Neutrino Oscillations Due to Nonuniversal Gauge Interactions in the Weak Sector

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Abstract

We discuss the structure of the neutrino mass matrix as derived from a nonuniversal electroweak gauge interaction model. We discuss two interesting patterns of neutrino masses. The first pattern is hierarchal which fits the LMA or LOW solutions of solar neutrino data. The second pattern gives rise to inverted mass spectrum which fits the currently not preferred SMA solution. The mechanism for generating a light sterile neutrino mass is interesting and is briefly discussed.

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I Introduction

New results from several experiments on neutrino physics [1, 2, 3, 4] have a convincing evidence of neutrino masses and mixings. A large body of information on the neutrino mass matrix structure has been established from solar [1], atmospheric [2], reactor [3], and accelerator [4] experiments. Still, the complete picture of the mass matrix, as derived from neutrino data, is far from complete. Further data are needed to constrain the large number of parameters in the mass matrix. Results on neutrinoless double-beta decay [5], if confirmed, provide further information that cannot be provided by oscillation experiments.

Extensions of the Standard Model (SM) and/or new symmetries, e.g., horizontal symmetries, have been discussed thoroughly in connection with neutrino oscillation data [6, 7, 8, 9, 10, 11]. A possibility to explain neutrino data through non-universal gauge interactions in the electroweak sector was discussed in Ref. [12], where the third generation of fermions are assumed to transform under a weak $SU(2)_h$ symmetry, while the first and second generations transform under a different $SU(2)_l$. The phenomenology of the scenario and similar versions were extensively discussed in Refs. [12, 13] and earlier in Ref. [14]. The basic idea is the assumption of a nonuniversal electroweak gauge symmetry $SU(2)_l \times SU(2)_h \times U(1)_Y$, where different generations of fermions transform under different $SU(2)$ symmetries. The scenario is phenomenologically well motivated and consistent with all low energy data as long as the heavy gauge boson masses are at least of the order of 1 TeV [13]. However, as pointed out in Ref. [12], the scenario fails to give rise to a maximal mixing in order to account for the atmospheric data. This is due to the fact that the second and third families can only mix weakly through the ratio of the two vacuum expectation values of the two Higgs doublets. In this work, we modify the previous scenario by assuming that the second and third generations of fermions transform under $SU(2)_h$, while the first generation transform under $SU(2)_l$. In this case we can produce maximal mixing in the atmospheric sector and be consistent with other neutrino data.

It could be argued that the phenomenology of the new scenario is less interesting,
This could be due to the precise low energy data on universality of the gauge interactions, e.g., regarding the electron and muon interactions [15]. This will simply have the effect of pushing the mass of the extra heavy gauge bosons further up. Still, a detailed study of the phenomenology of the model is highly needed to constrain the heavy gauge boson masses and couplings. Nevertheless, the scenario can survive all low energy data constraints provided the extra gauge bosons are heavy enough. On the other hand, the Yukawa sector is not constrained by low energy data and this is the motive for considering such a scenario. A study of the neutrino mass matrix is performed in this work independent of the gauge sector and low energy data.

The rest of the paper is organized as follows: In Sec. II, we briefly review the model and then extract the general form the mass matrix. In Sec. III, we discuss the structure of the mass matrix and discuss two possible patterns of neutrino masses. We also discuss briefly the possibility of including a light sterile neutrino.

II Structure of the model

The scenario we discuss is based on the gauge symmetry $G= SU(3)_c \times SU(2)_l \times SU(2)_h \times U(1)_Y$. Where, the left-handed second and third fermion generations are subjected to a weak gauge interaction described by $SU(2)_h$. On the contrary, the first generation is subjected to another $SU(2)_l$ gauge interaction. The $U(1)_Y$ group is the SM hypercharge group. The right-handed fermions only transform under the $U(1)_Y$ group as assigned by the SM. Finally, the QCD interactions and the color symmetry $SU(3)_c$ are the same as that in the SM.

The spontaneous symmetry-breaking of the group $G$ is accomplished by introducing the complex scalar fields $\Sigma$, $\Phi_1$, and $\Phi_2$, where $\Sigma \sim (1,2,2,0)$, $\Phi_1 \sim (1,2,1,1)$, and $\Phi_2 \sim (1,1,2,1)$. The group $G$ is then broken at three different stages. The first stage of symmetry breaking is accomplished once the $\Sigma$ field acquires a vacuum expectation value (vev) $u$, i.e., $\langle \Sigma \rangle = \begin{pmatrix} u \\ 0 \\ 0 \end{pmatrix}$, where $u$ is expected to be of the order of 1 TeV. The form of $\langle \Sigma \rangle$ guarantees the breakdown of $SU(2)_l \times SU(2)_h \rightarrow SU(2)$. Therefore, the unbroken symmetry is essentially the SM gauge symmetry
SU(3)\(_c\) \times SU(2) \times U(1)\(_Y\), where SU(2) is the usual SM weak group. At this stage, three of the gauge bosons acquire a mass of order \(u\), while the other gauge bosons remain massless. The second and third stage of symmetry breaking (the electroweak symmetry-breaking) is accomplished through the scalar fields \(\Phi_1\) and \(\Phi_2\) by acquiring their vacuum expectation values \(\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}\), and \(\langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}\), respectively. The electroweak symmetry-breaking scale \(v\) is defined as \(v \equiv \sqrt{v_1^2 + v_2^2} = 246\) GeV. Since the third fermion generation is heavier than the first generation, it is suggestive to conclude that \(v_2 \gg v_1\).

Fermions acquire their masses through Yukawa interactions via the \(\Phi_1\) and \(\Phi_2\) scalar fields. The full neutrino Yukawa interaction terms are given by the Lagrangian

\[
\mathcal{L}_{\text{Yukawa}} = \bar{\Psi}_L^1 \Phi_1 \left[ g_{11}^{\nu} \nu_{eR} + g_{12}^{\nu} \nu_{\mu R} + g_{13}^{\nu} \nu_{\tau R} \right] + \\
\bar{\Psi}_L^2 \Phi_2 \left[ g_{21}^{\nu} \nu_{eR} + g_{22}^{\nu} \nu_{\mu R} + g_{23}^{\nu} \nu_{\tau R} \right] + \\
\bar{\Psi}_L^3 \Phi_2 \left[ g_{31}^{\nu} \nu_{eR} + g_{32}^{\nu} \nu_{\mu R} + g_{33}^{\nu} \nu_{\tau R} \right] + \text{h.c.,} 
\]

where \(\Phi_{1,2} \equiv i\tau_2 \Phi_{1,2}^*\) and where

\[
\Psi_L^1 = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \quad \Psi_L^2 = \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \quad \text{and} \quad \Psi_L^3 = \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}. 
\]

The Dirac mass matrix derived from Eq. (1) is written as

\[
M_D = \begin{pmatrix} g_{11}^{\nu} v_1 & g_{12}^{\nu} v_1 & g_{13}^{\nu} v_1 \\ g_{21}^{\nu} v_2 & g_{22}^{\nu} v_2 & g_{23}^{\nu} v_2 \\ g_{31}^{\nu} v_2 & g_{32}^{\nu} v_2 & g_{33}^{\nu} v_2 \end{pmatrix}. 
\]

The right-handed neutrino Majorana mass matrix \(M_R\) is assumed to have a common mass scale of the order of the GUT scale, \(M_X \sim 10^{15}\) GeV. Therefore, the full neutrino mass matrix forms a \(8 \times 8\) matrix which can be written as

\[
M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}. 
\]

By invoking the seesaw mechanism the left-handed neutrino Majorana mass matrix is then given as

\[
M_L = M_D M_R^{-1} M_D^T. 
\]
Due to the seesaw mechanism all elements of $M_L$ are highly suppressed by the GUT scale $M_X$ of the right-handed Majorana mass matrix $M_R$. In this work we assume the charged lepton mass matrix is diagonal. In the next section we give further discussion of the derived neutrino mass matrix.

**III Neutrino masses and mixings**

The most general form of the Majorana mass matrix as given in Eq. (5) can be written as

$$M_L = m \begin{pmatrix} g_{11} \epsilon^2 & g_{12} \epsilon & g_{13} \epsilon \\ g_{12} \epsilon & g_{22} & g_{23} \\ g_{13} \epsilon & g_{23} & 1 \end{pmatrix}.$$  \hfill (6)

where, $\epsilon \equiv \nu_1/\nu_2$ and all other parameters inside the mass matrix are assumed of order 1. It is highly desirable to investigate what symmetries, discrete or continuous, could lead to a pattern that is consistent with the neutrino data. However, it is our intention in this work to discuss the mass matrix in a general way without regard to the underlying symmetry. There are two special forms of the mass matrix that could be compatible with data. The first form gives rise to a degenerate pattern in the neutrino masses and can be written as

$$M_L = m \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix}.$$  \hfill (7)

where coefficients of order 1 multiplying the small parameter $\epsilon$ has been omitted.

The mass matrix form has been discussed in Refs. [6, 7] in connection with other scenarios. In this case one finds two light eigenstates approximately given by $\pm m \epsilon$ and a heavy mass eigenstate approximately given by $2m$. Also one finds $\tan \theta_{23} \approx 1$, $\tan \theta_{12} \approx 1 - \epsilon$, $\Delta m^2_{\text{atm}} \approx 4m^2$, $\Delta m^2_{\text{sol}} \approx m^2 \epsilon^2$, and $\theta_{13} \approx \epsilon$. A large value of $\epsilon$ is needed to recover the LMA solution [16] which raises doubts to the perturbation approach. While, the result could fit the LOW solution. For example, taking $\epsilon = 0.1$ and other parameters around 1, we can easily recover the best fit of the LOW solution, while a $3\sigma$ is needed for the LMA solution. On the other hand, $m_{ee} \approx \Delta m^2_{\text{sol}}$ is beyond the reach of the next generation of experiments.
Next we consider a second form of the neutrino mass matrix which gives rise to what is called the inverted pattern, where we get two heavy eigenstates with masses of order \(O(v_2^2/M_X)\) and a third light mass eigenstate of order \(O(v_1^2/M_X)\). In order for the scenario to give the correct pattern of mass splittings, we impose the condition that the two heavy states are degenerate within a small gap to explain the solar anomaly. While their large splitting with the light state explains the atmospheric anomaly. Thus, we consider the special form

\[
M_L = m \begin{pmatrix} 
\epsilon^2 & \epsilon & \epsilon \\
\epsilon & -1 & a \\
\epsilon & a & 1 
\end{pmatrix},
\]

where the parameter \(a\) is written explicitly in the mass matrix and we omitted the order 1 coefficients multiplying \(\epsilon\). The mass eigenstates can be readily determined as

\[
m_1 \approx m \epsilon^2, \\
m_2 \approx -m \sqrt{1 + a^2} + O(\epsilon^2), \\
m_3 \approx m \sqrt{1 + a^2} + O(\epsilon^2).
\]

The atmospheric scale, \(\Delta_{\text{atm}}\), is then associated with the mass splitting \(\Delta_{13}\) (or \(\Delta_{12}\)), where

\[
\Delta_{\text{atm}} = \Delta_{13} = \Delta_{12} \approx m^2 \left(1 + a^2\right)
\]

While the solar mass scale, \(\Delta_{\text{sol}}\), is associated with the mass splitting \(\Delta_{23}\), where

\[
\Delta_{\text{sol}} = \Delta_{23} \approx m^2 \epsilon^2.
\]

For \(a \approx 1\) we find that \(\epsilon \approx 0.1\) is consistent with results on mass splittings. For the mixing angles we find We find that

\[
\tan \theta_{23} \approx \frac{a}{1 + \sqrt{1 + a^2}}.
\]

Note that for large \(a\)

\[
\lim_{a \to \infty} \tan \theta_{23} = 1.
\]

In fact for \(\sin^2 2\theta_{23} \geq 0.7\), as indicated by data, we require \(a \geq 1.5\). Hence, the scenario can account for the large mixing angle needed to explain the atmospheric
data. For the mixing angle $\theta_{13}$ we find $\theta_{13} \approx \epsilon$ which is also consistent with the CHOOZ bound [3]. For $a = 1.5$ and $\epsilon \approx 0.1$, we find that $\tan \theta_{13} \approx 0.1$. Finally, for the mixing angle $\theta_{12}$ we find $\theta_{12} \approx \epsilon$ with the limit

$$\lim_{a \to \infty} \tan \theta_{12} = \frac{\epsilon}{\sqrt{2a}}.$$  \hspace{1cm} (16)

The result gives rise to the SMA solution of the solar neutrino data. For $a = 1.5$ and $\epsilon \approx 0.1$, we find that $\sin^2 2\theta_{12} \approx 2 \times 10^{-3}$. Recent global analysis of all neutrino data disfavors the SMA solution [16]. However, more data is still needed to confirm such a result. If the SMA solution turns out to be excluded with high confidence then the inverted pattern as derived from this scenario would be discarded because it can only lead to the SMA solution of the solar neutrino data.

The scenario offers an interesting mechanism for embedding an extra light sterile neutrino which has been discussed in detail in Ref. [12]. In this case, one introduces an exotic bi-doublet fermion under the weak symmetries $SU(2)_{l,h}$. Once the first symmetry breaking is invoked, the bi-doublet fermion is split into two pieces; a triplet under the remaining weak $SU(2)$ and a singlet which is identified as the sterile neutrino. The mass matrix $M_D$ in Eq. (3) is then enlarged to a 4x4 matrix with an extra parameter $u \gg v_2 \gg v_1$. The new mass matrix $M_L$ gives rise to a light sterile neutrino compatible with LSND data [4]. This case has been discussed extensively in Ref. [12] which is similar to the case we have.

A final comment is that the flavor changing neutral currents (FCNC) are not expected to be significant in our scenario due to the tight constraints driven by low energy data [12].

References


