Universal Quantum Cloning in Cavity QED

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We propose an implementation of an universal quantum cloning machine [UQCM, Hillery and Buzek, Phys. Rev. A 56, 3446 (1997)] in a Cavity Quantum Electrodynamics (CQED) experiment. This UQCM acts on the electronic states of atoms that interact with the electromagnetic field of a high Q cavity. We discuss here the specific case of the $1 \rightarrow 2$ cloning process using either a one- or a two-cavity configuration.

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\textit{a. Introduction.} Quantum theory provides new and unexpected effects when compared to classical physics. Among them, the no-cloning theorem, derived in 1982 by Wooters and Zurek [1], plays a particularly important role: While classical information can be copied perfectly and many times, quantum information cannot. This fundamental difference is a consequence of the unavoidable creation of quantum correlations. Since perfect cloning is not possible, an important question naturally arises: What is the best quantum copying operation? The answer to this question is context-dependent. On the one hand, there is a single transformation that produces the best identical copies of a qubit prepared in any input states. This “universal quantum cloning machine” (UQCM) has been discussed for the first time in [2]. On the other hand, many other rules of the game can be considered, such as state dependent cloning [3], cloning of 3-dimensional states [4] and cloning of orthogonal qubits [5].

The quality of a copy is usually measured by the quantum fidelity [6]. This quantity is discussed, in the context of universal quantum cloning machine (UQCM), in [2] and [7]. When $M$ copies are produced from $N$ identical pure 2-dimensional states, the fidelity of the copies is given by $F(N,M) = (NM + N + M)/(M(N+2))$. For the simplest case of two copies produced from one input state, this expression reduces to $F(1,2) = 5/6$. The complete understanding of the fidelity behavior versus $N$ and $M$ is still a subject of debate, with connections to the measurement and state estimation problems [8]. Beyond these fundamental problems, the interest of quantum cloning machines also encompasses a wide area of quantum information processing, including quantum cryptography, teleportation [9], eavesdropping, state preservation and measurement-related problems, as well as quantum algorithm improvements [10].

The derivation of the optimal UQCM transformation led to several proposals [11] for its experimental implementation. Most of them, based on the Buzek and Hillery quantum logics network [12], use the quantum optics framework. Experimental quantum cloning has been realized up to now only with photons as the carriers of quantum information. This information was either encoded in different degrees of freedom of the same photon (polarization and position) [13] or in the photon polarization only [14]. An alternative network adapted to NMR-based quantum information processors has also been proposed and experimentally implemented [15].

In this paper, we propose an implementation of the $1 \rightarrow 2$ UQCM operating for atomic states in the Cavity QED (CQED) context [16]. The quantum information is coded on electronic levels of long-lived highly excited Rubidium (Rb) atoms. Our protocol realizes, with four atoms, the transformation described in [2], with an original quantum logics network based on the resonant interaction between the atoms and two high-Q niobium superconducting microwave cavities $C_a$ and $C_b$. We discuss, at the end of this paper, an adaptation of the scheme using two different modes of a single cavity [17], making the proposal implementation more realistic with the present cavity QED set-up. This paper focuses on the quantum logics protocol. The interested reader can find more details about the experimental techniques in [16].

Let us first recall the optimal $1 \rightarrow 2$ UQCM transformation [2]. When the qubits are encoded in the basis $\{|\pm\rangle\}$, the UQCM performs the transformations

\begin{align}
|\pm\rangle_B & \rightarrow \sqrt{\frac{2}{3}} |\pm\rangle_A, \\
|\mp\rangle_B & \rightarrow \sqrt{\frac{1}{3}} |\Phi\rangle_A, \\
|\theta\rangle_B & \rightarrow \sqrt{\frac{1}{3}} |\Phi\rangle_A, \\
|\psi\rangle_B & \rightarrow \sqrt{\frac{1}{3}} |\Phi\rangle_A,
\end{align}

\textcolor{red}{(1)}
The atomic qubit encoding is |\rangle, classical microwave pulses either from level coupled to them. However, it can be accessed via the Stark effect induced by an electric field applied between the Fabry Perot cavity mirrors [16]. The auxiliary level |i\rangle is far off-resonant from the cavity fields and is not coupled to them. However, it can be accessed via classical microwave pulses either from level |g\rangle (one photon transition) or from level |e\rangle (two-photon transition). The atomic qubit encoding is |+\rangle = 1/\sqrt{2}(|i\rangle + |g\rangle) and |−\rangle = 1/\sqrt{2}(|i\rangle − |g\rangle). The photon number states of each cavity mode are denoted as |n\rangle_i, where i = (a, b).

b. Description of the protocol: The sequence of operations achieving the UCQM transformation is pictorially depicted in Fig. (1). It presents, in a space-time diagram, the space lines of the two cavity modes and of the four Rydberg atoms, A1−4, involved in the process. The atom-cavity resonant interactions are represented by black lozenges. Classical microwave pulses mixing the atomic levels are represented as gray circles.

The cavity fields are initially prepared in the vacuum state |0\rangle_i [16]. The first atom, A1, initially in state |g\rangle_1, is prepared in state |
\Psi\rangle_1 = \sqrt{\frac{2}{3}}|g\rangle_1 + \sqrt{\frac{1}{3}}|e\rangle_1,
\tag{2}

by a classical pulse resonant with the |g\rangle → |e\rangle transition. The coefficients in the equation above are set adjusting the duration of the classical pulse to a \phi = \arcsin(\sqrt{3)/2} rotation (see Fig. 1). The production of (2) can be checked in auxiliary experiments measuring the population of states |g\rangle_1 and |e\rangle_1 and the quantum coherence. The atomic state (2) is then transferred to C_a, through a \pi pulse of resonant quantum Rabi oscillation [18]. Atom A1 finally leaves C_a in state |g\rangle_1 and the cavity field is left in state |
\psi\rangle_a = \sqrt{2/3}|0\rangle_a + \sqrt{1/3}|1\rangle_a. The first atom’s final state being factored out, it will no longer be considered here.

Atom A2, carrying the state to be cloned, crosses then C_a. It is prepared in the arbitrary state
|
\Psi\rangle_2 = \alpha|+\rangle_2 + \beta|−\rangle_2,
\tag{3}

where \alpha and \beta are complex coefficients. Note that the preparation of this state, which is not part of the quantum cloning process, is not represented in figure 1. This atom interacts with the cavity field, performing a 2\pi quantum Rabi pulse, amounting to a resonant quantum phase gate (QPG) described in [19]. The QPG produces a \pi phase shift of the atom-cavity quantum state if and only if the atom is in state |g\rangle and the cavity is in state |1\rangle_a. When expressed in the \{|+\rangle, |−\rangle\} basis, this QPG operation amounts to a controlled not gate (CNOT), where the control qubit is the field state. After this interaction the total entangled atom-field state becomes

\[ \sqrt{\frac{2}{3}}(\alpha|+\rangle_2 + \beta|−\rangle_2)|0\rangle_a + \sqrt{\frac{1}{3}}(\alpha|−\rangle_2 + \beta|+\rangle_2)|1\rangle_a. \]
\tag{4}

We then send a third atom, A4, prepared in state |g\rangle_3. It interacts resonantly with C_a, for a time interval corresponding to a \pi/2 quantum Rabi pulse, producing the state

\[ \sqrt{\frac{2}{3}}(\alpha|+\rangle_2 + \beta|−\rangle_2)|g\rangle_3|0\rangle_a \]
\tag{5}
\[ + \sqrt{\frac{1}{6}}(\alpha|−\rangle_2 + \beta|+\rangle_2)(|g\rangle_3 |1\rangle_a + |e\rangle_3 |0\rangle_a). \]

The state of C_a is finally transferred to a fourth atom, A4, initially in |g\rangle_4 via a resonant \pi quantum Rabi pulse, creating the three-atom entangled state:

\[ \sqrt{\frac{2}{3}}(\alpha|+\rangle_2 + \beta|−\rangle_2)|g\rangle_3 |g\rangle_4 \]
\tag{6}
\[ + \sqrt{\frac{1}{6}}(\alpha|−\rangle_2 + \beta|+\rangle_2)(|g\rangle_3 |e\rangle_4 + |e\rangle_3 |g\rangle_4), \]

and leaving C_a in the vacuum state, which factors out. Classical microwave pulses then address the three atoms, transforming |e\rangle into |i\rangle via a two-photon \pi pulse. This transformation does not affect state |g\rangle. Then, another classical \pi/2 pulse is applied on the three atoms, combining states |g\rangle and |i\rangle. The sequence of transformations produced by these classical pulses can be summarized as follows:

|g\rangle \leftrightarrow |g\rangle \leftrightarrow \sqrt{\frac{1}{2}}(|i\rangle − |g\rangle) = |−\rangle,
|e\rangle \leftrightarrow |i\rangle \leftrightarrow \sqrt{\frac{1}{2}}(|i\rangle + |g\rangle) = |+\rangle. \]
\tag{7}

The remaining part of the protocol involves the second cavity C_b. Atom A2 interacts resonantly with C_b for a time interval corresponding to a \pi quantum Rabi pulse, transferring its state to the field mode. The final state of A2 is |g\rangle_2 and also factorizes out. The total state of A_3, A_4 and C_b is then:

\[ \sqrt{\frac{2}{3}}(\alpha|1\rangle_b + \beta|0\rangle_b)|−\rangle_3 |−\rangle_4 \]
\tag{8}
\[ + \sqrt{\frac{1}{6}}(\alpha|0\rangle_b + \beta|1\rangle_b)(|−\rangle_3 |+\rangle_4 + |+\rangle_3 |−\rangle_4). \]

Atoms A_3 and A_4 interact then independently and successively with C_b. They perform a resonant QPG, corresponding to a CNOT in the \{|+\rangle, |−\rangle\} basis. The final
This is shorter than present cavity damping times (about of the order of 4 full atomic transit times, i.e. has left it. The total quantum information storage time is may enter the cavity immediately after the preceding one.

|A⟩second cavity field ensures the orthogonality of where |A⟩ and |A⊥⟩ and hence is the important qubit in the ancilla’s final state. This achieves the implementation of the optimal 1 → 2 cloning process.

Eq. (9) shows that this sequence actually implements the UCQM transformation given by Eq. (1). In this proposal, the blank state |B⟩ corresponds to the initial state of atoms A1, A3 and A4 and of both cavity fields. It writes thus

\[ |B⟩ = \left( \sqrt{\frac{1}{3}} |e⟩_1 + \sqrt{\frac{2}{3}} |g⟩_1 \right) |g⟩_3 |g⟩_4 |0⟩_a |0⟩_b. \] (10)

c. Discussion: We can now discuss the feasibility of an experimental implementation of the UQCM in a CQED system. The basic operations (quantum gates and classical field pulses) involved in the scheme have already been thoroughly tested [16]. Their implementation thus does not present any major difficulty. The availability of an experimental configuration with two cavities can be considered as natural development of the present configurations using a single cavity, and mainly a matter of time. Note also some other interesting proposals require at least a two-cavity system [20].

Atoms interact with C or Cb for a time corresponding at most to a 2π quantum Rabi pulse. The single photon Rabi frequency [18] being Ω/2π = 50 kHz, the atomic velocity should be ≈ 500 m/s, in the range used in present experiments. Cavity and atomic relaxation are of course important issues. The circular Rydberg atoms lifetime is much longer than the protocol duration and is not bound to be a limiting factor. The main cause of decoherence in the present set-up is the cavity mode relaxation. The quantum information is stored in C only during the time interval between the passage of A1 and A4. Each atom may enter the cavity immediately after the preceding one has left it. The total quantum information storage time is of the order of 4 full atomic transit times, i.e. ≈ 2.10^{-4}s. This is shorter than present cavity damping times (about 1 ms). The cavity C stores quantum information for an even shorter time interval. Note finally that the atomic transit time between the two cavities does not matter to evaluate the influence of damping, since the quantum information is then carried by long-lived atomic systems.

An alternative implementation of our UQCM scheme uses two modes of a single cavity. In the present experimental set-up, the cavity sustains two gaussian modes, M and M, with orthogonal linear polarizations. Due to mirrors imperfections, these two modes have slightly different resonant frequencies (splitting 130 kHz). Since this splitting is much larger than the atom-field coupling Ω, the atoms resonantly interact with one mode only at the same time. Stark tuning can be used to tailor atomic interactions with the two modes during the atomic transit time through the cavity.

In this scheme, A1 leaves its state in M. Then, A2 performs the CNOT operation in M. It is set off-resonance with both modes for a short time interval during which the microwave classical pulses are applied. Atom A2 it then tuned to resonance with M for its final quantum Rabi pulse. A3 and A4 interact first resonantly with M, undergo the classical pulses while being off-resonance from the two modes and finally interact with M as described above. This implementation of the UQCM, requiring a single cavity, would be much simpler to realize. Each atom should interact with the cavity for a total time corresponding at most to a 3π quantum Rabi pulse (the duration of the classical pulses is negligible). The atomic velocity should be about 330 m/s, still well within the available range. The quantum information is stored in the cavity modes for a slightly longer time than in the two-cavity arrangement (four times the full transit time of atoms at the slower 330 m/s velocity). Cavity damping should thus be somewhat smaller.

The UQCM operation verification can, in principle, be performed by the usual detection techniques [16]. As mentioned above, the fidelity is, ideally, 5/6 while the trivial production of a maximally mixed state gives an average fidelity of 2/3. This means that the fidelity should be measured with a precision greater than ≈ 92% (≈ 1 − 1/2(5/6 − 2/3)). Note that, in the NMR quantum cloning experiment [15], this degree of precision was not reached, so that the improvement due to the cloning process could not be verified.

In our proposal, all the elementary operations, quantum Rabi or classical field pulses, are prone to errors. The total number of these operations is sixteen if we take into account the detection and preparation process. The necessary precision could only be reached if each pulse has a fidelity greater than 0.995 is still out of the experimental reach (present pulse imperfections are between 3 and 10%). This figure, however, sets an interesting goal to be reached.

d. Conclusion. We described a protocol implementing the universal optimal copying transformation in CQED. Basic quantum information operations have already been implemented in this context [19], and proposals that could extend these experimental realizations to more elaborated quantum information algorithms [21] are naturally appealing.

The quantum logics network used in our scheme is simpler than previous ones by making use of auxiliary degrees of freedom which are discarded in the end of the process. Note also that the same protocol can be applied to the cloning of equatorial qubits [3].
i.e., $|\psi\rangle = \sqrt{1/2} (|0\rangle + e^{i\phi} |1\rangle)$ by sending $A_1$ in state $\sqrt{1/2}(|e\rangle + |g\rangle)$.

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