Cosmological Constraints from Compact Radio Source Angular Size versus Redshift Data

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ABSTRACT

We use the Gurvits, Kellermann, & Frey compact radio source angular size versus redshift data to place constraints on cosmological model parameters in models with and without a constant or time-variable cosmological constant. The resulting constraints are consistent with but weaker than those determined using current supernova apparent magnitude versus redshift data.

Subject headings: cosmology: cosmological parameters—cosmology: observation—large-scale structure of the universe

1. Introduction

Cosmological models now under consideration have a number of adjustable parameters. A simple way to determine whether a model provides a useful approximation to reality is to use many different cosmological tests to set constraints on cosmological-model-parameter values and to check if these constraints are mutually consistent (see, e.g., Maor et al. 2002; Wasserman 2002).

During the past few years much attention has been focussed on the Type Ia supernova apparent magnitude versus redshift test (see, e.g., Riess et al. 1998; Perlmutter et al. 1999; Podariu & Ratra 2000; Waga & Frieman 2000; Gott et al. 2001; Leibundgut 2001). This cosmological test indicates that the energy density of the current universe is dominated by a cosmological constant, \( \Lambda \), or a term in the material stress-energy tensor that only varies slowly with time and space and so behaves like \( \Lambda \).

In conjunction with dynamical estimates which indicate a low non-relativistic matter density parameter \( \Omega_0 \) (see, e.g., Peebles 1993), cosmic microwave background anisotropy measurements also suggest the presence of \( \Lambda \) or a \( \Lambda \)-like term (see, e.g., Podariu et al. 2001b; Wang, Tegmark, 1

1The proposed SNAP satellite should provide much tighter constraints on cosmological parameters from this test (see, e.g., Podariu, Nugent, & Ratra 2001a; Weller & Albrecht 2002; Wang & Lovelace 2001; Gerke & Efstathiou 2002; Eriksson & Amanullah 2002).
However, the observed rate of multiple images of radio sources or quasars, produced by gravitational lensing by foreground galaxies, appears to favor a smaller value for \( \Lambda \) (see, e.g., Ratra & Quillen 1992; Helbig et al. 1999; Waga & Frieman 2000; Ng & Wiltshire 2001) than is indicated by the observations mentioned above. It is therefore of interest to examine the entrails of other cosmological tests.

In this paper we consider the redshift-angular size test, using the Gurvits, Kellermann, & Frey (1999) compact radio source measurements. The redshift-angular size relation is measured by Buchalter et al. (1998) for quasars, and by Guerra, Daly, & Wan (2000) for radio galaxies; we do not use these data sets in our analysis here. Vishwakarma (2001), Lima & Alcaniz (2002), and references therein, use the Gurvits et al. (1999) data to set constraints on cosmological parameters; our results are consistent with, but as discussed next extend, their analyzes.

An application of the redshift-angular size test requires knowledge of the linear size of the “standard rod” used. Earlier analyzes of the Gurvits et al. (1999) data appear to assume that this linear size will be determined using additional data, and so quote limits on cosmological parameters (such as \( \Omega_0 \) or the cosmological constant density parameter \( \Omega_\Lambda \)) for a range of values of this linear size. Here we note that it is best to treat this linear size as a “nuisance” parameter (for the cosmologically relevant part of this test), that is also determined by the redshift-angular size data, and so marginalize over it (using a prior to incorporate other information about it, if needed).

In §2 we summarize our computation. Results are presented and discussed in §3. We conclude in §4.

### 2. Computation

For our analyzes here we use the redshift-angular size data of Fig. 10 of Gurvits et al. (1999), which are binned redshift-angular size data derived from measurements of 145 sources. These measurements are combined in twelve redshift bins, with about the same number of sources per bin, with the lowest and highest redshift bins centered at redshifts \( z = 0.52 \) and \( z = 3.6 \).

We consider two cosmological models as well as a currently popular parametrization of dark energy. These are low-density cold dark matter (CDM) dominated cases, consistent with current observational indications. The first model is parametrized by two “cosmological” parameters, \( \Omega_0 \) and \( \Omega_\Lambda \) (in addition to all the other usual parameters). This model includes, as special cases, two “one parameter” models: the currently popular \( \Lambda \)CDM case with flat spatial hypersurfaces

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2The situation here is similar to that for the redshift-magnitude test (e.g., Riess et al. 1998; Perlmutter et al. 1999) where one must marginalize over the magnitude of the standard candle used, treating it as a nuisance parameter.
and \( \Lambda > 0 \) (see, e.g., Peebles 1984; Efstathiou, Sutherland, & Maddox 1990; Stompor, Górski, & Banday 1995; Ratra et al. 1997; Sahni & Starobinsky 2000) and a model with open spatial hypersurfaces and no \( \Lambda \) (see, e.g., Gott 1982; Ratra & Peebles 1995; Cole et al. 1997).

We also derive constraints on the parameters of a spatially-flat model with a dark energy scalar field \((\phi)\) with scalar field potential energy density \(V(\phi)\) that at low \(z\) is \(\propto \phi^{-\alpha}\), \(\alpha > 0\). The energy density of the scalar field decreases with time, behaving like a time-variable \(\Lambda\) (see, e.g., Peebles & Ratra 1988; Ratra & Peebles 1988; Steinhardt 1999; Brax, Martin, & Riazuelo 2000; Carroll 2001).

In linear perturbation theory, a scalar field is mathematically equivalent to a fluid with time-dependent equation of state parameter \(w = p/\rho\) and speed of sound squared \(c_s^2 = dp/d\rho\), where \(p\) is the pressure and \(\rho\) the energy density (see, e.g., Ratra 1991). The XCDM parametrization for dark energy approximates \(w\) as a constant (see, e.g., Steinhardt 1999; Sahni & Starobinsky 2000; Carroll 2001; Huterer & Turner 2001), which is accurate in the radiation and matter dominated epochs but not in the current, dark energy scalar field dominated epoch. Nevertheless the XCDM parametrization is recommended by its simplicity, so we also determine redshift-angular size constraints on its parameters.

We want to determine how well the Gurvits et al. (1999) redshift-angular size measurements distinguish between different cosmological-model-parameter values. To do this we pick one of the above models or the XCDM parametrization, and a range of model-parameter values and compute the angular size distance \(r(z)\) (the coordinate position for the object considered at redshift \(z\) with the observer at the origin) for a grid of model-parameter values that span this range. An object of physical or proper length \(l\) transverse to the line of sight subtends an angle \(\theta(z) = l(1+z)/[a_0 r(z)]\), where \(a_0\) is the current value of the scale factor (Peebles 1993, §13). To determine the probability distribution of the cosmological model parameters \((P)\), we compute

\[
\chi^2(l, P) = \sum_{i=1}^{12} \left[ \frac{\theta(l, P, z_i) - \theta_{\text{obs}}(z_i)}{\sigma(z_i)} \right]^2, \tag{1}
\]

where \(\theta_{\text{obs}}(z_i)\) and \(\sigma(z_i)\) are the observed angles and errors for each of the twelve redshift bins centered at redshifts \(z_i\) of the Gurvits et al. (1999) data. \(P\) represents the model parameters, for instance \(\Omega_0\) and \(\Omega_\Lambda\) in the general two-dimensional constant \(\Lambda\) case. This representation (eq. [1]) is exact for the case where the correlated errors between redshift bins are negligible.

With a uniform prior for the physical length \(l\), the probability distribution (likelihood) of the cosmological model parameters is

\[
L(P) = \int dl \, e^{-\chi^2(l, P)/2}, \tag{2}
\]

where the integral is over a large enough range of \(l\) to include almost all of the probability. We typically integrate \(l\) over the range \(1 \, h^{-1}\) to \(60 \, h^{-1} \, \text{pc},^{3}\) sampled at 60 points, although the range

\(^3\)Here the Hubble constant \(H_0 = 100h \, \text{km s}^{-1} \, \text{Mpc}^{-1}\).
depends somewhat on the model under analysis. For the constant $\Lambda$ model we compute over the ranges $0 \leq \Omega_0 \leq 1$ and $-1 \leq \Omega_\Lambda \leq 1$, both sampled at 201 points; for the scalar field dark energy model we compute over the ranges $0.005 \leq \Omega_0 \leq 0.995$ and $0 \leq \alpha \leq 8$ sampled at 199 and 161 points, respectively; while for the XCDM parametrization we compute over the ranges $0 \leq \Omega_0 \leq 1$ and $-1 \leq w \leq 0$, both sampled at 201 points. The confidence limits are computed from the distribution of eq. (2), and we typically show 1, 2, and 3 $\sigma$ confidence contours which correspond to enclosed probabilities of 68.27, 95.45, and 99.73 $\%$, respectively.

Since $\Omega_0$ is a positive quantity, we also consider the non-informative or logarithmic prior $p(\Omega_0) \propto 1/\Omega_0$ (Berger 1985; Gott et al. 2001) and compute confidence regions for the probability distribution $L(P)/\Omega_0$, where $L(P)$ is given in eq. (2).

3. Results and Discussion

Figure 1 shows the Gurvits et al. (1999) redshift-angular size constraints on the general two-dimensional constant $\Lambda$ case. Apparently these have not previously been published. These constraints are consistent with, but not as constraining as those from Type Ia supernova redshift-magnitude data (e.g., Podariu & Ratra 2000, Fig. 5).

Figure 2 shows the redshift-angular size constraints on the XCDM parameters. Lima & Alcaniz (2002, Fig. 2) show related constraints computed at fixed physical length $l$ for the same Gurvits et al. (1999) data. While the shapes are similar, the Lima & Alcaniz (2002) contours are much more constraining than those found here, largely because our procedure of marginalizing over the physical length $l$ also accounts for the uncertainty in the determination of $l$. Podariu & Ratra (2000, Fig. 2) show corresponding constraints on the XCDM parameters from the Type Ia supernova redshift-magnitude data, which are significantly more constraining than those shown in Fig. 2 here.

Figure 3 shows the constraints on the dark energy scalar field model with potential energy density $V(\phi) \propto \phi^{-\alpha}$, $\alpha > 0$ (Peebles & Ratra 1988). They are consistent with, but not as constraining as those from the Type Ia supernova redshift-magnitude data (Podariu & Ratra 2000; Waga & Frieman 2000).

4. Conclusion

Constraints on cosmological model parameters derived from the redshift-angular size compact radio source data of Gurvits et al. (1999) are consistent with but less constraining than those derived from the redshift-magnitude Type Ia supernova data of Riess et al. (1998) and Perlmutter et al. (1999).

Higher quality redshift-angular size data will more significantly constrain cosmological models, and in combination with high quality redshift-magnitude data will provide a check of conventional
general relativity on cosmological length scales.

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REFERENCES


Berger, J. O. 1985, Statistical Decision Theory and Bayesian Analysis (New York: Springer-Verlag), 82


Carroll, S.M. 2001, Living Rev. Relativity, 4, 1


Leibundgut, B. 2001, ARAA, 39, 67

Vishwakarma, R.G. 2001, Class. Quant. Grav., 18, 1159
Fig. 1.— Contours of 1, 2, and 3 $\sigma$ confidence for the constant $\Lambda$ model. Solid lines are contours computed using the uniform prior $p(\Omega_0) = 1$ (with three contours bunched together in the upper left corner), while short dashed lines show the case using the logarithmic prior $p(\Omega_0) = 1/\Omega_0$ (with three contours lying on each other at the left edge). The horizontal dot-dashed line demarcates models with a vanishing cosmological constant, $\Lambda = 0$, and the diagonal dot-dashed line running from the top left corner to the point $\Omega_0 = 1$, $\Omega_\Lambda = 0$ indicates the spatially-flat $\Omega_0 + \Omega_\Lambda = 1$ case.
Fig. 2.— Contours of 1, 2, and 3 $\sigma$ confidence for the XCDM parametrization of dark energy, $p = \omega \rho$. Solid lines are contours computed using the uniform prior $p(\Omega_0) = 1$ (with three contours bunched together in the lower left corner), and short dashed lines show the case using the logarithmic prior $p(\Omega_0) = 1/\Omega_0$ (with three contours lying on each other at the left edge).
Fig. 3.— Contours of 1, 2, and 3 σ confidence for the dark energy scalar field model with inverse power-law potential energy density $V(\phi) \propto \phi^{-\alpha}$. Solid lines are contours computed using the uniform prior $p(\Omega_0) = 1$ (with three contours bunched together in the lower left corner), and short dashed lines show the case using the logarithmic prior $p(\Omega_0) = 1/\Omega_0$ (with three contours lying on each other at the left edge, at the boundary of parameter space).