BLACK HOLE PHYSICS FROM TWO DIMENSIONAL
DILATON GRAVITY BASED ON $SL(2, R)/U(1)$ COSET
MODEL

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Abstract

We analyze quantum two dimensional dilaton gravity model, which is described by \( SL(2, R)/U(1) \) gauged Wess-Zumino-Witten model deformed by (1, 1) operator. We show that the curvature singularity does not appear when the central charge \( c_{\text{matter}} \) of the matter fields is given by \( 22 < c_{\text{matter}} < 24 \). When \( 22 < c_{\text{matter}} < 24 \), the matter shock waves, whose energy momentum tensors are given by \( T_{\text{matter}} \propto \delta(x^+ - x^0) \), create a kind of wormholes, i.e., causally disconnected regions. Most of the quantum informations in past null infinity are lost in future null infinity but the lost informations would be carried by the wormholes.

We also discuss about the problem of defining the mass of quantum black holes. On the basis of the argument by Regge and Teitelboim, we show that the ADM mass measured by the observer who lives in one of asymptotically flat regions is finite and does not vanish in general. On the other hand, the Bondi mass is ill-defined in this model. Instead of the Bondi mass, we consider the mass measured by observers who live in an asymptotically flat region at first. A class of the observers finds the mass of the black hole created by a shock wave changes as the observers’ proper time goes by, i.e., they observe the Hawking radiation. The measured mass vanishes after the infinite proper time and the black hole evaporates completely. Therefore the total Hawking radiation is positive even when \( N < 24 \).
1 Introduction

When we try to construct the quantum theory of the gravity, black hole evaporation provides a serious problem [1]. A toy model proposed by Callan, Giddings, Harvey and Strominger (CGHS) [2] was expected to give a clue to solve this problem. The model describes the two-dimensional gravity coupled with a dilaton and conformal matters. Many authors have investigated this model [3]–[24] and it has been pointed out in several papers [7, 8, 10] that the consistent quantization of this theory in the conformal gauge

\[ g_{\mp} = -\frac{1}{2} e^{2\phi} , \quad g_{\pm} = 0 , \]  

requires that the theory should be a conformal field theory with the vanishing central charge. The quantum action of the theory is given by a sum of the actions of two free fields and the dilatonic cosmological term, which is a \((1,1)\)-operator [7, 8]. Recently the authors have proposed a new class of quantum dilaton gravity which is described by \(SL(2,R)/U(1)\) gauged Wess-Zumino-Witten model [25] deformed by \((1,1)\) operator [22]. In the weak coupling limit : \(e^{2\phi}\to 0\) (\(\phi\) is a dilaton field) the action reduces to CGHS’ classical action. The model was analyzed by \(1/k\) expansion \((k\) is the level of \(SL(2,R)\) Wess-Zumino-Witten model and we found the curvature singularity does not appear when \(k\) is negative and \(|k|\) is large.

In this paper, we analyze the previously proposed model [22] in detail.\(^1\)

We show that the curvature singularity does not appear when the central charge \(c_{\text{matter}}\) of the matter fields is given by \(22 < c_{\text{matter}} < 24\). When \(22 < c_{\text{matter}} < 24\), the matter shock waves, whose energy momentum tensor is given by \(T_{\text{matter}} \propto \delta(x^+ - x_0^+)\), create a kind of wormholes, \textit{i.e.}, causally disconnected regions. Most of the quantum informations in past null infinity are lost in future null infinity but the lost informations are carried by the wormholes.

We also discuss about the problem of defining the mass of quantum black holes. Recently Bilal, Kogan [23] and de Alwis [24] have proposed a definition of the mass based on the argument by Regge and Teitelboim [26] and they have found the ADM mass of the black holes vanishes. In spite of their interesting observations, we claim that problems are remained in order to obtain a well-defined mass and the ADM mass measured by the observer who lives in one of asymptotically flat regions does not vanish in general.

We also show that the Bondi mass is ill-defined in this model since the scalar curvature diverges at a point in the infinite future. This curvature singularity, however, would not give any other serious problem. Instead of the Bondi mass, we consider the mass measured by observers who live in an asymptotically flat region at first. A class of the observers finds the mass of the black hole created by a shock wave changes as the observers’ proper time goes by, \textit{i.e.}, they observe

\(^1\)This paper is the complete and revised version of [22].
the Hawking radiation. The measured mass vanishes after the infinite proper time and the black hole evaporates completely. Therefore the total Hawking radiation is positive even when $N < 24$.

In the next section, we review the model proposed in the previous paper [22]. We give the action of the model and solve the equations of motion explicitly. By using the solutions, we discuss the structure of the space-time in section 3. Especially, we consider the problem of the loss of the quantum informations. In section 4, we define the mass of the quantum black holes. Last section is devoted to summary and discussion.

## 2 A Model of Quantum Dilaton Gravity based on $SL(2, R)/U(1)$ Coset Model

In this section, we review the model proposed previously [22].

In the original paper by CGHS [2], the quantum effects were expected to be described by adding correction terms which only comes from the conformal anomaly to the classical action. It was, however, clarified [7, 8, 10] that we need more counterterms since the quantum action should have conformal symmetry when we choose the conformal gauge (1) [27]. By following de Alwis’ paper [17], we assume the kinetic term of the quantum action in the dilaton gravity coupled to $N$ free bosons is given by

$$S_{\text{kin}} = \frac{1}{2\pi} \int d^2x \left[ -8e^{-2\phi}(1 + h(\phi))\partial_+\phi\partial_-\phi ight. \tag{2}$$

$$+ 4e^{-2\phi}(1 + \bar{h}(\phi))(\partial_+\phi\partial_\rho - \rho + \partial_\rho\partial_-\phi) + 2\kappa(1 + \bar{h}(\phi))\partial_+\rho\partial_-\rho \right].$$

Here $h(\phi)$, $\bar{h}(\phi)$ and $\bar{h}(\phi)$ are $O(e^{2\phi})$ and $\kappa = \frac{24 - N}{6}$. Note that $e^{2\phi}$ plays the role of space-time dependent gravitational coupling [2]. In Ref.[17], it was only considered the case $\bar{h}(\phi) = 0$, where $S_{\text{kin}}$ can be rewritten by two free fields action. In Ref.[22], the authors considered $\bar{h}(\phi) \neq 0$ case.

If we define new fields $X$ and $Y$ by,

$$X \equiv \pm 2b \int d^2s e^{-2\phi}(1 + \bar{h}(s))(1 + \bar{h}(s))^{-1} + \kappa e^{2s}(1 + h(s)),$$

$$Y \equiv \pm a \left[ \rho + \frac{2}{\kappa} \int d^2s e^{-2\phi}(1 + \bar{h}(s))(1 + \bar{h}(s))^{-1} \right], \tag{3}$$

the kinetic term of the quantum action $S_{\text{kin}}$ in Eq.(2) can be rewritten by,

$$S_{\text{kin}} = \frac{k}{4\pi} \int d^2x \left[ \partial_+ X \partial_- X - \left( 1 + \bar{h}(\phi(X)) \right) \partial_+ Y \partial_- Y \right]. \tag{4}$$
Here $a$ and $b$ are defined by

\[ a \equiv \sqrt{-\frac{4\kappa}{k}}, \quad b \equiv \sqrt{-\frac{4}{kk}}, \]  

(5)

and $k$ is a constant satisfying $k\kappa < 0$ and we will identify $k$ with the level of $SL(2,R)$ Wess-Zumino-Witten model later. If $\tilde{h}(\phi)$ in the action (4) is a constant, this action describes a free field theory, which is the simplest conformal field theory. On the other hand, if we can choose $1 + \tilde{h}(\phi(X)) = \tanh^2 X$, the action (4) is nothing but the action of another conformal field theory, i.e., $SL(2,R)/U(1)$ gauged Wess-Zumino-Witten model in unitary gauge [28].

\[ S_{\text{kin}} = \frac{k}{4\pi} \int d^2x \left[ \partial_+ X \partial_- X - \tanh^2 X \partial_+ Y \partial_- Y \right]. \]  

(6)

In fact, if we choose, for example,

\[ 1 + h(\phi) = -\frac{e^{-2\phi}}{k} \left\{ \left(1 - \frac{ke^{2\phi}}{2}\right)^2 \tanh^2(be^{-2\phi}) - 1 \right\}, \]

\[ 1 + \tilde{h}(\phi) = \left(1 - \frac{ke^{2\phi}}{2}\right) \tanh^2(be^{-2\phi}), \]

\[ 1 + \tilde{h}(\phi) = \tanh^2(be^{-2\phi}), \]  

(7)

i.e., when $X$ and $Y$ are given by,

\[ X = be^{-2\phi}, \quad Y = a \left(\rho - \phi - \frac{1}{k} e^{-2\phi}\right), \]  

(8)

we find the action (4) is rewritten in the form of Eq.(6). Note that there appears non-perturbative contribution of $O(e^{-\frac{2\phi}{k}})$.

The cosmological term which is $(1,1)$-operator can be added to the action (6). The vertex operator $V_{lm}$, which is an $(l,m)$ representation of $SL(2,R)$, has the following form in the unitary gauge [29],

\[ V_{lm} = (\sinh^2 X)^i e^{-2m} F(m - l, -m - l, 1; \coth^2 X). \]  

(9)

Here $F(\alpha, \beta, \gamma; x)$ is Gauss’ hypergeometric function. If we require $V_{lm} \sim e^{i(X + Y)} \sim e^{c_1(X + \rho - \phi)}$ (c and $c'$ are positive constants.) when $X \rightarrow \infty$ i.e., $e^{2\phi} \rightarrow 0$ (weak coupling limit), which is expected from the $\beta$-function analysis [17], we find $l = -m$ and

\[ V_l \equiv V_{ll} = (\sinh^2 X)^i e^{2lY}. \]  

(10)

Then the total action including cosmological term should be given by,

\[ S = \frac{k}{4\pi} \int d^2x \left[ \partial_+ X \partial_- X - \tanh^2 X \partial_+ Y \partial_- Y + \frac{\alpha}{4\kappa} (\sinh X e^Y)^{2l} \right] 
\quad + (N \text{ free boson terms}). \]  

(11)
We choose \( l \) so that the conformal dimension \( \Delta_l \) of the operator \( V_l \) should be unity: \( \Delta_l = 1 \).

In order to consider the stress tensors, we rewrite the action (11) in a reparametrization invariant form,

\[
S = \frac{k}{8\pi} \int \sqrt{-g} d^2x \left[ g^{\mu\nu} \partial_\mu X(\phi) \partial_\nu X(\phi) + \tanh^2 X g^{\mu
u} \partial_\mu (a\dot{\rho} + \dot{\bar{Y}}(\phi)) \partial_\nu (\dot{a\rho} + \dot{\bar{Y}}(\phi)) + \frac{\alpha}{\kappa} \left( \sinh X e^{a\dot{\rho} + \dot{\bar{Y}}(\phi)} \right)^{2l} e^{-2\dot{\rho}} \right] + (N \text{ free boson terms}) .
\]

(12)

Here we have used the parametrization in Eq.(8) and we define \( \dot{Y}(\phi) \) and \( \dot{\rho} \) by \( (R \text{ is a scalar curvature}) \)

\[
\dot{Y} \equiv a \left( -\phi - \frac{1}{\kappa} e^{-2\phi} \right), \quad \dot{\rho} \equiv -\frac{1}{2} \left( \partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu \right)^{-1} \sqrt{-g} R .
\]

(13)

Then we find the stress tensors \( T_{\pm\pm} \) have the following forms,

\[
T_{\pm\pm} = k(\partial_{\pm} X \partial_{\pm} X - \tanh^2 X \partial_{\pm} Y \partial_{\pm} Y)
+ \frac{ka}{2} \partial_{\pm} \int_{\alpha}^\beta dy^\mp \left\{ \partial_-(\tanh^2 X \partial_+ Y) + \partial_+(\tanh^2 X \partial_- Y) \right\}
- \frac{\alpha}{4} (1 - a\ell) \partial_{\pm} \int_{\alpha}^\beta dy^\mp (\sinh X e^Y)^{2l} + T_{\pm\mp}^{\text{matter}} ,
\]

\[
T_{\pm\mp} = -\frac{ka}{2} \left\{ \partial_+(\tanh^2 X \partial_+ Y) + \partial_-(\tanh^2 X \partial_- Y) \right\}
- \frac{a\ell}{4} \alpha \sinh X e^Y)^{2l} .
\]

(14)

Here \( T_{\pm\mp}^{\text{matter}} \) is the stress tensor of \( N \) scalar fields. Note that there appear non-local terms in \( T_{\pm\pm} \) and it is ambiguous how to fix the boundary conditions in these terms. In the following, we consider the solutions corresponding to the following boundary conditions:

\[
T_{\pm} = k(\partial_{\pm} X \partial_{\pm} X - \tanh^2 X \partial_{\pm} Y \partial_{\pm} Y)
+ \frac{ka}{2} \partial_{\pm} \left\{ \tanh^2 X \partial_{\pm} Y + \int_{\alpha}^\beta dy^\mp \partial_{\pm} (\tanh^2 X \partial_{\pm} Y) \right\}
- \frac{\alpha}{4} (1 - a\ell) \partial_{\pm} \int_{\beta}^{\frac{\beta}{\alpha}} dy^\mp (\sinh X e^Y)^{2l} + T_{\pm\mp}^{\text{matter}} .
\]

(15)

The physical quantities like black hole mass etc. do not depend on the boundaries \( \alpha \pm \) and \( \beta \pm \) if we use the equations of motion. As we will see later, the solutions under the above boundary conditions give the solutions corresponding to CGHS’s classical solutions in the weak coupling limit.
We now solve the equations of motion and constraints of the system. In order to do this, it is convenient to define new fields $X^\pm$ by,

\[ X^\pm = \pm \sinh X e^{\pm Y}. \]  

(16)

Then the action (11) is rewritten by

\[ S = \frac{k}{4\pi} \int d^2 x \left[ -\frac{\partial_-^2 X^+ X^- + \partial_+ X^+ X^-}{2(1 - X^+ X^-)} + \frac{\alpha}{4k} (X^+)^{2l-1} \right] \]

+(N free boson terms).

(17)

and we obtain the following equations of motion

\[ 0 = \frac{X^-\partial_- X^+ \partial_+ X^+}{1 - X^+ X^-} + \frac{\partial_- \partial_+ X^+}{1 - X^+ X^-}, \]  

(18)

\[ 0 = \frac{X^+\partial_+ X^- \partial_- X^-}{1 - X^+ X^-} + \frac{\partial_- \partial_+ X^-}{1 - X^+ X^-} + \frac{\alpha}{2k} (X^+)^{2l-1}. \]  

(19)

Note that $X^+ = w$ ($w$ : constant) satisfies the first equation (18). Then we can solve the second equation (19),

\[ X^+ = w, \quad X^- = \frac{1}{w} \left( 1 - e^{-\Lambda x^+ x^- + u(x^+, x^-)} \right). \]  

(20)

Here $\Lambda \equiv -\frac{\alpha}{2k} (w)^{2l}$ and $u(x^+, x^-) = u_+(x^+) + u_-(x^-)$. Although we cannot obtain the general solution to Eqs.(18) and (19), it is important that this class of special solutions (20) includes the solutions corresponding to the linear dilaton vacuum, the classical black hole and the shock wave solutions in the weak coupling region and we can investigate the time-development which infalling matters lead to. From now on, we only consider the solutions given by Eq.(20). By using the solution (20), we find the stress tensors in Eqs.(14), (15) have the following forms,

\[ T_{\pm\mp} = 0, \quad T_{\pm\pm} = -\frac{k\alpha}{4} \partial_\pm^2 u_\pm (x^\pm) + T_{\pm\pm}^{\text{matter}}. \]  

(21)

Now let us impose the only a priori restriction [17] in the present formalism, which is given by,

\[ 0 = T_{\pm\pm} + t_{\pm\pm}, \quad t_{\pm\pm} = -\frac{N}{24} \frac{1}{(x^\pm)^2}. \]  

(22)

When $T_{\pm\pm}^{\text{matter}} = 0$, we find the following solutions

\[ -\frac{k\alpha}{4} u_\pm (x^\pm) = a_\pm + b_\pm x^\pm - \frac{N}{24} \ln |x^\pm|. \]  

(23)

\[ ^2 \text{There would be an ambiguity in the coefficient of } \ln |x^\pm| \text{ in } [17]. \text{ In the following, we assume, as a natural choice, the coefficient should be } -\frac{N}{24}. \text{ We can straightforwardly extend the results, which we will obtain in the following, to the cases of the general coefficient but most of the results do not be changed qualitatively.} \]
If \( b_+ = b_- = 0 \), the solution (23) corresponds to a static object. On the other hand, the solution corresponding to the shock wave which describes collapsing matters, \( T^\pm = m \delta(x^+ - x_0^+) \), is given by

\[
-k \frac{a}{4} u_+(x^+) = a_+ + b_+ x^+ - m(x^+ - x_0^+) \theta(x^+ - x_0^+) - \frac{N}{24} \ln |x^+| ,
\]

\[
-k \frac{a}{4} u_-(x^-) = a_- + b_- x^- - \frac{N}{24} \ln |x^-| .
\] (24)

Since the action (11) is correct for large \( k \) \cite{28}, it is convenient to consider the corresponding action in Landau gauge \cite{29} in order to clarify what kind of conformal theory describes the system. The Landau gauge action corresponding to (11) is given by

\[
S = \frac{k}{4 \pi} \int d^2 x \left[ \partial_+ X \partial_- X - \cosh^2 X \partial_+ Z \partial_- Z - \sinh^2 X \partial_+ \hat{Y} \partial_- \hat{Y} \right. \\
\left. + \left( \cosh^2 X - \frac{1}{2} \right) (\partial_+ Z \partial_- \hat{Y} - \partial_+ \hat{Y} \partial_- Z) + \frac{1}{4} \partial_+ \varphi \partial_- \varphi \\
+ \frac{\alpha}{4k} (\sinh X e^{\hat{Y}})^2 \right] + \text{(ghost terms)} \\
+ (N \text{ free boson terms}) .
\] (25)

Here \( Z \) is a degree of freedom in \( SL(2, R) \) Wess-Zumino-Witten model which is independent of \( X \) and \( Y \). A real scalar field \( \varphi \) comes from the \( U(1) \) gauge fields and \( \hat{Y} \) is defined by \( \hat{Y} = Y - \frac{\varphi}{2} \). By using new fields \( T \) and \( S \), we can rewrite the action (25) in a reparametrization invariant and local form,

\[
S = \frac{k}{8 \pi} \int d^2 x \left[ \sqrt{-g} \left\{ -g^{\mu\nu} \partial_\mu X(\phi) \partial_\nu X(\phi) - \cosh^2 X g^{\mu\nu} \partial_\mu Z \partial_\nu Z \\
+ \sinh^2 X g^{\mu\nu} \partial_\mu \left( aT + \hat{Y}(\phi) \right) \partial_\nu \left( aT + \hat{Y}(\phi) \right) \\
- \left( \cosh^2 X - \frac{1}{2} \right) e^{\mu\nu} \partial_\nu \left( aT + \hat{Y}(\phi) \right) + \frac{1}{4} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \\
+ \frac{\alpha}{4k} \left( \sinh X e^{\hat{Y}} + \frac{\varphi}{2} \right)^2 \right] e^{-2T} + \frac{1}{2} g^{\mu\nu} \partial_\mu S \partial_\nu T - \frac{1}{4} S R \right] \\
+ (U(1) \text{ ghost term}) + (N \text{ free boson terms}) .
\] (26)

Here \( \hat{Y}(\phi) \) is defined by \( \hat{Y} \equiv \frac{\varphi}{2} + a \left( -\phi - \frac{1}{k} e^{-2\phi} \right) \). (See Eq.(13).) By using the equation of motion which we obtain by the variation of \( S \), we find \( T = \frac{1}{2} (\partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu) ^{-1} \sqrt{-g} R \). Therefore if we integrate \( S \) and \( T \) fields we obtain

\[
S = \frac{k}{8 \pi} \int d^2 x \left[ \sqrt{-g} \left\{ -g^{\mu\nu} \partial_\mu X(\phi) \partial_\nu X(\phi) - \cosh^2 X g^{\mu\nu} \partial_\mu Z \partial_\nu Z \\
+ \sinh^2 X g^{\mu\nu} \partial_\mu (a \hat{\rho} + \hat{Y}(\phi)) \partial_\nu (a \hat{\rho} + \hat{Y}(\phi)) \right. \\
\right.
\]

\[
+ \left. \frac{\alpha}{4k} \left( \sinh X e^{\hat{Y}} + \frac{\varphi}{2} \right)^2 \right] e^{-2T} + \frac{1}{2} g^{\mu\nu} \partial_\mu S \partial_\nu T - \frac{1}{4} S R \right] + (U(1) \text{ ghost term}) + (N \text{ free boson terms}) .
\]
\[ - \left( \cosh^2 X - \frac{1}{2} \right) e^\nu \partial_\nu Z \partial_\nu (a \hat{\rho} + \hat{\Phi}) + \frac{1}{4} g^\nu \partial_\nu \varphi \partial_\nu \varphi \\
+ \frac{\alpha}{4k} \left( \sinh X e^{\hat{\Phi}(\phi) + \frac{\pi}{2}} \right)^2 e^{(2k-2)\hat{\rho}} \\
+ (U(1) \text{ ghost term}) + (N \text{ free boson terms}) \] (27)

Here \( \hat{\rho} \) is defined by Eq.(13): \( \hat{\rho} \equiv -\frac{1}{2} (\partial_\nu \sqrt{-g} g^\nu \partial_\nu)^{-1} \sqrt{-g} R \). Note that the kinetic terms of the resulting action contain the product of two inverse of \( \partial_\mu \sqrt{-g} g^\mu \partial_\nu \) as in Eq.(12), which is the reason why we need two fields \( S \) and \( T \) in order to localize the action (27).

In the conformal gauge, this action (26) has the following form

\[ S = \frac{k}{4\pi} \int d^2 x \left[ \partial_+ X \partial_- X - \cosh^2 X \partial_\pm Z \partial_\pm Z + \sinh^2 X \partial_+ \hat{Y} \partial_- \hat{Y} \\
+ \left( \cosh^2 X - \frac{1}{2} \right) (\partial_+ Z \partial_- \hat{Y} - \partial_+ \hat{Y} \partial_- Z) + \frac{1}{4} \partial_+ \varphi \partial_- \varphi \\
+ \frac{\alpha}{4k} (\sinh X e^{\hat{\Phi} + \frac{\pi}{2}})^2 e^{-2\hat{T}} - \frac{1}{4} (\partial_+ S \partial_- \hat{T} + \partial_+ \hat{T} \partial_- S) \right] \\
+ \text{(ghost terms)} + (N \text{ free boson terms}) \] (28)

Here we have redefined new fields \( \hat{Y} \) and \( \hat{T} \) by \( \hat{Y} \equiv a T + \hat{\Phi} (\phi) \) and \( \hat{T} \equiv T - \rho \). Then we find that the stress tensors are given by

\[ T_{\pm \pm} = k \left( \partial_{\pm} X \partial_{\pm} X - \cosh^2 X \partial_{\pm} Z \partial_{\pm} Z + \sinh^2 X \partial_{\pm} \hat{Y} \partial_{\pm} \hat{Y} \\
+ \frac{1}{4} \partial_{\pm} \varphi \partial_{\pm} \varphi + \frac{1}{2} \partial_{\pm} S \partial_{\pm} T + \frac{1}{4} \partial_{\pm}^2 S \right) + T_{\pm \pm}^{\text{matter}}, \]

\[ T_{\pm \mp} = \frac{\alpha}{4k} (\sinh X e^{\hat{\Phi} + \frac{\pi}{2}})^2 e^{-2\hat{T}} - \frac{k}{4} \partial_{\pm} \partial_{\pm} S. \] (29)

When \( \alpha = 0 \), the system is a direct product of those of \( SL(2,R) \) Wess-Zumino-Witten model, free boson \( \varphi, \) ghosts, \( N \) free bosons and \( S-\hat{T} \) fields which contribute to the central charge by 2. Since the total central charge \( c^{\text{total}} \) does not depend on \( \alpha \), we find the total central charge \( c^{\text{total}} \) is given by

\[ c^{\text{total}} = c^{SL(2,R)/U(1)} + 2 - 26 + N \]
\[ = \frac{3k}{k-2} - 1 + 2 - 26 + N \]
\[ = \frac{6}{k-2} - 6\kappa + 2. \] (30)

Here \( c^{SL(2,R)/U(1)} \) is the central charge of \( SL(2,R)/U(1) \) gauged Wess-Zumino-Witten model with level \( k \) \( c^{SL(2,R)/U(1)} = \frac{3k}{k-2} - 1 \). Since the total central charge \( c^{\text{total}} \) should vanish, we obtain

\[ \kappa = \frac{1}{3} + \frac{1}{k-2}. \] (31)
Since $k \kappa < 0$, we find $k < -1$ or $0 < k < 2$ i.e., we find the restriction for the number of matter fields $N$ i.e., $22 < N < 24$ or $25 < N$.

The operator $\hat{V}_l$ which corresponds to $V_l$ in Eq.(10) is given by,

$$\hat{V}_l = (\sinh X e^{\frac{r}{2}})^{2l} e^{-2\tilde{r}}.$$  \hspace{1cm} (32)

Since $e^{-2\tilde{r}}$ has the conformal dimension $\frac{1}{2}$, $V_l$ has the dimension $\Delta_l$ as follows,

$$\Delta_l = -\frac{l(l+1)}{k-2} - \frac{l^2}{k} - \frac{1}{2}.$$  \hspace{1cm} (33)

We can find $l$ by solving the equation $\Delta_l = 1$.

Since the equation of the motion, which is obtained by the variation of $S$, is given by $\partial_+ \partial_- T = \partial_+ \partial_- (T - \rho) = 0$, we can fix the residual symmetry of the reparametrization invariance by choosing the condition $\tilde{T} = 0$, by which we select a special coordinate system. Under this gauge condition, the solutions of the equations of motion in the Landau gauge corresponding to the solutions (20), (23), (24) in the unitary gauge are given by

$$X = \frac{1}{2} \ln(e^f + \sqrt{e^{2f} - 1}),$$

$$Y = \tilde{Y} + \frac{\varphi}{2} = \frac{1}{2} \ln \frac{u^2}{e^f - 1},$$

$$Z = 0,$$

$$\varphi = -f,$$

$$S = -2\alpha f.$$  \hspace{1cm} (34)

Here $f(x^+, x^-)$ is defined by

$$f \equiv \Lambda x^+ x^- + u(x^+, x^-).$$  \hspace{1cm} (35)

By the action (11) or (28), we only consider the quantum correction which comes from the measure with respect to the reparametrization invariance and the action does not include any other counterterms. In this sense, this action is classical. By using the knowledge of current algebra, however, we can calculate amplitudes from the action (28). We can also calculate S-matrix by solving the equations of motion which can be obtained from the action (11) or (28). Note that we cannot obtain anything from the equations of motion which are obtained from quantum effective action, which are given in terms of a sum of one particle irreducible vertices times semi-classical fields.

3 The Structure of Space-Time and the Problem of the Loss of Quantum Informations

In this section, we analyze the structure of the space-time which is given by solutions (34) and we show that the curvature singularity does not appear when
the central charge of the matter fields $c_{\text{matter}} = N$ is given by $22 < c_{\text{matter}} < 24$. When $22 < c_{\text{matter}} < 24$, the matter shock waves, whose energy momentum tensors are given by $T_{\text{matter}} \propto \delta(x^+ - x_0^+)$, create a kind of wormholes, i.e., causally disconnected regions. We also discuss about the problem of the loss of quantum informations and we claim that most of the quantum informations in past null infinity are lost in future null infinity but the lost informations would be carried by the wormholes.

By using the equations (8), we obtain

$$\rho = \frac{1}{a}Y - \frac{1}{2} \ln \frac{X}{b} + \frac{1}{b^{2}k}X. \quad (36)$$

Substituting the solutions in the equations (34), we find $\rho$ is given by

$$\rho = \frac{1}{2a} \ln \frac{w^2}{e^f - 1} - \frac{1}{2} \ln \left( \frac{1}{2b} \ln(e^f + \sqrt{e^{2f} - 1}) \right) + \frac{1}{2b^2k} \ln(e^f + \sqrt{e^{2f} - 1}). \quad (37)$$

Here $f$ is defined in the equation (35). Note that $f$ should be positive since $\rho$ is real. Since $\rho$ is given by a monotonously decreasing smooth function with respect to $f$, the scalar curvature $R \sim e^{-2f} \partial_\pm \partial_\pm \rho$ can be singular when $f \rightarrow +0$ or $f \rightarrow +\infty$. The limit $f \rightarrow +\infty$ corresponds to spatially infinite asymptotically flat region. When $f \rightarrow +0$, $\rho$ is given by,

$$\rho = -\frac{1}{2} \left( \frac{1}{a} + \frac{1}{2} \right) \ln f + O(1). \quad (38)$$

Then the scalar curvature behaves like

$$R \sim f^{\pm + \frac{1}{2}} \left\{ \frac{\partial_\pm \partial_\pm f}{f} - \frac{\partial_+ f \partial_- f}{f^2} \right\}, \quad (39)$$

when $f \rightarrow 0$. Since $f$ is a smooth function with respect to $x^\pm$, except on the trajectory of matter shock waves, $\partial_\pm f$ or $\partial_+ \partial_- f$ does not diverge on the $f = 0$ line. Therefore if $\frac{1}{a} - \frac{3}{2} \geq 0$, there does not appear curvature singularity.

Because $a$ is given by $a = \sqrt{\frac{4\kappa}{k}}$ (5) and $\kappa = \frac{1}{3} + \frac{1}{k-2}$ (31), the curvature singularity does not appear when $k < -1$ ($k \neq -1$ since $\kappa k \neq 0$), i.e., the number of matter fields $N$ is restricted to $22 < N < 24$ or more generally, the central charge $c_{\text{matter}}$ of the matter fields is given by $22 < c_{\text{matter}} < 24$.

Since we have obtained a model where the curvature singularity does not appear, it is important to consider the problem of the loss of quantum informations etc by using this model. For this purpose, we assume $22 < c_{\text{matter}} < 24$ in the following.

Since the metric $g_{\pm \pm} = -\frac{1}{2} e^{2\rho}$ diverges as $f^{-\frac{3}{2} + \frac{1}{2}}$ when $f \rightarrow +0$, it takes infinite proper time for anyone to approach to the $f = 0$ line. In Fig.1, the structure of the space-time corresponding to a static solution (23) is depicted. In Fig.1, $p$ and $q$ are defined by $x^\pm \equiv \pm e^{\frac{1}{2}(p \pm q)}$. The asymptotically flat region
corresponding to \( f \to +\infty \), which is depicted like a cylinder, is connected with another asymptotically flat region, which corresponds to \( f \to +0 \) and is depicted like a plane.

In the dynamical solution in the equation (24), we define a function \( g(x^+) \) so that \( x^- = g(x^+) \) satisfies \( f(x^+, x^-) = 0 \). We also define \( x_0^- \) which satisfies the equation \( \partial_+ g(x_0^-) = 0 \). Then we find that it takes infinite proper time if anyone who lives in the region I where \( x^- < g(x^+) \) and \( x^- > x_0^- \) (depicted in Fig.2) tries to get out of this region since his world line always crosses the \( f = 0 \) line. Therefore this region I is causally disconnected with asymptotically Minkowski region II. In this sense, this region can be regarded as a kind of wormhole.

We now consider the problem of the loss of the quantum informations. First we consider a solution corresponding to two matter shock waves whose stress tensor is given by

\[
T^{++} = \mu_1 \delta(x^+ - x_1^+) + \mu_2 \delta(x^+ - x_2^+) \quad (x_1^+ > x_2^+) \tag{40}
\]

and another solution corresponding to one shock wave whose stress tensor is

\[
T^{++} = (\mu_1 + \mu_2) \delta\left(x^+ - \frac{\mu_1 x_1^+ + \mu_2 x_2^+}{\mu_1 + \mu_2}\right). \tag{41}
\]

These two solutions have a same form when \( x^+ > \max(x_2^+, \frac{\mu_1 x_1^+ + \mu_2 x_2^+}{\mu_1 + \mu_2}) \):

\[
- \frac{ka}{4} u(x^+, x^-) \equiv - \frac{ka}{4} (u_+(x^+) + u_-(x^-))
= - (\mu_1 + \mu_2)x^+ \mu_1 x_1^+ + \mu_2 x_2^+ - \frac{N}{24} \ln |x^+ x^-| \tag{42}
\]

This tells that we cannot distinguish these two solutions in future null infinity \((x^+ \to +\infty, x^- \text{ fixed})\). If we define \( S \)-matrix between future null infinity and past null infinity \((x^+ \text{ fixed, } x^- \to +\infty, )\) by these solutions, we find the \( S \)-matrix cannot be unitary since the \( S \)-matrix does not have inverse. Therefore the quantum information in the past null infinity is lost in the future null infinity. These two kinds of shock waves, however, create different kinds of wormholes. The two shock waves solution creates two (Fig.3a) or one (Fig.3b) wormhole. (\( n \) shock wave solution often creates \( n \) wormholes.) This suggests that if we consider the Fock space which includes wormholes, we might possibly construct an \( S \)-matrix which is unitary, which might tell that we need to quantize spacetime.

### 4 The Mass of Black Hole and the Hawking Radiation

In this section, we discuss about the definition of the mass of two dimensional black holes in the basis of the argument of Regge and Tetelboim [26].

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In four dimensional gravity theory, if we fix the degrees of general coordinate transformation completely and choose one of coordinate systems, we can define a Hamiltonian $H$ with respect to the coordinate system. This Hamiltonian $H$ generates the translation of the “time” $t$ in this coordinate system. The “time” has not always any relation with the physical (in some sense) time and the direction of the “time” is not always that of any time-like Killing vector. (It often happens that there is not any time-like Killing vector.) The operator $\epsilon H$ ($\epsilon$ is an infinitely small constant) generates a “time”-translation $t \rightarrow t + \epsilon$ but the physical distance between $t$ and $t + \epsilon$ is, of course, not $\epsilon$ but $\sqrt{g_{00}\epsilon}$.

Therefore what kind of “time” translation a Hamiltonian generates depends on the choice of the coordinate system and the Hamiltonian or its value i.e., mass, has not universal meaning. At least in four-dimension, however, we can define the mass almost uniquely by according to Regge and Teitelboim.

Let’s suppose three dimensional space is a sphere and a black hole lies on its center. According to Regge and Teitelboim, the Hamiltonian is given by a sum of the volume integral of the stress tensor and the surface term. Since the stress tensor vanishes due to the constraints, only the surface term can contribute to the value of the Hamiltonian, i.e., the mass of the black hole. Therefore the mass only depends on the value of the fields on the surface of the sphere i.e., the boundary value of the fields. If the sphere is large enough, the space-time is asymptotically flat and we can find an asymptotically time-like Killing vector even if the black hole is not static. Since the mass only depends on the boundary value, the mass is independent of the choice of the coordinate system inside the sphere. On the surface, we can uniquely (up to the rotation and the translation of the space coordinates) choose a asymptotically Minkowski coordinate whose time direction coincides with that of the asymptotically time-like Killing vector. By using this coordinate system, we can evaluate the mass uniquely from the boundary values of the fields.

In the papers by Bilal, Kogan [23] and de Alwis [24], it was considered the boundary value which corresponds to that on the surface of the sphere in four-dimensional space time. Since the two dimensional space is a line, its boundaries are two ends of the line. Bilal, Kogan and de Alwis have claimed that it is necessary to sum up these two boundary values in order to get the mass. We have questions about this point. If we change the coordinate system, the time-slice i.e., the line of $t = \text{constant}$ and the positions of the end points of the line are also changed. Different coordinate choice gives different pair of two points which are the boundaries of the space. The direction and the scale of time at the two end points can be also changed by the change of the coordinate. This imply that the operation of the summing up two boundary values does not have universal meaning. Note that any observer lies on one of two end points. If the mass which is measured by the observer depends on the boundary value on another end point, which is space-like disconnected with him, it seems that the causality would be violated. In this paper, we propose that the boundary value on the point where the observer lives should be the
mass of black hole which he observes. Therefore the mass depends on which
asymptotically flat region the observer lives in. More rigorously, we propose
an observer-dependent mass as follows: We choose a local Lorentz coordinate
whose time-direction coincide with the tangent of the observer’s world line. We
choose a local Lorentz coordinate, where metric is given by $g^{\mu\nu} = \text{diag}(-1,1)$,
on the point where the observer lives. We define the mass of the black hole by
evaluating the boundary value in this local Lorentz coordinate and we neglect
the boundary value on another point.\(^3\)

We now construct the Hamiltonian $H$ corresponding to the action (28). The
conjugate momenta $\Pi_A \equiv 4\pi \frac{2\alpha}{5\lambda A}$ ($A = X, \bar{Y}, Z, \varphi, S, \bar{T}, \bar{x}^\pm = t \pm r$ and $A \equiv \partial_\mu A$) are given by

\[
\Pi_X = \frac{k}{2} \partial_t X, \\
\Pi_{\bar{Y}} = \frac{k}{2} \left\{ -\sinh^2 X \partial_t \bar{Y} + \left( \cosh^2 X - \frac{1}{2} \right) \partial_t \bar{Z} \right\}, \\
\Pi_Z = \frac{k}{2} \left\{ \cosh^2 X \partial_t Z - \left( \cosh^2 X - \frac{1}{2} \right) \partial_t \bar{Y} \right\}, \\
\Pi_{\varphi} = -\frac{k}{8} \partial_t \varphi, \\
\Pi_S = -\frac{k}{8} \partial_t \bar{T} , \\
\Pi_{\bar{T}} = -\frac{k}{8} \partial_t S. \tag{43}
\]

In terms of $\Pi_A$ and $A'$ ($A' \equiv \partial_\mu A$), the Hamiltonian density $T_{00} \equiv \frac{1}{2}(T_{++} + T_{--} + 2T_{+-})$ is given by

\[
T_{00} = k \left[ \frac{2}{k^2} \Pi_X^2 + \frac{1}{2} \lambda^2 + \frac{2}{k^2} \cosh^{-2} X \Pi_Z^2 + \frac{2}{k} \left( 1 - \frac{1}{2} \cosh^{-2} X \right) \Pi_{\bar{Y}} \Pi'_{\bar{Y}} + \frac{1}{8} \cosh^{-2} X \Pi_{\bar{Y}}^2 - \frac{1}{8} \sinh^{-2} X \Pi_Z^2 - \frac{1}{8} \sinh^{-2} X \Pi_{\bar{Z}}^2 + \frac{2}{k} \sinh^{-2} X \left( \cosh^2 X - \frac{1}{2} \right) \Pi_{\bar{Y}} \Pi_{\bar{Y}}' - \frac{8}{8} \Pi_{\varphi}^2 - \frac{1}{8} \varphi^2 + \frac{16}{k^2} \Pi_S \Pi_{\bar{T}} + \frac{1}{4} \Pi'_{\bar{T}} - \frac{1}{4} \Pi'_{\bar{T}} \right] \\
+ \frac{\alpha}{2} \left( \sinh X e^{\xi + \frac{\lambda}{2}} \right)^{2I} e^{\gamma \Pi_{\bar{T}}^2} + T_{00}^{\text{matter}} + T_{00}^{\text{ghost}}. \tag{44}
\]

\(^3\)If we define the mass by summing up two boundary value, the mass should be positive
semi-definite by using the argument of supersymmetry [20, 21]. We could also show that the
mass defined in this paper should be also bounded from below if we could choose a coordinate
system where an end of the space is fixed to a point in the space-time. If the mass measured
by the fixed point is $m_b$, any mass should be greater than $-m_b$ since the sum of the masses
should be positive due to the supersymmetry. The result about the Bondi mass, which will
be given later, might tell that we cannot always choose the fixed point where the measured
mass is finite.
Then the infinitesimally small variation of the bulk Hamiltonian $H_0 = \int dr T_{00}$ with respect to $A$ and $\Pi_A$ ($A = X, \vec{Y}, Z, \varphi, S, \vec{T}$) is given by a bulk part $\delta H$, which gives the equations of motions, and the surface term $-\delta H_\theta$ (the minus sign comes from the later convenience.)

$$\delta H_0 = \delta H - \delta H_\theta.$$  \hspace{1cm} (45)

The explicit form of $-\delta H_\theta$ is given by,

$$-\delta H_\theta = k \left[ X' \delta X + \frac{2}{k} \left( 1 - \frac{1}{2} \cosh^{-2} X \right) \Pi_Z \delta \vec{Y} + \frac{1}{4} \cosh^{-2} X \vec{Y}' \delta \vec{Y} \right.$$  
$$\left. \frac{1}{4} \sinh^{-2} X Z' \delta Z + \frac{2}{k^2} \sinh^{-2} X \left( \cosh^2 X - \frac{1}{2} \right) \Pi_\varphi \delta \varphi - \frac{1}{4} \varphi' \delta \varphi \right.$$  
$$+ \frac{1}{4} \left( \vec{T}' \delta S + S' \delta \vec{T} \right) + \frac{1}{4} \delta S' \right] \text{the end points of t=constant line}, \hspace{1cm} (46)$$

By using the solution (34) of the equations of motion, the equation (46) can be rewritten by

$$-\delta H_\theta = \left. \frac{k}{4} (\vec{T}' \delta S + \delta S') \right| \text{the end points of t=constant line}, \hspace{1cm} (47)$$

in the general coordinate system. (Note that $\vec{Y}$ is a scalar but $\vec{T}$ is not scalar.) Therefore the total Hamiltonian $H$ should be given by

$$H = H_0 + H_\theta, \hspace{0.5cm} H_\theta = \left. -\frac{k}{4} (\vec{T}' \Delta S + \Delta S') \right| \text{the end points of t=constant line}, \hspace{1cm} (48)$$

Here $\Delta S$ is a finite variation around some reference solution. We need the above surface Hamiltonian $H_\theta$ in order to cancel the surface term which comes from the variation of the bulk Hamiltonian $H_0$. Since $T_{00}$ vanishes identically due to the constraints, $H_0$ also vanishes and only $H_\theta$ can contribute to the value of the total Hamiltonian $H$. By assuming that an observer lives in one of two end points, we regard one of the above two boundary values as a mass $M$ which the observer measures.

$$M = \left. -\frac{k}{4} (\vec{T}' \Delta S + \Delta S') \right| \text{the end point where the observer lives}, \hspace{1cm} (49)$$

In the neighborhood around the observer, we need to choose the local Lorentz frame whose time direction is parallel to the tangent of the world line of the observer. Or equivalently, we can multiply $\sqrt{-g^{00}}$ to the expression in (49).

$$M = \left. -\frac{k}{4} \sqrt{-g^{00}} (\vec{T}' \Delta S + \Delta S') \right| \text{the point where the observer lives}, \hspace{1cm} (50)$$

The above expression (50) can be used for any observer who does not always live in an asymptotically flat region. In this sense, we can define local mass by the equation (50).
We now calculate the ADM mass of our model. We choose the asymptotically Minkowski coordinates $x^\pm$ for the static solution (23) ($b_+ = b_- = 0$) by

$$x^\pm = \pm \frac{1}{\lambda} \ln(\pm \lambda x^\pm) \,. \tag{51}$$

and for the shock wave solution (24) ($a_\pm = b_\pm = 0$) by

$$x^+ = \frac{1}{\lambda} \ln(\lambda x^+) \,, \quad x^- = -\frac{1}{\lambda} \ln(-\lambda x^- + \frac{4m\lambda}{\Lambda k a}) \,. \tag{52}$$

The ADM mass corresponds to the observer who lives in an asymptotically flat region where $x^+ + x^- : \text{fixed}$ and $x^+ - x^- \to +\infty \ i.e., \ f \to +\infty \ (35)$. By using Eqs.(1) and (37), we find the metric $g^{00}$ in this region is given by

$$g^{00} = \frac{1}{2} g^{\pm \pm} = -e^{-2\rho}$$

$$\to - \frac{f e^{-\lambda (x^+ - x^-)} }{2b(2w^2)^{1/2}}$$

$$\to - \frac{\Lambda}{2b(2w^2)^{1/2} \lambda^2} \,. \tag{53}$$

Note that conformal mode $\rho$ transforms as $\rho(x^\pm) \to \rho(x^\pm) + \frac{\lambda}{2}(x^+ - x^-)$ by the coordinate transformation (51) or (52). In the following, we choose $\lambda$ so that $g^{00} \to -1 \ i.e.,$

$$\lambda^2 = \frac{\Lambda}{2b(2w)^{1/2}} \,. \tag{54}$$

Then by using the mass formula (50) and by choosing a reference solution by

$$-\frac{ka}{4} u_0^0 (x^\pm) = -\frac{N}{24} \{ \lambda (x^+ - x^-) - \ln \lambda^2 \} \,, \tag{55}$$

which gives a quantum analogue of the linear dilaton vacuum, we find the ADM mass for the static solution

$$M = \frac{\lambda}{2} (a_+ + a_-) \,, \tag{56}$$

and for the shock wave solution,

$$M = 2me^{x_0^+} \,, \tag{57}$$

where $x_0^+ = \frac{1}{\lambda} \ln(\lambda x_0^+)$. Note that $\bar{T}$ in this coordinate system is given by

$$\bar{T} = -\frac{\lambda}{2}(x^+ - x^-) \,. \tag{58}$$

The above values of masses (56) and (57) do not vanish in general and finite.
Besides \( f \to +\infty \) region, there is another asymptotically flat region where \( f \to 0 \) and we can also consider the mass measured by an observer who lives in this region. Due to Eq.\((38)\), \( g^{00} = -\frac{1}{6} \) is given by,

\[
g^{00} \sim - f^{\mp + \delta}. \tag{58}\]

Therefore we find the mass has the following form when \( f \to 0 \):

\[
M \sim f^{\mp}\left(\frac{\mp + \delta}{\ln f}\right). \tag{59}\]

Then we find the mass vanishes in the limit of \( f \to 0 \).

We now consider the Bondi mass. The Bondi mass is measured by the observer who lives in the region where \( x^- \) fixed and \( x^+ \to +\infty \). Then the formula \((50)\) gives the following expression for the shock wave solution:

\[
M = 2\lambda \left[ \frac{\ln e^{\pm \gamma \pm \delta}}{24} - \frac{N}{24} \ln \left( 1 + \frac{a}{\lambda} e^{\pm \gamma} \right) - \frac{N}{24} \frac{\lambda}{2 + \frac{e^{\pm \gamma}}{3}} \right]. \tag{60}\]

The expression diverges when \( x^- \to +\infty \). The divergence, however, would just tell that the Bondi mass is ill-defined and would not give any other serious problem. \( x^- \to +\infty \) means that the observer approaches to \( f = 0 \) line. In the following we show that the scalar curvature \( R \) diverges near \( f = 0 \) line in the limit of \( x^+ \to +\infty \) although the curvature vanishes just on the \( f = 0 \) line. Since the space-time is strongly curved, the Bondi mass becomes ill-defined in the limits \( x^+ \to +\infty \) and \( x^- \to +\infty \). The limit \( x^- \to +\infty \) does not commute with the limit \( x^+ \to +\infty \). The mass formula gives different answers depending on different limiting procedures.

By defining \( M, X, p \) and \( q \) by

\[
M \equiv -\frac{4m x^+}{kn}, \quad X \equiv -\frac{4m}{\Lambda kn}, \quad x^+ = e^{-\Phi(p+q)}, \quad x^- = -e^{-\Phi(p-q)} - X, \tag{62}\]

\( f \) in Eq.\((35)\) corresponding to the shock wave solution \((24)\) has the following form when \( x^+ > x^- \)

\[
f(p, q) = \Lambda e^p + M - \frac{N}{24} \ln \left( e^p \left( 1 + e^{-2\Phi(p-q)} \right) \right). \tag{63}\]

By putting \( f = 0 \) and solving \( q \) with respect to \( p \), we obtain

\[
q = q_o(p) \equiv -p + 2 \ln \left( \frac{e^{2\Phi(\Lambda e^p + M)} - e^p}{X} \right). \tag{64}\]

This expression is different from that in the previous paper \([22]\), where it was considered the first variations of the stress tensor with respect to \( X^\pm \). These fields, however, would be inadequate for the calculation of the Bondi mass since they do not damp in the spacial infinity.
When $p$ is sufficiently large, $q$ becomes monotonously increasing function with respect to $p$ and behaves $q \propto \frac{\Lambda e^p}{X}$. Therefore in the limit $p \to +\infty$ on the $f = 0$ line, $x^\pm \to x^- = -X$, i.e., $x^\pm \to +\infty$.

We now calculate the scalar curvature $R = 8e^{-2\rho} \partial_{\rho} \partial_{-\rho} = -8e^{-2\rho} \rho''$. Here $\rho$ is given in Eq.(37). By defining

$$P \equiv p - p_0, \quad Q \equiv -q + q_0(p_0),$$

we consider the limit $p_0 \gg |P|, |Q|$ and calculate the scalar curvature $R$ on the line where $Q = 0$. On the $Q = 0$ line, $P \geq 0$. When $p_0 \gg |P|, |Q|$, $f$ has the following form

$$f = \Lambda e^{p_0}(e^p - 1) - \frac{N}{48}(P + Q) + o(1),$$

which tells that the derivative with respect $Q$ can be neglected compared with the derivative with respect to $P$. Then we obtain

$$R \sim -8e^{-2\rho - (P + p_0)}(\partial^2_{\rho} f \rho' + (\partial_{\rho} f)^2 \rho'').$$

Here $\rho' = \frac{dp}{d\phi}$. (See Eq.(37).)

When $0 \leq P \ll e^{-p_0}$, $f$ is small : $f \ll 1$. By using Eq.(38), we find

$$R \sim -4\Lambda \frac{1}{e^P - 1} \left( \frac{1}{2} \right)^{p_0} P^{\frac{1}{2}}.$$ 

When $22 < c_{\text{matter}} < 24$, $R$ vanishes when $P = 0$ but the derivative of $R$ with respect to $P$ exponentially increases, due to the factor $e^{\left(\frac{1}{2} - \frac{1}{2}\right)p_0}$, as $p_0$ increases.

On the other hand, when $p$ is finite and $p_0 \gg 1$, $f$ is large $f \gg 1$. Then, by using Eq.(37), we find that $\rho$ behaves as

$$\rho \sim -\frac{1}{2} \ln f + O(1),$$

and the scalar curvature $R$ is given by

$$R \sim -\frac{4\Lambda}{e^P - 1}.$$ 

The expression (70) tells that $R$ diverges in the limit $P \to 0$, where the expression (70) breaks down and the expression (69) becomes valid. By combining these results Eq.(70) and Eq.(69), we find that $R$ has a peak $|R| \sim e^{p_0}$ when $P \sim e^{-p_0}$ and the scalar curvature in the neighborhood of the $f = 0$ line diverges in the limit of $p_0 \to +\infty (x^+ \to +\infty, x^- \to -X)$. At present we do not fully understand the meaning of this curvature singularity.

Instead of defining Bondi mass, we consider the mass measured by an observer who is depicted in Fig.4. He lives in an asymptotically flat $f \to +\infty$
region at first and he approaches to \( f = 0 \) line. For example, we assume the
world line of the observer is given by \( r \equiv \frac{1}{2}(x^+ - x^-) = r_0 \) (constant) and
the constant \( r_0 \) is sufficiently large. When he lives in the asymptotically flat
region, the mass measured by him is given by Eq.(57). As he approaches to
\( f = 0 \) line, the space-time becomes to have a curvature and the mass measured
by him becomes to vary and he would consider he is observing the Hawking
radiation. After infinite proper time, he reaches to the \( f = 0 \) line, where the
measured mass vanishes and he would consider the black hole has evaporated
completely. Of course, since the space-time is curved in the neighborhood of the
\( f = 0 \) line, the mass measured there has not universal meaning. The masses,
however, measured by at first (in the asymptotically flat region) and at last (on
the \( f = 0 \) line) are universal. Therefore a class of observers who live in the
asymptotically flat region at first and whose world lines cross the \( f = 0 \) line in
finite \( x^+ \) (but after infinite proper time) observes the black hole evaporation.

5 Summary and Discussion

In this paper, we have analyzed a quantum two dimensional dilaton gravity
model, which is described by \( SL(2, R)/U(1) \) gauged Wess-Zumino-Witten model
deformed by \( (1, 1) \) operator. It has been shown that the curvature singularity
does not appear when the central charge \( c_{\text{matter}} \) of the matter fields is given
by \( 22 < c_{\text{matter}} < 24 \). When \( 22 < c_{\text{matter}} < 24 \), the matter shock waves,
whose energy momentum tensors are given by \( T_{\text{matter}} \propto \delta(x^+ - x^+_0) \),
create a kind of wormholes, i.e., causally disconnected regions. Most of the quantum
informations in past null infinity are lost in future null infinity but the lost
informations would be carried by the wormholes.

Recently Hawking and Hayward [30] have pointed that there might be a
close connection between wormholes and the formation and evaporation of black
holes. In their scenario, informations falling into black holes pass into another
universe through the wormholes. It is interesting to consider the relation be-
tween their works and the present one.

We have also discussed about the problem of defining the mass of quantum
black holes. On the basis of the argument by Regge and Teitelboim, we find
the ADM mass measured by the observer who lives in one of asymptotically
flat regions is finite and does not vanish in general. Instead of the Bondi mass,
we have considered a mass measured by a class of observers who live in an
asymptotically flat region at first and approaches to \( f = 0 \) line. They observe
the change of mass, i.e., the Hawking radiation. The measured mass vanishes
finally and the black hole evaporates completely. Therefore the total Hawking
radiation is positive even when \( N < 24 \).

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References


Figure Captions

**Fig. 1** The structure of the space-time corresponding to a static solution is depicted. $p$ and $q$ are defined by $x^\pm \equiv \pm e^{(p\pm q)}$. The asymptotically flat region corresponding to $f \to +\infty$ is depicted like a cylinder. And the asymptotically flat region corresponding to $f \to +0$ is depicted like a plane.

**Fig. 2** Wormhole creation by a matter shock wave. The region I is causally disconnected with asymptotically Minkowski region II.

**Fig. 3** Worm holes created by two matter shock waves. The two shock wave solution creates two (Fig.3a) or two (Fig.3b) wormhole.

**Fig. 4** The world line of the observer is depicted. His world line is given by $\tau \equiv \frac{1}{4}(\bar{x}^+ - \bar{x}^-) = \tau_0$ (constant). He lives in an asymptotically flat $f \to +\infty$ region at first and he approaches to $f = 0$ line.