Fine tuning of parameters of the universe

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Abstract

The mechanism of production of a large number of universes is considered. It is shown that universes with parameters suitable for creation of life are necessarily produced as a result of quantum fluctuations. Fractal structures are formed provided fluctuations take place near a maximum of the potential. Several ways of formation of similar fractal structures within our universe are discussed. Theoretical predictions are compared with observational data.

1 Introduction

Within many years it was supposed that we live in a space with Friedman-Robertson-Walker metric. From the astrophysical point of view it means an expanding universe with small negative acceleration. From the point of view of modern field theory it means the vacuum energy density being strictly zero or, equivalently, a vanishing cosmological constant. There were no clear theoretical reasons for this, but there was speculation about a hidden symmetry, implying this strict equality (see, for example, the review [1]).

Four years ago observations [2] indicated some positive value of the cosmological constant $\Lambda \approx 0.7 \rho_M$, which is only a little less than the average density of matter $\rho_M$ in the universe. All quantum effects, which give a contribution to the vacuum energy, surpass this value by many orders of magnitude. The mechanism of almost complete cancellation of different contributions is still not understood. And at the same time, if the cosmological constant would be approximately 200 times larger as its present value, galaxies would not have been formed [3] and life would have been impossible. The impression is that the universe is specially arranged to create life.

The bound of the cosmological constant described above is not the only case where the existence of life implies a constraint on parameters in nature. In elementary particle physics there are a number of similar examples. I recall here only one - the smallness of the electron mass. The electron mass is about 2000 times smaller than the nucleon mass. One might suppose that it would not to matter if it would be several times larger than its value $0.511 MeV/c^2$. But in this case neutrons would be stable and the process $p^+ + e^- \rightarrow n + \bar{\nu}$ would result in a sharp decrease of proton abundance in the universe with adverse consequences for the existence of life. We see that the universe is "adjusted to life" by a set of parameters and the cosmological constant is only one of those parameters (for a recent review see e.g. [4]). It looks like that nature has in store a large number of universes and only a small number of it is suitable for life. The question is how to find a mechanism to select those.

In this paper a mechanism of production of a large number of universes is considered. The universes differ from each other in physical parameters. It is shown that universes with parameters suitable for creation of the life are necessarily produced as a result of quantum fluctuations. The distribution of the universes has fractal character provided fluctuations take place near the maximum of a potential. The ways of observation of fractal structures inside our universe are discussed in Section 3.
2 Basic postulates

Usually, when a theoretical model is set up, first a concrete Lagrangian is postulated. Coupling constants are assumed to be small such that quantum corrections to the original Lagrangian are considered to be small as well. Nevertheless, corrections are small only for weak fields, while for strong fields this does not hold. To be more specific, let us consider the Lagrangian of a scalar field

$$L = \frac{1}{2} (\partial_{\mu} \phi)^2 - \frac{m^2}{2} \phi^2 - \frac{\lambda}{4} \phi^4.$$  \hspace{1cm} (1)

One can compute one-loop quantum corrections to the potential and finds [5]

$$\delta V = \frac{(3\lambda \phi^2 + m^2)^2}{64\pi^2} \ln \left( \frac{3\lambda \phi^2 + m^2}{2m^2} \right) - a\phi^2 - b\phi^4.$$  \hspace{1cm} (2)

The last two terms renormalize the mass and coupling constant of the Lagrangian and depend on the scheme of renormalization. The first term changes the form of the potential. This is the most important term for the following considerations. Multi-loop corrections as well as interaction with other fields may add new terms to the potential. It is important to note that any simple interaction causes an infinite number of additional terms to the original Lagrangian.

It is easy to see, comparing expressions (1) and (2), that new terms are small in comparison with the original terms if $\varphi << m \cdot \exp(1/\lambda)$. To get an estimate, one may choose $m = 100 GeV$, $\lambda = 0.1$, then quantum corrections to the potential become large at $\varphi \sim 10^6 GeV$. It is a rather large energy for an accelerator. However, at an early inflationary stage of our universe the average value was rather large, $\varphi > 10^{19} GeV$. Hence, it is necessary to take into account an infinite number of additional terms in the Lagrangian (1). Moreover, the amplitude of a scalar field is restricted even more stringently. The logarithm in expression (2) is the result of the summation of an infinite number of terms [6], which converges only when $\varphi < m/\sqrt{3}\lambda$. Besides, one can see directly from Lagrangian (1) that the interaction term is of order of the mass term when $\varphi \sim m/\sqrt{2}\lambda$. Two last estimations are in good agreement with each other and give a much smaller value of the field when quantum corrections are really small. A similar problem was discussed in the framework of hybrid inflation [7].

Thus, when considering phenomena in strong fields, i.e. $\varphi > m/\sqrt{\lambda}$, it is necessary to take into account all additional terms, inevitably arising due to quantum corrections. The potential becomes much more complex, based on the low energy limit of the theory. This can be visualized by the picture of mountains and valleys. In a mountain area it is possible to have smooth surfaces with small curvature only in valleys, i.e., in minima of the potential energy. After climbing to some height, it becomes obvious that the shape of the terrain is much more complex.

Usually the potential of interaction of a scalar field is assumed to be of the most simple form. The property of renormalizability of the theory is not required if one supposes that gravitational effects on Plank scale will regularize integrals. Usually, the fields are weak and quantum corrections are reduced to the renormalization of parameters of a Lagrangian under the assumption that the final corrections are small. As consequence of the previous discussion, at the moment of formation of our universe, i.e., at large amplitudes of a field, quantum corrections most likely were comparable with original terms of the Lagrangian, and its form was much more complex than the Lagrangian considered above.

The main conclusion is that the choice of any simple form of Lagrangian with specific parameters leads to difficult problems: One must explain ab initio the origin of both the form of Lagrangian and numerical values of parameters and finally manage to prove that quantum corrections are small at high energies. In addition, the field is only a dynamical variable which has no physical meaning. It is not clear why we should single out the value $\varphi = 0$ when postulating the form of a potential.

Let us take the opposite point of view and limit ourselves to the minimal number of specific assumptions about the form of a potential. Namely, let us postulate some kind of "democracy" - all terms are possible - and consider consequences of this assumption. More accurately, I suppose:

- The potential of a scalar field is a polynomial with an infinite number of terms. Coefficients of polynomial terms are uncorrelated numbers and are normalized by the Plank mass $M_{pl}$. As was discussed above, this postulate does not contradict conclusions of the quantum field theory at low energy near the bottom of the potential. At high energy it leads to qualitatively new results.
As an example, let us consider the following Lagrangian of a scalar field

\[ L = \frac{1}{2} (\partial_{\mu} \varphi)^2 - V(\varphi). \]

The field \( \varphi \) is determined in the interval \((-\infty, +\infty)\). The typical behavior of the potential is represented in 1. It should also be stated that the Lagrangian (2) is a special case of a more general Lagrangian, in which quantum corrections to the kinetic term would be taken into account.

The universe is located in one of the minima, where the potential \( V(\varphi) \) can be approximated in a simple way: \( V(\varphi) \approx V(\varphi_m) + a\phi^2 + b\phi^4, \quad \phi = \varphi - \varphi_m. \) Usually a similar potential is postulated from the beginning with specific constants \( a \) and \( b \). The constant \( a \) is connected with mass of a quanta of the field \( \varphi, a = m_\varphi^2/2, \) if \( a > 0 \). Other universes occupy other minima which are characterized by a potential with different parameters \( a \) and \( b \). The next section is devoted to cosmological consequences of the above postulates.

3 Quantum fluctuations as the generator of the universes

All (quasi) stationary states are located in minima of a potential and our universe, not being an exception, is located in such one minimum as well. As there is an enumerable set of minima (remind that the potential in question is the polynomial with infinite number of terms), each of which is characterized by some specific energy density, it seems unlikely that our universe has appeared just in the minimum with a very small energy density suitable for life. For an estimate of this probability let us assume that the probability to end up in a minimum of the potential with energy density \( \rho_V^{(m)} = V(\varphi_m) \) in an interval \( d\rho_V \) is given by

\[ dP(\rho_V^{(m)}) = d\rho_V^{(m)}/M_{pl}^4 \]

(i.e. the uniform distribution of \( \rho_V^{(m)} \) is assumed in the whole interval \((0, M_{pl}^4))\). The observational value of energy density in our universe is \( \rho_V \sim 10^{-123} M_{pl}^4 \). Thus, we come to the conclusion that the fraction of universes with vacuum energy density similar to ours is \( \approx 10^{-123} \). It is hard to believe, given an infinite number of such universes, that an event with such small probability has happened in nature. We conclude that a mechanism of a ”sorting” of the universes is necessary, and such mechanism really exists.

To proceed, let us show that if we have a set of potential minima where life is possible, the field ends up in one of those minima starting from an arbitrary initial value of the field. Consider Fig. 1. Let the field start at a value represented by point A in Fig. 1. The spatial area which is characteristic of the size of fluctuation, is chosen to equal the Planck scale \((\sim 10^{19} GeV \approx 10^{-33} cm)\). It is a lower limit where the concept of time can be used. The further destiny of the area strongly depends on field configurations within this area. Configurations being important for our considerations are those where spatial derivatives are small, i.e. \( (\partial\varphi_{\mu})^2 \ll V(\varphi) \). In this case we can use well developed methods of inflation theory and especially chaotic inflation [5]. The inflation paradigm is developing during more than 20 years [8, 12, 16, 17]. It successfully solves the basic problems of cosmology of our universe starting from the earliest stage since its creation and ending by the stage of galaxy formation. Here it worth to mention at least the horizon problem, the flatness problem and the problem of magnetic monopole absence.

It is important to note that the size of spatial area corresponding to fluctuation is growing exponentially. A physical distance \( R \) between two points increases like \( R(t) \sim exp(Ht) \), while the size of horizon \( 1/H \) remains constant. The Hubble parameter \( H \) is connected to the energy density of the potential \( H = \sqrt{8\pi V(\varphi)/3m_{pl}^2}. \) It is obvious that an initial causally connected volume of size \( 1/H \) is divided into a number \( \approx e^3 \) of causally disconnected areas with the same size \( 1/H \) in characteristic time \( 1/H \). The field values in different areas may differ from each other due to quantum fluctuations.
Figure 1: A part of the potential in a finite range of field $\varphi$.

It should be noted that this picture represents a look 'from inside' of the domain. An external observer would detect only field fluctuations of the size $1/H$ [4, 5]. Thus, the initial area is divided into an increasing number of causally disconnected areas with various values of the field $\varphi$. This process is the main process of the inflationary scenario.

Thus, the size of the originally chosen area grows, new areas with slightly different field values arise inside it. In some of the areas the field tends to the nearest local minimum of the potential, while in some other areas field values approach its local maximum due to fluctuations.

A universe with specific vacuum energy density is formed inside the domain where the field reached a potential minimum. According to the above estimate, the probability that this density favors life of our type, is of the order of $\sim 10^{-123}$. New quantum fluctuations in any given universe produce new spatial domains with high energy density $\sim M_{pl}^4$ and size $\sim 1/M_{pl}$. The process of size expansion in these causally disconnected volumes repeats itself in the manner discussed above [5]. Some of these domains contain field values corresponding to the slope of the potential at point B in Fig. 1. Thus, quantum fluctuations allow a field in some domains to pass through minima of the potential.

The destiny of spatial areas where the field overcomes potential maxima is much more interesting. Consider the fluctuation of the field near such maximum (point C in 1). The initial spatial size of this fluctuation is $\sim 1/H$. When some time of the order of $\sim 1/H$ has passed this spatial area will be separated into $e^3$ causally disconnected domains with different field values. The average value of the field $\varphi$ inside some of these domains will appear at the other side of the maximum (point C' in Fig. 1). Each of these domains will be divided in $e^3$ subdomains of the size of $\approx 1/H$ in time $1/H$ and some of them will pass back through the maximum of the potential. This process continuously reproduces itself and already after several steps a picture of a fractal structure will be observed. Until now we have not considered the motion of the classical field which is governed by the classical equation [5]

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{dV}{d\varphi}.$$  \hspace{1cm} (5)

According to this equation of motion the classical field moves away from the maximum what could prevent the formation of the fractal structure. Hence, the development of fractal structure in a final stage can take place only if the fluctuations are large. More specifically, let us assume that the classical field changes its value by $\Delta\varphi_{cl}$ in the time $1/H$. Then a fractal structure arises if the condition of the fluctuation dominance $\Delta\varphi_{fluct} >> \Delta\varphi_{cl}$ is satisfied. It gives enough time for formation of a fractal
structure due to the fluctuations around a maximum. An average value of fluctuations is well known, given by \( \Delta \varphi_{\text{fluct}} \approx H/2\pi \). The classical motion can be computed explicitly if one approximates the potential around a maximum by the function

\[
V(\varphi) \approx V_0 - (\varphi - \varphi_{\text{Max}})^2/2.
\]

An approximate solution of Eq.(5) has the form

\[
\varphi(t) \approx \varphi_{\text{Max}} + [\varphi(t = 0) - \varphi_{\text{Max}}]e^{\left(-t^2M_{\text{pl}}^2V_0\right)^{1/2}},
\]

where the second time derivative is neglected as is usually done at the inflation stage. The initial field value \( \varphi(t = 0) \approx \varphi_{\text{Max}} + \Delta \varphi_{\text{fluct}}/2 \) and the condition of quantum fluctuation dominance is easily found to be

\[
\eta = \frac{\Delta \varphi_{\text{fluct}}}{\Delta \varphi_{\text{cl}}} \approx \frac{H}{a^2M_{\text{pl}}} > 1.
\]  

The number of fractals increases with the parameter \( \eta \). For an estimate let us take the Planck scale: \( V_0 = M_{\text{pl}}^4, a = M_{\text{pl}} \). It leads to the value \( \eta \approx 16\pi \) and hence to a rich fractal structure in the final stage.

It is well known that two domains with field values separated by a potential maximum, are separated by a wall [10]. Classically, fields in such domains tend to various (neighboring) minima, and hence the energy density of the wall grows relative to the rest of the space. We come to the conclusion that neighbor universes are separated by field walls with large energy density.

Thus, quantum fluctuations continuously produce spatial domains with various values of the field \( \varphi \). Among these set of domains it is always possible to find a sequence of domains with monotonously growing field inside them. The fields in such domains will consistently pass all minima and maxima on its way. Eventually, a minimum with energy density suitable for life of our type will be found.

### 4 Fractal structures in our universe

Let us consider in more detail the process of production of the closed walls. As was already discussed above, if a causally connected area is placed near a maximum of the potential, for example \( \varphi \geq \varphi_{\text{max}} \), then several subdomains with average field value \( \varphi \leq \varphi_{\text{max}} \) will appear inside during the time \( 1/H \).

Moving along any line, connecting internal and external points of a subdomain, we necessarily pass through the maximum of the potential. Therefore, this subdomain is limited by the surface where the potential has the maximal value, i.e., by the closed field wall with definite surface energy density.

Just after formation the subdomain is placed near the potential maximum, which allows to repeat the process. Hence, closed walls of smaller scale will appear already inside this subdomain. Below it will be shown that this process results in the formation of fractal structures.

Suppose for a characteristic time \( 1/H \) several closed walls appear in a causally connected area of size \( R \) near a maximum of the potential. Denote the number of walls by \( N \) and its average size by \( \xi R \), \( \xi > 1/e \) (\( \xi \neq 1/e \) due to a possible merging of causally disconnected subdomains with one common wall). In each of these subdomains, \( N \) new smaller closed walls of size \( \xi^2 R \) arise during the next time step. Denote by \( ^n a^n \) the minimal size of such a wall that we are able to distinguish. This means that we may terminate the process after a step \( n \) such that \( a = \xi^n R \). The total area of the closed walls in the initial volume is the sum of areas with closed walls of size greater than \( a \). The simple summation leads to the following result

\[
S \approx R^2 q(q^n - 1)/(q - 1), \quad q = \xi^2 N.
\]  

This expression can be written in the form

\[
S \approx (R/a)^D,
\]  

where \( D \) is the fractal dimension. Equating these two expressions, one obtains

\[
D = 2 + \ln\left(q^2\frac{q^n}{q^n a^{-1}}\right) \quad \text{ln}(R/a).
\]
This quantity is constant only when the ratio $R/a$ is large, it is different for $q < 1$ and $q > 1$. It can be easily verified that $D \rightarrow 2$ for $q < 1$, while for $q > 1$, $D \rightarrow 2 + 3\ln(q)/\ln(4N)$. To get an estimate, suppose that the number of closed domains is $N \approx 4$, and $\xi \approx 1/e$. The value of the parameter $q$ can be easily calculated, $q \approx 0.5$. Hence, the fractal dimension of the system of closed walls $D \approx 2$.

So, if quantum fluctuations lead to the formation of spatial areas with the field taking a value near a potential maximum, its further evolution results in a system of enclosing walls. The characteristic size of the next generations of walls differs from the previous one approximately by a factor of $e$. The fractal dimension of such system is $D \approx 2$. The analytical calculations were done using approximations and hence the expression (9) has to be considered as an estimate.

According to the above postulates and based on the framework of chaotic inflation, our universe is a part of a meta-universe which was formed from one domain surrounded by a closed wall. The inflationary mechanism provided an exponential increase of its size from the point of view of an internal observer. The size of the universe, as presently observed, is estimated to be $\sim 10^{28}cm$, being smaller by many orders than the characteristic scale of the meta-universe $\sim 10^{10^{12}}cm$, [11]. Hence, the walls surrounding our meta-universe are not observable. Nevertheless, it turns out that the mechanism of generation of fractal structures appears in a natural way in many models of inflation. Below three different models which could give rise to observable consequences are considered.

The first mechanism of the formation of the observable structure is based on the main postulates of Section 2. Suppose that the potential $V(\varphi)$ has a local minimum, which is placed close to the main minimum, as shown in Fig. 1, point D. The potential can be approximated as follows

$$V(\varphi) = \begin{cases} \frac{1}{2}m^2 (\varphi - \varphi_m)^2; & |\varphi - \varphi_D| >> \Delta \varphi_D \\ \frac{1}{2}m^2 (\varphi_D - \varphi_m)^2 + \frac{1}{2}M^2 (\varphi - \varphi_D)^2; & |\varphi - \varphi_D| << \Delta \varphi_D \end{cases},$$

where $\varphi_D$ is field value at the local minimum. Inflation takes place when $H >> m$ which is supposed to hold in this case.

In the vicinity of the local minimum, the equation of motion (5) becomes simpler,

$$\ddot{\varphi} + 3H(\varphi_D) \dot{\varphi} + M^2 (\varphi - \varphi_D) \approx 0 .$$

If $H(\varphi_D) >> M$, dissipation of energy is large and the field could be located in the local minimum for a long time. We encounter serious problems, which were discussed in connection with first inflationary models [12] where our universe is formed from the domain in a local potential minimum.

In the case $H(\varphi_D) \leq M$ the situation differs from the previous one. The field slowly decreases, according to equation (11) until it appears in the vicinity of the local minimum $\varphi = \varphi_D$, where the equation of motion can be reduced to

$$\ddot{\varphi} + M^2 (\varphi - \varphi_D) \approx 0 ,$$

and the total energy of the field is approximately conserved. In this case the value of the classical field could overcome the local maximum and approach the nearest deeper minimum of the potential. In the meantime, the fluctuations described in previous section, occur in some domains near the local maximum, which leads to the formation of fractal structure. If this local maximum is deep enough, the expansion of space increases their sizes not very much. These fractal structures being small in comparison with the size of our universe could result in observable consequences.

Let us consider briefly other mechanisms of formation of similar fractal structures. They are based on a multicomponent or a complex field instead of a scalar one, which was studied so far. In this case the mechanism of closed wall production is the same as discussed above. The only difference is that fluctuations ought to be investigated near saddle points of the potential rather than near maxima.

To be specific, let us choose a complex field and, following Ref.[13] consider the process of formation of our universe in the framework of natural inflation on the basis of the Lagrangian

$$L = \partial_{\mu} \Phi^* \partial^{\mu} \Phi - \lambda \left( |\Phi|^2 - f/2 \right)^2 - \Lambda^4 (1 - \cos \theta) ,$$

where $\theta$ is a phase of the complex field $\Phi$. The last term is an approximation of rather complex expression for the contribution of quantum corrections.

The complex field moves, according to the equations of motion, to a minimum of the potential at a point $\theta = 0$. At the same time, due to quantum fluctuations, some part of causally disconnected
In the standard version of hybrid inflation the potential contains two fields

\[ V = V_0 + \frac{1}{2} m_\varphi^2 \varphi^2 + \frac{1}{2} \lambda_1 \varphi^2 \psi^2 - \frac{1}{2} m_\psi^2 \psi^2 + \frac{1}{2} \lambda_2 \psi^4. \]  

(14)

During inflation, the field \( \varphi \) rolls down along a valley \( \psi = 0 \). Just after passing the critical point \( \varphi = m_\varphi^2 / \lambda_1 \) the state \( \psi = 0 \) becomes unstable and field \( \psi \) moves (in average) to one of the new stable minima. In the meantime field fluctuations around the critical point \( \psi = 0, \varphi = m_\varphi^2 / \lambda_1 \) lead to the formation of fractal structure.

The inflationary mechanisms described above lead to the occurrence of fractal structure of the closed walls. After the end of inflation, as soon as the size of horizon becomes larger than the characteristic size of closed walls, the walls begin to shrink. The energy of each wall is proportional to the area of their surface and concentrates in small spatial domains (in the following they are considered as pointlike objects)\[20\]. These high density clots of energy could serve in the following for star and/or galaxy formation \[16\]. Hence, according to the given models, the distribution of stars and galaxies should carry fractal character as well. It is important to note that the total surface of walls in specific volume is proportional to the total energy within the volume, while the number of walls is equal to the number of dense clots.

According to this scenario, it is interesting to find the number of walls inside a sphere of radius \( R \) given by

\[ N_{\text{tot}} = \sum_{i=1}^{n} N^i = N \frac{N^n - 1}{N - 1} \approx \frac{N^{n+1}}{N}. \]  

(15)

By analogy with the previous calculations and using Eq.(15), one obtains the distribution of pointlike dense objects with fractal dimension \( D' \approx \ln N / \ln(1/\xi) \). For realistic values \( N \approx 4, \xi \approx 1/e \) we find \( D' \approx 1.4 \) which differs somewhat from the value \( D \approx 2 \) previously obtained. This is not surprising because in the first case we measure the area of surfaces of walls within a certain volume while in second case we measure the number of walls.

Let us compare our calculations with observational data of spatial distribution of galaxies and of stars in those galaxies. Recent data indicate that the distribution of stars and galaxies really carries fractal character. So, the number of galaxies inside a sphere of radius \( R \) is \( N(R) \sim (R)^{2.2 \pm 0.2} \) up to the sizes of 200 Mpc \[18\].

The distribution of stars inside galaxies also carries fractal character. In Ref.\[19\] this fractal dimension was determined by averaging observational data of ten galaxies and was found to be equal to \( D \sim 2.3 \).

Evidently, the observable fractal dimension \( D \) in distributions of stars and galaxies are in agreement with predictions of the given model. Of course, other mechanisms at a later stage may contribute to the distribution and change the fractal dimension somewhat, but the model discussed gives a primordial reason of fractality in the galaxy and star distribution.

For the sake of completeness it is worth to note another observational consequence following from the assumption of the existence of closed walls at an early stage of formation of the universe. If the mass of a wall is rather large, it can collapse into a black hole, when shrinking. This process was studied in Ref.\[20\]. Hence, the considered model predicts existence of massive black holes at the centers of galaxies. This conclusion is in good agreement with observations. The presence of black holes with masses of order \( 10^7 M_\odot \) at the centers of galaxies is an established fact by now \[21\].

5 Interaction with fermions

The interaction of a scalar field with fermions is usually considered in the form of Yukawa coupling

\[ V_F = g \varphi \bar{\psi} \psi. \]  

(16)
In this case we arrive at a serious problem. The minimum of the potential, guaranteeing conditions suitable for life, can appear far from the value \( \varphi = 0 \). Hence, the term contributing to the fermion mass \( M_F = g \varphi_m \) will be huge comparing with experimentally observed fermion masses. The problem can be solved by noticing that the choice (16) selects the field value \( \varphi = 0 \) which contradicts the main postulates of Section 2. Let us suppose the interaction has the form

\[
V_F = G(\varphi)\bar{\psi}\psi,
\]

which is a generalization of expression (16). The function \( G(\varphi) \) is chosen to be a polynomial with random factors in analogy with the scalar potential \( V(\varphi) \). In this case the fermion mass \( M_F \) and the constant \( g \) of interaction with the field \( \phi = \varphi - \varphi_m \) depend on the number \( m \) of the universe,

\[
M_F = G(\varphi_m); \quad g = G'(\varphi_m).
\]

This expression is obtained by expansion of Eq.(17) in a power series around the minimum \( \varphi_m \). Because we have an infinite number of universes, it is obvious that for any given interval of fermion mass \( (\mu_F; \mu_F + \delta) \) and function \( G(\varphi) \), one can find an appropriate universe such that the value of the potential at the minimum \( V(\varphi_m) \) satisfies the equality \( \mu_F \approx G(\varphi_m) \) with desired accuracy.

It becomes now possible to use this mechanism for fine tuning of another parameters of a universe, but not only vacuum energy density. For example, the existence of life is possible if the fermion mass lies in an interval \( (\mu_{life}; \mu_{life} + \delta) \). Then from an infinite set of universes with energy density suitable for life, one can always choose universes with suitable values \( G(\varphi_m) \), such that the fermion mass appears in the given interval. Moreover, this new restricted set of universes still contains an infinite number of universes and we can choose a subset of universes with other parameters suitable for life. Let us introduce a finite set of physical parameters \( \ell_k \) which are necessary for creation life in a universe and enumerable set of universes \( \mathcal{R}\{\ell\}_n \). Here \( \{\ell\}_n \) is a set of \( n \) parameters \( \ell_1, \ell_2, ..., \ell_n \). Then the process of finding of suitable universe looks like

\[
\mathcal{R}\{\ell\}_0 \Rightarrow \mathcal{R}\{\ell\}_1 \Rightarrow \mathcal{R}\{\ell\}_2 \Rightarrow ... \Rightarrow \mathcal{R}\{\ell\}_{N_{life}}.
\]

Here \( N_{life} \) is a minimal number of parameters which leads to conditions suitable for life in the universe. Thus, quantum fluctuations supply a permanent "search" of universes, suitable for life by all parameters.

6 Conclusion

In this paper the mechanism of creation of universes with given set of microscopic parameters is developed. The process of formations of each universe is unique, because the form of potential is unique in the vicinity of each minimum. The formation of universes is described by different types of inflationary models. Presently, a large number of models with a wide range of different potentials are considered as potentially realistic. Apparently, each of them describes some subset of the universes of our type. It is shown, that at an early stage of formation of our universe primordial fractal structures are created in natural way. Three different scenarios of fractal creation are considered here. Two of them are based on the well known natural and hybrid models of inflation. These structures could be the germs of galaxies and stars. The fractal dimension \( (D \approx 2) \) of galaxy distribution calculated in the paper is in agreement with observations.

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