physics reach of rare $b$-decays

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I discuss the theoretical and empirical status of $b \to s\gamma$, $b \to s\ell^+\ell^-$ decays, as well as their future prospects. I emphasize those observables in rare $b$-decays which can potentially establish new physics and distinguish between extensions of the Standard Model. I briefly review current models of electroweak symmetry breaking, all of which can carry interesting flavor characteristics accessible with $b$-physics experiments.

1. $b \to s\gamma$ STATUS

Rare radiative $b \to s\gamma$ decays are both theoretically and experimentally well studied and reached attention as an important constraint on extensions of the Standard Model (SM). The branching ratio is known to NLO (see e.g. [1]) and depends only at 2-loop (the 1-loop matrix element vanishes for an on-shell photon) on the charm mass. But even it appears only at 2-loop, different choices of $m_c$ do numerically matter [1]

$$B(B \to X_s\gamma)_{SM} = (3.35 \pm 0.30) \cdot 10^{-4} \quad (1)$$

if the pole mass $m_c^{pole}/m_b = 0.29 \pm 0.02$ is used and

$$B(B \to X_s\gamma)_{SM} = (3.73 \pm 0.30) \cdot 10^{-4} \quad (2)$$

for the Ms-bar mass renormalized at a scale $\sim m_b$ $\bar{m}_c(\mu)/m_b = 0.22 \pm 0.04$. The difference between $m_c^{pole}$ and $\bar{m}_c(\mu)$ is a higher order in $\alpha_s$ issue. In the absence of a 3-loop NNLO calculation to identify the correct $m_c$ prescription we combine the above branching ratios, inflate errors and obtain

$$B(B \to X_s\gamma)_{SM} = (3.54 \pm 0.49) \cdot 10^{-4} \quad (3)$$

Comparison with the data by Cleo, Aleph and Belle (Ref. [3-5] in [2])

$$B(B \to X_s\gamma)_{worldave} = (3.22 \pm 0.40) \cdot 10^{-4} \quad (4)$$

shows that the theory error exceeds the experimental one and unless there is progress in theory (or the experimental central value moves a lot), we cannot establish new physics (NP) with the $B \to X_s\gamma$ branching ratio alone.

Model independent constraints from $B(B \to X_s\gamma)$ have been obtained in the effective Hamiltonian theory $\mathcal{H}_{eff} = -4G_F/\sqrt{2}V_{tb}V_{ts}^* \sum C_i O_i$ with effective vertices $O_i$ and Wilson coefficients $C_i$ [2]. Important here are the operators $O_7 \propto \bar{s}_L\sigma_{\mu\nu}b_R F^{\mu\nu}$ and $O_8 \propto \bar{s}_L\sigma_{\mu\nu}b_R G^{\mu\nu}$. The LO branching ratio $B(B \to X_s\gamma)_{LO} \propto |C_7(m_b)|^2$ fixes the modulo of $C_7$, which illustrates that one can measure the $C_i$. Constraints on $C_{7,8}$ have been worked out at NLO in terms of the ratios $R_i(\mu) \equiv (C_{7,8}^{SM}(\mu)/C_{7,8}^{NP}(\mu))$ [2]. The result is shown in Fig. 1 at $\mu = m_W$. The bands are the allowed regions, the SM is $R_{7,8} = 1$. The solid (dashed) lines denote the bound using a pole (Ms-bar) mass prescription for the charm quark. Future expectations are such that by 2005 the $B$-factories have collected $500 fb^{-1}$ and measured the $b \to s\gamma$ branching ratio precisely $\sigma(\text{stat, sys}) = 1.8\%, 3\%$ [3]. Hence, the 2 bands would be very narrow, approximately of the size of the difference between the solid and dashed lines given by todays dominant theory error, $m_c$.

The scatter plot results from a scan over the minimal supersymmetric model (MSSM) parameter space with minimal flavor violation (MFV). This is defined as no more flavor violation than in

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The first mode mediated by TeV, 2 m O Z points and both branches are allowed and can be of the MSSM-MFV scan obey gauge and anomaly mediation. The parameters degeneracy of squark masses. MFV examples are \( \sim \) and enforces proportionality of A-terms \( \sim Y \) and the SM, i.e. in the yukawas \( Y \). In supersymmetry (SUSY) this is a condition on the SUSY breaking and enforces proportionality of A-terms \( \sim Y \) and degeneracy of squark masses. MFV examples are gauge and anomaly mediation. The parameters of the MSSM-MFV scan obey \( m_{\tilde{t}, m_{\chi}} > 90 \text{ GeV}, m_{\tilde{g}} > 50 \text{ GeV}, m_{H^\pm} > 78.6 \text{ GeV}, |\mu|, M_2 < 1 \text{ TeV}, 2.3 < \tan \beta < 50 \) and stop mixing angle \( |\Theta_{\tilde{t}}| < \pi/2 \). The solution with the sign of \( C_7 \) flipped w.r.t. the SM needs a large chargino-stop contribution, since both SM and charged Higgs ones interfere constructively, thus large \( \tan \beta \) and/or a light stop is required. Fig. 1 demonstrates that \( B(b \to s\gamma) \) data cut out many points and both branches are allowed and can be reached by NP. To distinguish them we need additional constraints such as from \( b \to s\ell^+\ell^- \) decays.

\[ B(B \to K\ell^+\ell^-)_{SM} = 0.35 \pm 0.12 \cdot 10^{-6} \quad (7) \]

The SM calculation [2] is performed at NNLO (see below the discussion of inclusive decays) assuming factorization. Corrections from spectator interactions have been ignored, because this is a sub leading effect compared to the dominant source of theoretical uncertainty, i.e. the form factors. The reduction of \( B(B \to K\ell^+\ell^-) \) with respect to earlier calculations obtained at NLO by 39% (central values) [6] has 2 sources, first the effects of the NNLO calculation which are also active in the inclusive decays and secondly the lower central value of form factors, as suggested by related analyses in \( B \to K^+\gamma \) decays [2]. To be specific, the minimum set of form factors from light cone QCD sum rules from Ref. [6] plus a \( \pm 15\% \) error has been used to obtain Eq. (7).

Inclusive \( b \to s\ell^+\ell^- \) decays are known to NNLO accuracy for low dilepton mass [7]-[9]. The effective coefficients \( C_i^{eff} = \left[ 1 + \frac{4s}{9s} \omega_i(\hat{s}) \right] A_i + \frac{4s}{9s} C_j F_i(\hat{s}) \) include virtual \( \alpha_s \)-corrections in the functions \( F_i \) and bremsstrahlung and \( \alpha_s \)-corrections to the matrix elements \( <O_i> \) in the \( \omega_i \). The \( b \to s\ell^+\ell^- \) decay rate can then be written as a function of the normalized dilepton mass \( \hat{s} \equiv q^2/m_b^2 \) as [2]

\[
\frac{dT}{d\hat{s}} \sim (1 - \hat{s})^2 \left[ \left( 1 + 2\hat{s} \right) |C_7^{eff}|^2 + |C_{10}^{eff}|^2 \right] f_1(\hat{s}) + 4(1 + 2\hat{s}) |C_7^{eff}|^2 f_2(\hat{s}) + 12 Re(C_7^{eff} C_{10}^{eff} f_3(\hat{s}) + f_c(\hat{s})) \quad (8)
\]

including \( 1/m_c^{10} \) and \( 1/m_b^{11} \) corrections in the functions \( f_c \) and \( f_{1,2,3} \). Experimental information on inclusive \( B \to X_s\ell^+\ell^- \) decays from Belle [12,13] is compiled in Tab. 1 and is consistent with the SM branching ratios [2].

\[
B(B \to X_s e^+e^-) = 6.89 \pm 0.37 \pm 0.25 \pm 0.91 \cdot 10^{-6} \quad (5)
\]

\[
B(B \to X_s \mu^+\mu^-) = 4.15 \pm 0.27 \pm 0.21 \pm 0.62 \cdot 10^{-6} \quad (6)
\]

where the errors correspond to varying \( m_b/2 < \mu < 2m_b, m_t^{pole} = (173.8 \pm 5) \text{ GeV} \) and \( m_c/m_b = 0.29 \pm 0.04 \), respectively. Adding them in quadrature the total errors are estimated as \( \delta B_{X_s e^+e^-} = \pm 15\% \) and \( \delta B_{X_s \mu^+\mu^-} = \pm 17\% \). Going from NLO

![Figure 1. Constraints from \( B(b \to s\gamma) \) at 90\% C.L. on \( R_{7,8}(m_W) \). Figure taken from [2].](image-url)
to NNLO decreases $B(B \to X_s \ell^+ \ell^-)$ by 12% for dielectrons and 20% for dimuons. The full NNLO calculation, i.e. the functions $F_{ij}$ are only available for $\hat{s} < 0.25$ below the $cc$ threshold. The above branching ratios are obtained from naive extrapolation of the $F_{ij}$, which gives a spectrum that is well approximated for all $\hat{s}$ by the partially NNLO one with $F_{ij} \equiv 0$ for $\mu \simeq m_b/2$ [2]. Contributions from charmonium vector mesons via $b \to sV \to s\ell^+\ell^-$ should be removed from the data by cuts in the dilepton mass around $q^2 = m_{J/\Psi}^2$. The biggest source of theory uncertainty in the $B \to X_s \ell^+\ell^-$ branching ratios is $m_c$ [2]. It appears already at 1-loop and is conservatively varied as $m_c/m_b = 0.29 \pm 0.04$ [9]. Study of the parametric dependence of the decay rates on $z = m_c/m_b$ such that $\epsilon$ denotes the percental correction to $\Gamma$ if $z$ changes from 0.29 by $-0.04$ as

$$\Gamma(z) = \frac{\Gamma(0.29)}{0.29 - z + O((0.29 - z)^2)}$$

yields $\epsilon = 2\%$, $6\%$, $16\%$ for $b \to s\ell^+\ell^-$, $b \to s\gamma$, $b \to c\ell^+\bar{\nu}_\ell$ decays, respectively. This explains the shift of 11% in $B(b \to s\gamma)$ see Eqs. (1),(2) when going from the pole mass to the $\overline{MS}$ charm mass. Further, the bulk of the $m_c$ dependence in $B(b \to s\ell^+\ell^-)$ does not result from the $b \to s\ell^+\ell^-$ decay rate, but from the normalization to $\Gamma(b \to c\ell^+\bar{\nu}_\ell)$, which is employed to remove the $m_b^5$ dependence. The decay rates of exclusive $B \to (K, K^*)\ell^+\ell^-$ decays are normalized to the $B$-lifetime, hence there the theoretical error due to $m_c$ is small. By 2005 $B$-factories are expected to have collected a few hundred events of both $b \to s e^+e^-$, $s \mu^+\mu^-$ decays and measured the branching ratios with $\sigma(stat,sys)_{e^+e^-} \simeq 7\%$, $7\%$ and $\sigma(stat,sys)_{\mu^+\mu^-} \simeq 9\%$, $12\%$ [3].

With data available on $B \to (X_s, K)\ell^+\ell^-$ decays the model independent analysis can be extended to include constraints on NP contributions to $C_{9,10}$ for fixed sign of the effective $bs\gamma$ coupling, see Fig. 2. The left plot corresponds to the SM-like sign, i.e. $C_7 < 0$. Unlike the the $\tan \beta$ enhanced dipole coefficients $C_{7,8}$ the MSSM-MFV reach in $C_{9,10}$ is small, i.e. the SM is corrected by $\lesssim 20\%$ at $\mu = m_W$. The scatter plot represents a MSSM scenario with additional flavor violation, i.e. mixing between up-squarks of the 2nd and 3rd generation encoded in $\delta^{23}_{LL}, \delta^{23}_{LR}$. The Forward-Backward asymmetry $A_{FB}$ in $b \to s\ell^+\ell^-$ decays is defined as

$$A_{FB}(\hat{s}) \sim \left(\int_0^1 d\cos \Theta - \int_{-1}^0 d\cos \Theta\right) \frac{d^2 \Gamma}{ds d\cos \Theta}$$

$$\sim - C_{10} [C_7 + \beta(\hat{s}) Re(C_9)]$$

where $\Theta$ is the angle between $\ell^+$ and $b$ in the dilepton CMS, see [14] for a discussion of the $A_{FB}$ sign and CP properties. It tests unique combinations of Wilson coefficients [11] and is an ideal NP counter, as illustrated in Fig. 3. In the SM $A_{FB}$ is negative for very low and positive for large $\hat{s}$.

Table 1

<table>
<thead>
<tr>
<th>mode</th>
<th>branch fraction Belle’02 [13]</th>
<th>signif.</th>
<th>upper bound Belle’01 [12]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B \to X_s\mu^+\mu^-$</td>
<td>$8.9^{+2.3+1.6}_{-1.7-2.1} \cdot 10^{-6}$</td>
<td>4.4$\sigma$</td>
<td>$&lt; 19.1 \cdot 10^{-6}$ $\oplus$ 90% C.L.</td>
</tr>
<tr>
<td>$B \to X_s e^+ e^-$</td>
<td>$5.1^{+2.6+1.3}_{-2.4-1.3} \cdot 10^{-6}$</td>
<td>2.1$\sigma$</td>
<td>$&lt; 10.1 \cdot 10^{-6}$ $\oplus$ 90% C.L.</td>
</tr>
</tbody>
</table>

Figure 2. Constraints from $B \to (X_s, K)\ell^+\ell^-$ data [4,12] on $C_{9,10}^{NP}(m_W)$, $C_{9}^{NP}$ for each solution allowed by $B(b \to s\gamma)$ @90% C.L., figure from [2].
The zero disappears (curve 2) for the non-SM solution $C_7 > 0$. With NP in $C_{10}$ e.g. induced by non-SM Z-penguins [14] the sign of $A_{FB}$ can be flipped (curves 1,3) and also a flat $A_{FB}(s) \sim 0$ is possible. The regions labeled in the left plot of Fig. 2 match the corresponding numbers and $A_{FB}$ shapes in Fig. 3. The $A_{FB}$ in exclusive collider or even before at the Tevatron, with technicolor already being disfavored by precision electroweak data. Why do we expect to see NP in low energy signals? This is related to the question of how much flavor (and CP) violation is in the model besides the one present in the SM, i.e. MFV vs. non-MFV, as illustrated in the model survey in Fig. 4. In the left part are models with SM like

\[
B \rightarrow K^{0}\mu^{+}\mu^{-} \text{ decays has analogous behaviour [6]. The expected event yield for } 2fb^{-1} \text{ at CDF, BTeV, ATLAS, CMS, LHCb is } 59, 2240, 665, 4200, 4500 [3], \text{ respectively and suggests that an experimental study of } A_{FB} \text{ is an opportunity for hadron colliders, too.}
\]

3. FLAVOR/CP AND ELECTROWEAK SYMMETRY BREAKING

Realistic extensions of the SM have to address the hierarchy problem, i.e. why is the Higgs mass stable against quadratic corrections arising at 1-loop $\delta m_h^2 \sim \Lambda^2/16\pi^2$ and does not get renormalized up to the Planck scale $\Lambda \sim 10^{19}$GeV? Theorists created several frameworks to explain this, which are SUSY, models with extra dimensions (ED) [15,16], little Higgs (theory space) [17] and technicolor theories plus hybrids. In all of them we expect to see NP participating in the mechanism of electroweak symmetry breaking (EWKSB) at/below 1 TeV at the LHC, a linear $B$-physics, on the right those with NP in $B$-data. They are separated by a range which size and position depends on how well we can measure and on the theoretical uncertainties.

Flavor violation arises generically in SUSY GUTs from running above the GUT scale, because of the large yukawas of the third generation, e.g. [18]. The reported large atmospheric neutrino $\mu - \tau$ mixing angle in the context of $SO(10)$ embeddings of the MSSM [19,20] and extended MSSM [19] has NP consequences for rare processes, in particular for the $B$-system. Effective SUSY with first 2 generations of sfermions heavy but the third below a TeV [21,22] does predict NP effects in $B$-data. The supersoft proposal [23] with all squarks above 1 TeV and highly degenerate SUSY breaking however will escape low energy searches. In ED scenarios the flavor yield is model dependent, i.e. depends on the location of the SM fields: if they -or part of the SM- live in the bulk, new sources of flavor changing neutral currents (FCNC) arise [24]. Generic little Higgs models do have anomalous top couplings which
can yield interesting flavor physics because of the low cut off $\Lambda \sim 10 \text{ TeV}$, see [25]. This might be evaded by a clever choice of UV completion [17]. All above EWKSB frameworks can lead to NP signals in rare decays. Hence, experimental study at the $b$-factories and the Tevatron – if we are lucky even before the LHC – can establish NP and as shown in Sec. 5 distinguish between models. Particularly interesting (experimentally feasible, theoretically clean SM interpretation) observables are the top10 beyond $\mathcal{B}(b \rightarrow s\gamma)$ given in the next Sec. 4, which probe different aspects of the underlying theory, e.g. CP, $bsg$, chirality.

4. TOP10 OBSERVABLES SEEKING NP

1. The CP asymmetry in $b \rightarrow s\gamma$ decays. In the SM direct CP violation in $b \rightarrow s$ transitions is small $a_{CP} = \frac{|A|^2 - |A^*|^2}{|A|^2 + |A^*|^2} \sim a_s(m_b)Im\frac{V_{ts}V_{tb}^*}{V_{tb}V_{ts}^*} \sim \alpha_s(m_b)\lambda^2 \lesssim \mathcal{O}(1\%)$, e.g. [26]. This is experimentally probed at the 10 % level $a_{CP} = (-0.079 \pm 0.108 \pm 0.022)(1 \pm 0.03)$ by Cleo [27]. 2. Wrong helicity contributions to $\bar{s}_R \sigma_{\mu\nu} b_L F^{\mu\nu}$ in $b \rightarrow s\gamma$ decays. In the SM this is small $C'_7 = m_s/m_b C_7$. It can be tested e.g. with polarization studies in $\Lambda_b \rightarrow (\Lambda \rightarrow p\pi)\gamma$ at hadron colliders and GigaZ [26]. 3. Time dependent study in $B, \bar{B} \rightarrow \Phi K_{S,L}$ decays. The difference $|\sin 2\beta_{(J/\Psi K)} - \sin 2\beta_{(J/\Psi K)}|$ is $\lesssim \mathcal{O}(\lambda^2)$ in the SM and probes direct CP violation in $b \rightarrow s\bar{s}s$ decays even in the absence of strong phases. The precision expected at the $B$-factories is $\sigma_{\Phi K_{S,L}}(stat) = 0.56, 0.18$ for 0.1, 0.1$b^{-1}$ [28]. 4. Precision study of the inclusive $b \rightarrow s\ell^+\ell^-$ branching ratio for low $q^2$ below the charm threshold [2]. 5. Sign and shape of $A_{FB}(B \rightarrow (X_s, K^*)\ell^+\ell^-)$ [26]. 6. If it exists, the position of the $A_{FB}$ zero [6]. 7. The Forward-Backward-CP asymmetry $A_{FB}^{CP} = \frac{A_{FB}^+ - A_{FB}^-}{A_{FB}^+ + A_{FB}^-} \sim \frac{Im(C_9)}{Re(C_{10})}$ above the $\Psi'$ to have sizeable strong phase probes non-SM CP violation in $C_{10}$. The SM background is tiny $A_{FB}^{CP} < 10^{-3}$ [14]. 8. $B_s - \bar{B}_s$ mixing. 9. $B(B_{d,s} \rightarrow \mu^+\mu^-)$ is sensitive to neutral Higgs exchange [29]. 10. The non-observation of nucleon electric dipole moments (nEDMs) created the strong CP problem, i.e. why is $\Theta < 10^{-10}$ while $\delta_{CKM} \sim \mathcal{O}(1)$? nEDMs are sensitive to flavor blind CP violation in case of the PQ-axion solution. In models with spontaneously broken CP tight constraints on the flavor structure arise, suggesting that nEDMs could be close to the current bounds [30].

5. NEW PHYSICS PATTERN

If the SM is extended by adding either more a) symmetry, b) Higgs, c) matter or d) gauge interactions distinct pattern of NP signals in low energy observables arise. This is illustrated in Tab. 2 for the MSSM with MFV [6,29], the 2 Higgs doublet model (2HDM) III [31], which contains an extra source of CP violation, a model with a vector-like down quark (VLDq) [32] and anomalous top couplings [33]. All except the MSSM are toy models, but they can be part of a complete model of EWKSB and mimik e.g. the enlarged Higgs sector of little Higgs theories or the Kaluza-Klein states in models with EDs. Note that a non-SM $sZb$ vertex includes NP in $A_{FB}, A_{FB}^{CP}$, $b \rightarrow s\nu\bar{\nu}$ decays and $B_s - \bar{B}_s$ mixing [14]. Also in the last row of Tab. 2 lives the MSSM without R parity [34] and the MSSM with generic soft terms. A direct (non-FCNC) determination of $V_{tq}$ is important since extra quarks generally violate CKM unitarity $\sum_{i=u,c,t} V_{ib}V_{iq}^* \neq 0, q = d, s$.

6. SUMMARY

Running and upcoming $b$-facilities allow for an extension of the program of FCNC tests which started a decade ago with $B \rightarrow K^*\gamma$ decays. Further flavor/CP sensitive observables are in reach now and rare $b \rightarrow s\ell^+\ell^-$ decays have began to be measured. They complement the radiative modes and collider searches, with particularly clean observables $B(B \rightarrow X_s\ell^+\ell^-)$ for low dilepton mass and the Forward-Backward-asymmetry in inclusive and exclusive $B \rightarrow K^*\ell^+\ell^-$ decays (Fig. 3).

The analysis presented here to search for NP in the short distance coefficients $C_{7,8,9,10}$ is only model independent as long as the operator basis in the effective Hamiltonian is complete. Certain observables listed in the top10 such as wrong helicity contributions require additional operators beyond those present in the SM. With NP at a
Table 2
New physics pattern in $b$-decay observables. The $!$ indicates drastic non-SM effect possible.

<table>
<thead>
<tr>
<th>Example</th>
<th>$\text{flip } C_7$</th>
<th>$\delta_{CP}^{S+}$</th>
<th>$\arg (\frac{\lambda}{m})_{K,S,L}$</th>
<th>$C_7^b$</th>
<th>$sZb$</th>
<th>$B \rightarrow \mu^+\mu^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) MSSM+MFV</td>
<td>!</td>
<td>$\lesssim 1%$</td>
<td>–</td>
<td>–</td>
<td>$\lesssim \frac{m_s}{m_b} \lesssim 20%$</td>
<td>! for large $\tan \beta$</td>
</tr>
<tr>
<td>b) 2HDM III</td>
<td>–</td>
<td>$\lesssim 1%$</td>
<td>!</td>
<td>$\lesssim \frac{m_s}{m_b}$</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>c) VLdQ</td>
<td>–</td>
<td>$\lesssim 1%$</td>
<td>!</td>
<td>$\lesssim \frac{m_s}{m_b}$</td>
<td>!</td>
<td>–</td>
</tr>
<tr>
<td>d) anomal. coupl.</td>
<td>!</td>
<td>!</td>
<td>!</td>
<td>!</td>
<td>!</td>
<td>!</td>
</tr>
</tbody>
</table>

TeV as suggested by models of EWKSB there is a good chance (Fig. 4) that it will show up in one or the other observables @ 5GeV. In analogy with the determination of the parameters of the CKM matrix only a global analysis of all FCNC and low energy data might reveal NP and whether it violates also CP. This procedure is able to distinguish between models (Tab. 2). Constraints from rare $K,D$ and lepton flavor violating processes and neutrino physics complete the picture.

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