^R_{\gamma^0 \gamma^0}$, where $\Gamma^R$ is the one loop corrected vertex, keeping only the $\ln m^2$ terms. It is found that this correction is proportional to the tree level vertex:

$$\Gamma^R_{\gamma^0 \gamma^0} = \Gamma^0_{\gamma^0 \gamma^0} \cdot \delta_{\gamma^0 \gamma^0},$$

with

$$\delta_{\gamma^0 \gamma^0} = -\delta_g - \frac{5}{12} \cdot \frac{g^2}{16\pi^2} \cdot \ln m^2. \quad (5.10)$$

Note that we left $\delta_g$ unspecified; possible values are given in eqs.(3.20) and (3.21). The one loop correction to the diagram of fig.9a is thus given by

$$C_{\alpha\beta\mu
u}(a) = A_{\alpha\beta\mu
u}(a) \cdot 2\delta_{\gamma^0 \gamma^0},$$

and is proportional to the tree level diagram.

Similarly, for the Higgs ghost exchange diagrams of fig.9b, given by

$$A_{\alpha\beta\mu
u}(b) = \left\{ \Gamma^0_{\gamma^0 \gamma^0} \cdot P_\phi \cdot \Gamma^0_{\gamma^0 \gamma^0} \right\}_{\alpha\beta\mu
u},$$

we must replace the tree vertex $\Gamma^0_{\gamma^0 \gamma^0}$ by $\Gamma^R_{\gamma^0 \gamma^0}$. Again the corrected vertex is proportional to the tree level vertex, i.e. $\Gamma^R_{\gamma^0 \gamma^0} = \Gamma^0_{\gamma^0 \gamma^0} \cdot \delta_{\gamma^0 \gamma^0}$, with

$$\delta_{\gamma^0 \gamma^0} = -\delta_g - \frac{5}{12} \cdot \frac{g^2}{16\pi^2} \ln m^2 \]

$$= \delta_{\gamma^0 \gamma^0}. \quad (5.13)$$

This result may be easily understood by considering the on-shell Ward identity for the $\gamma^0 \gamma^0$ vertex. Thus

$$C_{\alpha\beta\mu
u}(b) = A_{\alpha\beta\mu
u}(b) \cdot 2\delta_{\gamma^0 \gamma^0}. \quad (5.14)$$

As a result of the shifts of the fields and the parameters, the $\gamma^0 \gamma^0$ four-point vertex of fig.9c receives the counter term:

$$\Gamma^c_{\gamma^0 \gamma^0} = \Gamma^0_{\gamma^0 \gamma^0} \cdot \delta_{\gamma^0 \gamma^0},$$

with

$$\delta_{\gamma^0 \gamma^0} = -2\delta_g - \frac{3}{4} \cdot \frac{g^2}{16\pi^2} \cdot \ln m^2$$

$$= 2\delta_{\gamma^0 \gamma^0} + \frac{1}{12} \cdot \frac{g^2}{16\pi^2} \cdot \ln m^2. \quad (5.16)$$

All that is left to be done is the evaluation of the irreducible $\gamma^0 \gamma^0$ diagrams. The diagrams that contribute to the leading $\ln m^2$ terms are shown in fig.11. The result is

$$\Gamma^1_{\gamma^0 \gamma^0} = -\Gamma^0_{\gamma^0 \gamma^0} + \frac{1}{12} \cdot \frac{g^2}{16\pi^2} \cdot \ln m^2. \quad (5.17)$$
The one loop corrected $\gamma\gamma WW$ four point vertex is thus given by

$$C_{\alpha\beta\mu\nu}(c) = \{\Gamma^c_{\gamma\gamma WW} + \Gamma^1_{\gamma\gamma WW}\}_{\alpha\beta\mu\nu} = \{\Gamma^0_{\gamma\gamma WW} \cdot 2\delta_{\gamma\gamma WW}\}_{\alpha\beta\mu\nu}. \quad (5.18)$$

When we add up all the contributions given by eqs. (5.11), (5.14) and (5.18), we find for the one loop corrected amplitude $A^w_1 = A^{ww} + C^{ww}$,

$$A^{ww}_1(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = A^{ww}(\lambda_1, \lambda_2, \lambda_3, \lambda_4) \cdot (1 + 2\delta_{\gamma\gamma WW}). \quad (5.19)$$

This result may also be derived by considering on-shell Ward identities, without actually performing the exact one loop calculation for the $\gamma\gamma WW$ vertex. From eq.(3.21) we observe that the process $\gamma\gamma \rightarrow WW$ receives the same ln $m^2$ correction as Coulomb scattering.

The ln $m^2$ term needs to be interpreted as an unknown parameter and must be fixed by experiment. According to the heavy Higgs model the parameter measured in the process $\gamma\gamma \rightarrow WW$ at a center of mass energy less than 1 TeV, is the same as the parameter measured in low energy processes like the $\rho$ parameter, muon decay or Coulomb scattering. A discrepancy would indicate that higher order corrections are important.

6. The process $\gamma\gamma \rightarrow t\bar{t}$

The top quark has yet to be discovered. From the CDF experiment at Fermilab, the lower bound on the top quark mass is given by 100 GeV. From the precision measurements performed at the LEP experiments, the upperbound is derived to be 200 GeV when assuming the validity of perturbation theory.

The Higgs exchange diagram of fig.1 with a $t\bar{t}$ pair in the final state is a next-to-leading order contribution to the amplitude for the process $\gamma\gamma \rightarrow t\bar{t}$. Nevertheless it may lead to a significant effect since the Higgs $t\bar{t}$ coupling is given by the Yukawa coupling and is proportional to $m_t/M = O(1)$. For a given finite Higgs mass value, this contribution has been examined in ref.[30] for the purpose of testing the Yukawa coupling.

6.a The amplitude and the cross-section in lowest non-zero order

The tree level Feynman diagram for the process $\gamma\gamma \rightarrow t\bar{t}$ is shown in fig.12. The corresponding expression for the amplitude in lowest non-zero order in perturbation theory is given by

$$A^{t\bar{t}}(\lambda_1, \lambda_2) = 2\pi\alpha Q^2 \cdot \epsilon^\lambda_1(k_1)\epsilon^\lambda_2(k_2) \cdot \left\{ \frac{1}{p_1k_1} \{ \bar{u}(p_2)\gamma^\beta[-i(\not\!p_1 - \not\!p_2) + m_t][\gamma^\alpha u(-p_1)] \ight.$$  

$$+ \frac{1}{p_1k_2} \{ \bar{u}(p_2)\gamma^\alpha[-i(\not\!p_2 - \not\!p_1) + m_t][\gamma^\beta u(-p_1)] \} \right\}. \quad (6.1)$$

with $Q = 2/3, p_1k_1 = s/4 \cdot (-1 + \beta_t \cos \theta)$ and $p_1k_2 = s/4 \cdot (-1 - \beta_t \cos \theta)$. Using eq.(2.15), we find for the $J = 0$ differential cross-section

$$\frac{d\sigma^{t\bar{t}}_0(++)}{d\cos \theta} = \frac{16\pi\alpha^2 Q^4 N_c \beta_t}{(1 - \beta_t^2 \cos^2 \theta)^2} \cdot \left( \frac{m_t^2}{s^2} \right) \cdot (1 + \beta_t^2). \quad (6.2)$$

21
When the initial photons are in the $J = 2$ state, we have

$$\frac{d\sigma_{0}^{\ell}(\pm \pm)}{d \cos \theta} = \frac{4\pi \alpha^2 Q^4 N_c \beta_i}{s(1 - \beta_i^2 \cos^2 \theta)^2} \cdot \beta_i^2 \sin^2 \theta \cdot (2 - \beta_i^2 \sin^2 \theta),$$

where $N_c = 3$ and $1 - \beta_i^2 = 4m_i^2/s$. The subscript 0 indicates the lowest non-zero order contribution only. At large scattering angles the cross-section behaves like $1/s$ in the $J = 2$ state and behaves like $m_i^2/s^2$ in the $J = 0$ state. The integrated cross-sections are found to be

$$\sigma_{0}^{\ell}(\pm \pm) = \frac{8\pi \alpha^2 Q^4 N_c \beta_i}{s} \cdot \frac{m_i^2}{s^2} \cdot (1 + \beta_i^2) \cdot \left\{ \frac{2c_m}{1 - \beta_i^2 c_m^2} + \frac{1}{\beta_i} \ln \left( \frac{1 + \beta_i c_m}{1 - \beta_i c_m} \right) \right\},$$

and

$$\sigma_{0}^{\ell}(\pm \pm) = \frac{8\pi \alpha^2 Q^4 N_c \beta_i}{s} \cdot \left\{ -c_m - \frac{4m_i^2}{s} \cdot \left( 1 + \frac{2m_i^2}{s} \right) \cdot \frac{c_m}{1 - \beta_i^2 c_m^2} \right\} + \frac{1}{\beta_i} \left( 1 + \frac{2m_i^2}{s} - \frac{4m_i^4}{s^2} \right) \ln \left( \frac{1 + \beta_i c_m}{1 - \beta_i c_m} \right),$$

where $\cos \theta$ is integrated from $-c_m$ to $c_m$.

6.b Finite Higgs mass effects

We only consider the next-to-leading order effects due to the Higgs exchange diagram of fig.1. Although we neglect many other diagrams we expect that near the resonance, i.e. when $-s + m^2 = \mathcal{O}(g^2)$, it will give the leading correction for not too high Higgs mass values. Furthermore, at $-s + m^2 = 0$, this diagram is gauge invariant. The correction will only contribute when the initial photons have the same helicity. As may be derived easily from the discussion of section 3.b, the corresponding expression for the amplitude is given by

$$A_H^{\ell}(\pm \pm) = \alpha \alpha_w \cdot \frac{m_i}{2M^2} \{ \bar{u}(p_2) u(-p_1) \} \cdot \mathcal{H}(s),$$

with

$$\mathcal{H}(s) = P_H \cdot A(H).$$

$P_H$ is given by eq.(3.12), and is gauge invariant because the width is gauge invariant. $A(H)$, given by eq.(3.9), is gauge invariant when evaluated at $-s + m^2 = 0$. Including this correction, the differential cross-section may be written as

$$\frac{d\sigma(\pm \pm)}{d \cos \theta} = \frac{d\sigma_{0}^{\ell}(\pm \pm)}{d \cos \theta} + \frac{d\sigma_{1}^{\ell}(\pm \pm)}{d \cos \theta} + \frac{d\sigma_{2}^{\ell}(\pm \pm)}{d \cos \theta}.$$
\[ d\sigma_0^{Tf}(++)/d\cos\theta \] is the cross-section in lowest non-zero order and is given by eq.(6.2). The expression for the interference term \[ d\sigma_1^{Tf}(++)/d\cos\theta \] is given by

\[
\frac{d\sigma_1^{Tf}(++)}{d\cos\theta} = \frac{\alpha^2\alpha_w Q^2 N_c \beta_i}{1 - \beta_i^2 \cos^2\theta} \cdot \left( \frac{-\beta_i^2 m_i^2}{2sM^2} \right) \cdot \{\mathcal{H}(s) + \mathcal{H}^*(s)\}. \tag{6.9}
\]

The third and last term is of \( \mathcal{O}(g^8) \) and clearly will only give a significant contribution for a sufficiently narrow width of the Higgs. We have

\[
\frac{d\sigma_2^{Tf}(++)}{d\cos\theta} = \frac{\alpha^2\alpha_w^2 N_c \beta_i}{64\pi} \cdot \left( \frac{c_m\beta_i^2 m_i^2}{M^4} \right) \cdot \{\mathcal{H}(s) \cdot \mathcal{H}^*(s)\}. \tag{6.10}
\]

Writing \( \mathcal{H}(s) = (a + ib)/\{s - m^2 - im\Gamma(s, g^2)\} \), we have near the Higgs resonance

\[
\mathcal{H}(s) + \mathcal{H}^*(s) = \frac{-2b}{m\Gamma(g^2)} + \mathcal{O}(1), \tag{6.11}
\]

and

\[
\mathcal{H}(s) \cdot \mathcal{H}^*(s) = \left( \frac{a^2 + b^2}{m^2\Gamma^2(g^2)} \right) + \mathcal{O} \left( \frac{1}{g^2} \right). \tag{6.12}
\]

Here

\[
a = 6M^2 + m^2 + 6M^2 \cdot (m^2 - 2M^2) \cdot \Re \left( \frac{C_0^w}{1\pi^2} \right)
+ Q^2 N_c m_i^2 \cdot \left\{-4 + (8m_i^2 - 2m^2) \cdot \Re \left( \frac{C_0^t}{1\pi^2} \right) \right\}, \tag{6.13}
\]

and

\[
b = 6M^2 \cdot (m^2 - 2M^2) \cdot \Im \left( \frac{C_0^w}{1\pi^2} \right)
+ Q^2 N_c m_i^2 \cdot (8m_i^2 - 2m^2) \cdot \Im \left( \frac{C_0^t}{1\pi^2} \right). \tag{6.14}
\]

The integrated cross-section is given by

\[
\sigma^{Tf}(++) = \sigma_0^{Tf}(++) + \sigma_1^{Tf}(++) + \sigma_2^{Tf}(++), \tag{6.15}
\]

with \( \sigma_0^{Tf}(++) \) as given by eq.(6.4). Furthermore

\[
\sigma_1^{Tf}(++) = \frac{\alpha^2\alpha_w Q^2 N_c \beta_i}{s} \cdot \frac{m_i^2}{M^2} \cdot \left\{-\frac{\beta_i}{2} \ln \left( \frac{1 + \beta_i c_m}{1 - \beta_i c_m} \right) \right\} \cdot \{\mathcal{H}(s) + \mathcal{H}^*(s)\}, \tag{6.16}
\]

and

\[
\sigma_2^{Tf}(++) = \frac{\alpha^2\alpha_w^2 N_c \beta_i}{32\pi} \cdot \frac{m_i^2}{M^4} \cdot \beta_i^2 c_m \cdot \{\mathcal{H}(s) \cdot \mathcal{H}^*(s)\}. \tag{6.17}
\]
6.c The large Higgs mass limit

We are interested in the contribution of the Higgs exchange diagram of fig.1 to the \( \gamma \gamma \to t\bar{t} \) cross section in the limit of a heavy Higgs. It is derived from eq.(6.16) by taking the limit of a large Higgs mass in the expression for \( \mathcal{H}(s) \). Let us write

\[
\mathcal{H}_\infty(s) = \{\mathcal{H}(s)\}_{m \to \infty} = 1 + \mathcal{O}\left( \frac{M^2}{s} \right).
\] (6.18)

The \( U \)-particle contribution to \( \mathcal{H}_\infty(s) \) is given by

\[
\mathcal{H}_U(s) = -\frac{\alpha_w}{4\pi c_w^2} \cdot \left( \frac{\beta_U s}{8M^2} \right) \cdot \left\{ 1 + \mathcal{O}\left( \frac{M^2}{s} \right) \right\}.
\] (6.19)

Including these corrections, we have

\[
\sigma^{t\bar{t}}(++) = \sigma_0^{t\bar{t}}(++) + \sigma_\infty^{t\bar{t}}(++) + \sigma_U^{t\bar{t}}(++),
\] (6.20)

where \( \sigma_0^{t\bar{t}}(++) \), given by eq.(6.4), is the cross section in lowest non-zero order. The corrections according to the heavy Higgs model are given by

\[
\sigma_\infty^{t\bar{t}}(++) = \frac{\alpha^2 \alpha_w Q^2 N_c \beta_t}{s} \cdot \frac{m_t^2}{M^2} \cdot \left\{ -\frac{\beta_t}{2} \ln \left( \frac{1 + \beta_t c_m}{1 - \beta_t c_m} \right) \right\} \cdot \{\mathcal{H}_\infty(s) + \mathcal{H}^*_\infty(s)\} \] (6.21)

and

\[
\sigma_U^{t\bar{t}}(++) = \frac{\alpha^2 \alpha_w Q^2 N_c \beta_t}{s} \cdot \frac{m_t^2}{M^2} \cdot \left\{ -\frac{\beta_t}{2} \ln \left( \frac{1 + \beta_t c_m}{1 - \beta_t c_m} \right) \right\} \cdot \{\mathcal{H}_U(s) + \mathcal{H}^*_U(s)\}. \] (6.22)

It is seen that the cross section for the \( J = 0 \) state corresponds to the following expansion in \( s/M^2 \):

\[
\sigma^{t\bar{t}}(++) = \alpha^2 \cdot \frac{m_t^2}{s^2} \cdot \left\{ c_1^t + \alpha_w \cdot \frac{s}{M^2} \cdot c_2^t + \alpha_w^2 \cdot \frac{s^2}{M^2} \cdot \left\{ c_3^t \cdot \ln \frac{m_t^2}{s} + c_4^t + \ldots \right\} \right\}, \] (6.23)

where we must make the replacement

\[
c_4^t \to c_4^t + c_4^{tU}. \] (6.24)

From eqs.(6.4), (6.21) and (6.22) the expressions for \( c_1^t, c_2^t \) and \( c_4^{tU} \) are found in a very straightforward way. We therefore do not explicitly write them down. As it would require a two loop calculation, we have not evaluated the expression for \( c_3^t \). Let us compare the above expression to the \( s/M^2 \) expansions for the \( \gamma \gamma \to ZZ_{L} \) and \( \gamma \gamma \to WW_{L} \) cross sections. These are obtained by squaring the amplitudes, given by eq.(3.17), and
subsequently multiplying with $1/s$. We see that the $\gamma\gamma \to t\bar{t}$ process is suppressed by an overall factor $m_t^2/s$. Thus although the heavy Higgs effects are expected to be smaller in the $t\bar{t}$ channel than in the $Z_LZ_L$ or $W_LW_L$ channel, here the $t\bar{t}$ channel is relatively background free.

According to the heavy Higgs model the leading Higgs mass correction to the $J = 2$ cross section may be written as

$$\sigma_{t\bar{t}}^{(+)} = \alpha \cdot \frac{1}{s} \left\{ c_5^t + \alpha_w \cdot c_6^t \cdot \ln \frac{m^2}{M^2} + \mathcal{O}(1) \right\}.$$  

(6.25)

The term $c_5^t$ may be obtained by considering eq.(6.5). The term $c_6^t$ is found by performing a one loop calculation in the limit of a large Higgs mass. This has been done in ref.[12] and the result is

$$\sigma_{t\bar{t}}^{(+)} = \sigma_{t\bar{t}}^{(+)} \cdot \left\{ 1 + 4\delta_{\gamma w} \right\}.$$  

(6.26)

The correction is identical to the correction for the unpolarized $\gamma\gamma \to WW$ cross section.

6.d Results

In fig. 13 we show the total cross-section in lowest non-zero order, with an angular cut of $c_m = 0.7$ and for a top quark mass value of 150 GeV. The $J = 0$ cross-section is more rapidly vanishing for large $s$ then the $J = 2$ cross section.

In fig.14a we have plotted the effect due to the Higgs exchange diagram of fig.1 for the two different models that are considered in this paper. The top quark mass is taken to be 120 GeV. The existence of the elementary Higgs boson as predicted by the Standard Model Lagrangian can lead to a significant effect, notably for Higgs mass values below 500 GeV. For Higgs mass values above 700 GeV the cross section becomes very insensitive to the exact value of the Higgs mass. On the other hand, there is a significant difference with respect to the effect as predicted by the heavy Higgs model for $\beta^U \leq 5$. Assuming the validity of the partial wave analysis, $\beta^U = 5$ corresponds to a 2 TeV resonance in the $I = 1$ channel for $W_LW_L$ scattering; a smaller value for $\beta^U$ would indicate that the resonance is located at a higher energy.

Figs.14b and 14c show similar curves for top quark mass values of 150 GeV and 180 GeV.

7. Summary and concluding remarks

For each of the processes $\gamma\gamma \to ZZ$, $\gamma\gamma \to WW$ and $\gamma\gamma \to t\bar{t}$ we have investigated the level of sensitivity to the Higgs sector of the Standard Model Lagrangian in the energy region between 200 GeV and 1 TeV.

We have considered the Standard Model Lagrangian containing the elementary Higgs boson with a mass less than 1 TeV. The elementary Higgs boson may be produced through
\[ \gamma\gamma \text{ fusion, given that the initial photons are in the } J = 0 \text{ state. For Higgs mass values up to} \]
\[ 400 \text{ GeV a resonance signal may be observed in the } ZZ \text{ channel. This conclusion has also} \]
\[ \text{been reached in refs.}[21-23]. \text{ For higher Higgs mass values the resonance signal disappears} \]
\[ \text{and detecting the Higgs has turned into a precision measurement. This is because the} \]
\[ \text{Higgs width has become very broad and furthermore the transverse polarized vector boson} \]
\[ \text{production is a factor 100 larger than the longitudinal vector boson production, and is} \]
\[ \text{a severe background. For example for a Higgs mass value of 500 GeV there is a 10\%} \]
\[ \text{enhancement. For even higher Higgs mass values effects are less than 5\%. In the } WW \]
\[ \text{channel the background due to transverse vector boson production is even more substantial.} \]
\[ \text{In addition, the Higgs exchange diagram of fig.1 contributes to the amplitude only in next-} \]
\[ \text{to-leading order. For Higgs mass values higher than 300 GeV the effect is less than a few} \]
\[ \text{percent [17]. In the } t\bar{t} \text{ channel the Higgs exchange diagram of fig.1 also contributes to the} \]
\[ \text{next-to-leading order only. However the } t\bar{t} \text{ channel is relatively background free. For Higgs} \]
\[ \text{mass values above 400 GeV the size of the effect in the } t\bar{t} \text{ channel is about the same as the} \]
\[ \text{size of the effect in the } ZZ \text{ channel. Also here, for Higgs masses larger than 600 GeV the} \]
\[ \text{effect is reduced to less than a few percent.} \]

\text{If the elementary Higgs boson has not been found below an energy of 1 TeV, we assume} \]
\text{there is new physics which must show up in the TeV region. To examine this possibility we} \]
\text{have considered the heavy Higgs model. This model corresponds to the Standard Model} \]
\text{Lagrangian plus an additional Higgs interaction with a fictitious } U \text{ particle. The} \]
\text{corresponding coupling strength is denoted by } g_U. \text{ The mass of the } U \text{ particle is taken to be} \]
\text{equal to the mass of the Higgs particle. Subsequently, after a typical one loop calculation} \]
\text{the limit of a heavy Higgs mass is taken. In this way we are left with terms proportional to} \]
\[ \ln m^2 \text{ and } g_U^2. \] \text{These terms are to be interpreted as arbitrary parameters as may be easily} \]
\text{understood within the framework of effective field theory. Assuming the validity of the one} \]
\text{loop approximation, the heavy Higgs model contains just these two arbitrary parameters.} \]
\text{If higher order corrections are important, the } \ln m^2 \text{ and } g_U^2 \text{ terms obtained for one process} \]
\text{cannot be compared to the } \ln m^2 \text{ and } g_U^2 \text{ terms obtained for another process: they are then} \]
\text{independent sets of unknown parameters. Sensitivity to these arbitrary parameters implies} \]
\text{sensitivity to the new physics. A possible manifestation is the occurrence of a resonance in} \]
\text{the } I = 1 \text{ channel for } W_L W_L \text{ scattering. Assuming the validity of partial wave analysis,} \]
\text{the location of this resonance depends on the Lehmann } \beta \text{ parameter. The } \beta \text{ parameter} \]
\text{depends on } g_U^2 \text{ only. If } \beta = 5 (g_U^2 = 27) \text{ the resonance is located at 2 TeV, as predicted} \]
\text{by QCD-like models.} \]

\text{We have evaluated the cross sections for the processes } \gamma\gamma \rightarrow ZZ, \gamma\gamma \rightarrow WW \text{ and} \]
\[ \gamma\gamma \rightarrow t\bar{t}, \text{ according to the heavy Higgs model. We have examined the sensitivity that} \]
\[ \text{these processes may have to the } \beta \text{ parameter. Unfortunately, in the } ZZ \text{ and } WW \text{ channels} \]
\[ \text{effects are insignificant due to the enormous } Z_T Z_T \text{ and } W_T W_T \text{ background.} \]

\text{The effects in the } t\bar{t} \text{ channel are more significant. It may be possible to establish the} \]
\text{non-existence of the elementary Higgs boson with a mass of 1 TeV or less, thus favouring} \]
\text{the heavy Higgs model. Sensitivity to the } \beta \text{ parameter, i.e. sensitivity to the new physics,} \]
\text{requires an } \mathcal{O}(1\%) \text{ accuracy at } \sqrt{s} = 1 \text{ TeV. At this energy the } \gamma\gamma \rightarrow t\bar{t} \text{ cross section with} \]
\text{the initial photons in the } J = 0 \text{ state is about } 50-100 \text{ fb (depending on the top quark} \]
\text{mass).}
We have not discussed the role of the $\ln m^2$ term in this paper. We only wish to remark that this term, within the framework of partial wave analysis, may be correlated to a possible occurrence of a resonance in the $I = 0$ channel for $W_L W_L$ scattering [9-11].

References

[26] M. Veltman, Formf, a program that calculates one loop form factors.

Figure Captions

Fig.1  Higgs production through $\gamma\gamma$ fusion.
Fig.2  Kinematics for the process $\gamma\gamma \rightarrow XX$; $XX$ represents a ZZ, a $W^+W^-$ or a $t\bar{t}$ pair.
Fig.3  $\gamma\gamma \rightarrow ZZ$ lowest order Feynman diagrams
        (when labelling the external lines, include crossings).
        The dashed lines represent charged W and charged Higgs ghost $\phi$ lines.
        The dotted lines represent, in addition to $W$ and $\phi$, Faddeev-Popov ghost $\psi$
        and fermion $f$ lines. The $\psi$ and $f$ lines have a direction and
        need to be supplemented with an arrow.
Fig.4  The process $\gamma\gamma \rightarrow ZZ$ at a center of mass energy $\sqrt{s} = 200$ GeV.
        4a  $J = 0$ differential cross section as a function of $\cos \theta$.
        4b  $J = 0$ : Ratio $R(m, \theta, \sqrt{s} = 200$ GeV) as a function of $\cos \theta$ in \%.
        The value of the Higgs mass is as indicated.
        $R(m, \theta, \sqrt{s})$ is defined in eq.(4.18).
        4c  $J = 2$ differential cross section as a function of $\cos \theta$.
Fig.5  Same as fig.4, but here at $\sqrt{s} = 300$ GeV.
        The value for the top quark mass is as indicated.
Fig.6  Same as fig.4, but here at $\sqrt{s} = 500$ GeV.
        The value for the top quark mass is as indicated.
Fig.7  Same as fig.4, but here at $\sqrt{s} = 1$ TeV.
        The value for the top quark mass is as indicated.
Fig.8  $U$ particle correction to the $J = 0$ differential cross section for the process
        $\gamma\gamma \rightarrow Z_LZ_L$. The correction, in \%, is plotted as a function of $\cos \theta$
        for $\beta_U \approx 5$.
Fig.9  $\gamma\gamma \rightarrow W^+W^-$ lowest order Feynman diagrams
        (when labelling the external lines, include crossings).
Fig.10 The unpolarized differential cross section for the process $\gamma\gamma \rightarrow W^+W^-$
        as a function of $\cos \theta$.  

28
Fig. 11  One loop Feynman diagrams that give a leading $\ln(m^2)$ contribution to the process $\gamma \gamma \rightarrow W^+ W^-$ (when labelling the external lines, include crossings).

Fig. 12  $\gamma \gamma \rightarrow t\bar{t}$ lowest order Feynman diagram (when labelling the external lines, include crossings).

Fig. 13  The process $\gamma \gamma \rightarrow t\bar{t}$ for $m_t = 150$ GeV.

The $J = 0$ and $J = 2$ total cross sections are shown as a function of the center of mass energy $\sqrt{s}$. An angular cut of $c_m = 0.7$ is applied.

Fig. 14  Next-to-leading order correction in % to the $\gamma \gamma \rightarrow t\bar{t}$ cross section.

14a  $m_t$ is taken to be 120 GeV. The different curves show the effects according to the different models considered in this paper.

14b  Same as fig. 14a but here $m_t = 150$ GeV.

14c  Same as fig. 14a but here $m_t = 180$ GeV.