In superstring theory, starting with a non-BPS brane or a brane-anti-brane pair, a tachyon is always present. This tachyon rolls down its potential and tachyon matter appears as energy density without pressure, as pointed out by Sen [1]. In the cosmological context [2], it is easy to estimate that the pressureless tachyonic matter density is generically many orders of magnitude too big to be compatible with present day cosmological observations [3]. This implies either that tachyon rolling does not happen (i.e., irrelevant) in the early universe after inflation, or, instead of pressureless tachyon matter density, the tachyon potential energy goes somewhere else. In this paper, we argue for the latter.

In superstring theory, defects, or stable solitons (i.e., lower-dimensional BPS D-branes), are produced during tachyon rolling, also first pointed out by Sen [4]. Cosmologically, these defects will be produced [5]. Averaged over a region large compared to the typical defect separation, this defect density becomes spatially uniform, and may be identified as a part of the tachyonic matter.

In the brane world realization of superstring theory, where the standard model particles are open string modes that live on the brane while the graviton and other closed string modes live in the bulk, the brane inflationary scenario [6], where the inflaton is simply an interbrane separation, looks very robust [7–9]. In this scenario, tachyons invariably emerge towards the end of brane inflation, when the branes approach each other and collide. (Defects produced before or during inflation are inflated away.) As the tachyon rolls down its potential, it was shown that the only defects copiously produced are those that appear as cosmic strings [9,10]. So we expect all of the tachyon potential energy goes to heating the universe (via its coupling to the inflaton and other fields) and producing the cosmic string network. Interactions of the cosmic strings will reduce the very high initial cosmic string network density to an acceptable level [11], provided the cosmic string tension $\mu$ satisfies $G\mu < 10^{-6}$, which seems to be always satisfied in brane inflation [9,10]. The presence of cosmic strings will still give detectable signatures in the cosmic microwave background and the gravitational wave spectral density [10]. To summarize, the over-abundance of the tachyon matter density problem can be solved rather naturally by draining the tachyon potential energy to (re-)heating and to the cosmic string production. Below, we discuss these two mechanisms.

Consider a general Lagrangian of the form ($\alpha' = 1$)

$$\mathcal{L} = -V(T)F(X) \equiv g^{\mu\nu}T_{\mu\nu}, \quad (1)$$

where $T(x)$ is the tachyon mode, $T_{\mu\nu} = \partial_{\mu}T_{\nu}$, and $F(X)$ can be any function at this stage. In Ref. [1], $V(T) \propto e^{-T}$ and $F(X) = \sqrt{1+X}$.

The energy-momentum of Eq.(1) is

$$T_{\mu\nu} = V(T)[2F_{X}T_{\mu\nu} - g_{\mu\nu}F(X)] \quad (2)$$

where $F_{X} = \partial F/\partial X$. We can write this in the form of the energy-momentum tensor for a perfect fluid if we identify

$$\rho = V(T)[F(X) - 2XF_{X}X] \equiv V(T)D(X) \quad (3)$$

$$p = -V(T)F(X).$$

We shall mostly use the tachyon Langrangian suggested by the boundary (or background-independent) string field theory [12], in particular, its superstring version [13,14], which has been used to study tachyon matter recently [15]. The non-BPS brane model (with brane tension $\sqrt{T_{\mu\nu}}$) is

$$V(T) = \sqrt{T_{\mu\nu}}e^{-T^{2}/2}$$

$$F(X) = \frac{\sqrt{\Gamma(1+X)}}{\Gamma(X+1/2)} = \frac{X^{4}\Gamma(X)^{2}}{2\Gamma(2X)}.$$  (4)

Here, $F(X) = 1 + X \ln 4$ for small $X$. In this model, for spatially independent solutions, the pressure starts out negative when $X = -T^{2}$ is small and becomes positive when $X < -1/2$. As $1 + X = 1 - T^{2} \rightarrow 0$, $F(X) \simeq -1/2(X + 1)$, $D(X) \simeq (X + 1)^{-2}$, we see that

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\[ w = \rho/p = -F(X)/D(X) \simeq (1 + X)/2 \to 0 . \] (5)

As \( 1 + X \to 0 \), \( V(T) \to 0 \), but the energy density \( \rho \) stays constant, as required by energy conservation. The effective tachyon sound speed \( v_T = \sqrt{-F_{,X}/D_{,X}} \) also vanishes in this limit, allowing efficient clustering.

Next, let us consider the brake-anti-brake pair, which has a complex tachyon \( T = T_1 + iT_2 \). The tachyon action is [14]

\[ S_T = -2\tau_p \int dx^{p+1} e^{-(T_1^2 + T_2^2)/2} F(X_1)F(X_2) \] (6)

where \( X_1 = g_{\mu\nu}\partial_\mu T_1 \partial_\nu T_1 \). Again, let us consider spatially independent solutions. The energy conservation requires that \( \dot{\rho} = 0 \), where

\[ \rho = 2\tau_p e^{-(T_1^2)/2} [F_1 F_2 - 2X_1 F_{,X_1} F_2 - 2X_2 F_1 F_{,X_2}] \] (7)

where \( F_i = F(X_i) \) and \( F_{,X_1} = F_X(X_i) \). First, consider a simple ansatz of linear tachyon profile \( T_1(t) = t - \epsilon_1(t) \) and \( T_2 = 0 \). One finds that \( \dot{\epsilon}_1(t) \simeq e^{-t^2/4} \to 0 \) and \( 1 + X_1 \to 0 \), so \( w = 0 \) (i.e., zero pressure). Since \( X_2 = 0 \) and \( F(X_2) = 1 \), this essentially reduces to the single \( T \) case. In this solution, pressure approaches zero from the positive side [15].

Suppose we consider a more symmetric ansatz:

\[ T_1(t) = t - \epsilon_1(t), \quad T_2(t) = t - \epsilon_2(t) . \] (8)

An examination of the equations of motion yields \( \dot{\epsilon}_1 \propto \dot{\epsilon}_2 \propto e^{-t^2/3} \). However, when these are substituted into \( \rho \), we find

\[ \rho \simeq -\frac{\tau_p e^{-t^2}}{8} \left( \frac{1}{\dot{\epsilon}_1^2} + \frac{1}{\dot{\epsilon}_2^2} \right) . \] (9)

A positive constant \( \rho \) implies \( \dot{\epsilon}_i < 0 \) (i.e., \( 1 + X_i < 0 \)), a solution that cannot be reached starting from small \( \dot{T}_i \).

The expansion rate of the Universe is simply \( H^2 = 8\pi G \rho/3 \) where \( a(t) \) is the cosmological scale factor and \( H = \dot{a}/a \). For a spatially uniform tachyon field evolving in an expanding Universe, we have \( \dot{\rho} = -3H (\rho + p) \). With \( w = 0 \), \( \rho a^3 \) is constant, and since \( v_T = 0 \) too, tachyon matter is dust-like [1,15].

Let us estimate the tachyon matter density today as a fraction \( \Omega_T \) of the total density \( 3H_0^2M_{Pl}^2 \) of our universe. We assume \( \rho_{\text{initial}} \simeq M_s^2 \), where the string scale \( M_s^2 \) is \( 1/\alpha' \). Using \( H_0^2/M_{Pl}^2 \simeq 10^{-123} \) and \( a_{\text{initial}}/a_{\text{today}} \simeq 2.7^9K/M_s \), we find

\[ \Omega_T = \left( \frac{\rho_{\text{initial}}}{3H_0^2M_{Pl}^2} \right)^{a_{\text{initial}}/a_{\text{today}}} \simeq 10^{28}\rho_s/M_{Pl} \] (10)

where \( 2.7^9K/M_{Pl} \simeq 10^{-31} \). Since \( \Omega_T \) must be less than 1, and \( M_s > 1 \) TeV, we see that the tachyon matter density will be many orders of magnitude too big to be compatible with our universe [3]. Fortunately, as we shall argue below, this tachyon energy can be completely drained by cosmic string production and (re-)heating in a realistic early universe.

One may also consider a stable solitonic solution of Eq.(6), with \( T_1(x) = u_1x_1 \) and \( T_2(x) = u_2x_2 \) [14]. Since both the second derivatives of \( T_1(x) \) and the mixed derivatives \( \partial_i T_1 \partial_j T_2 \) with \( i = j \) vanish in this ansatz, the equations of motion reduce to

\[ T_i D(X_i) = 0 \quad I = 1, 2 . \]

Since \( D(X_i) \) only vanishes at \( X_i = u_i^2 \to \infty \) (this means infinitely thin brane), the codimension-2 brane tension \( (F(X) \to \sqrt{\pi}X) \) becomes

\[ \tau_{p-2} = 2\tau_p \int dx_1 dx_2 e^{-|T|^2/2} F(u_1^2)F(u_2^2) = 2\tau_p F(u_1^2)F(u_2^2)2\pi/\rho_{u_1}u_2 \to \tau_p (2\pi)^2 \] (11)

as expected. This is a vortex solution.

A non-BPS \((p-1)\)-brane may also be produced (\( u_1 \to 0, u_2 \to \infty \)), but it will quickly decay to BPS \( D(p-2) \)-branes. Multi-soliton solutions may also be constructed via the introduction of Chan-Paton factors. A moving brane with constant velocity \( v \) may also be constructed by generalizing \( T_1 \) to \( T_1(x) = u_1(x_1 - vt) \), which gives the energy \( E = \tau_{p-2}/\sqrt{1 - v^2} \), as expected.

If we start with a brake-anti-brake pair, the tachyon will roll with non-trivial spatial dependence. This will result in a collection of \( D(p-2) \)-branes. However, if we average over a region much larger than the typical spacing between the \( D(p-2) \)-branes, we expect to obtain a spatially uniform energy density and isotropized pressure, that is, a perfect fluid. In this sense, we may interpret the defect density as part of the tachyon matter. However, the fraction of energy that goes to defect formation is sensitive to the details and difficult to calculate.

The cosmological production of defects in superstring theory has been studied [5]. In particular, D3-brane-anti-D3-brane annihilation yields D1-branes [4], which appear as cosmic strings in our universe. Actually, following the fact that vortices (instead of domain walls or monopoles) are formed, the appearance of only cosmic strings is much more general [9,10]. As an illustration, consider D5-branes with two of its dimensions compactified on a two-cycle, while the remaining 3 dimensions span our universe. D5-brane-anti-D5-brane annihilation will produce D3-branes, which will also wrap around the two-cycle, so they also appear as cosmic strings. If there is no one-cycle inside the two-cycle, (e.g., topology of a sphere, as in orbifolding a torus), then no domain wall or monopole-like solitons will appear in our universe.

Naively, the cosmic string network has a density that is comparable to and decreases like (for cosmic string loops) that of the tachyon matter (10), or, for cosmic strings stretching across the horizon, even slower (i.e.,
For brane inflation, the inflaton potential is known to have the form \( U(\phi) \approx U_0 - U_1 \phi^{d-2} \) for large \( \phi \), when there are \( d \) extra dimensions transverse to the brane [7,16]. For a particular brane pair, \( U_1 \) is easily calculable [17]. For example, for a brane-anti-brane pair, \( U_0 = 2\tau_p V_{p-3} \) where \( V_{p-3} \) is the compactification volume of the extra \((p-3)\) dimensions of the branes and \( U_1 \) is simply twice that of the gravitational force. When the brane separation \( \phi \) is comparable to the compactification size, the compactification effect becomes important and it flattens the potential further [7,9], allowing inflation to take place for many e-foldings.

When the branes are far apart, \( T \) is a normal scalar field with positive mass squared \( m_T^2 \). As the branes move closer, \( m_T^2 \) decreases, and becomes negative at the bifurcation point \( \phi_c \), where \( T \) becomes tachyonic. This suggests the following potential:

\[
V(T, \phi) = U(\phi) \exp \left[ -|T|^2 \left( \frac{1}{2} - \lambda \phi^2 \right) \right]
\]

where the bifurcation point \( \lambda \phi_c^2 = 1/2 \).

Although the potential \( U(\phi) \) for small \( \phi \) remains to be calculated, it is expected to be positive and finite as \( \phi \to 0 \). In terms of open strings, \( U(\phi = 0) \) is a quantum one-loop correction, so \( 2\tau_p \) is renormalized to \( 2\tau_p v_0 \), where we shall take \( 1 > v_0 > 0 \). So, for \( \phi = 0 \), we recover the tachyonic model (6). Notice that the minimum of the potential \( V(T, \phi) \) (\( \phi < \phi_c \) and \( T \to \infty \)) is automatically zero, independent of the details of \( U(\phi) \). As a consequence, the dynamics is insensitive to the particular form of \( U(\phi) \) for \( \phi < \phi_c \), since tachyon rolling happens very fast, and \( V(T, \phi) \to 0 \) rapidly. For phenomenology, we choose a particularly simple form of \( U(\phi) \), where \( U(\phi) \) has a simple harmonic form for small \( \phi \),

\[
U(\phi) = U_0 \left[ 1 - \frac{(1 - v_0)}{(\phi^2/\phi_0^2 + 1)^{2(d-2)}} \right].
\]

In summary, Eqs. (12,13,14) comprise the model, which may be considered as a novel version of hybrid inflation. For fixed \( \phi > \phi_c \), \( T \) has positive mass \( m_T^2 > 0 \), so the minimum of the potential is at \( T = 0 \) and classically \( \phi \to 0 \), so \( X_1 = 0 \) and \( F(X_1) = 1 \). The model reduces to \( \phi \), \( T = 0 \) = \(-U(\phi) - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi/2 \). This describes the inflationary epoch for \( \phi > \phi_c \).

Based on the above analysis, let only one of the tachyons roll, say \( T = T_1 \), while \( T_2 \) is essentially frozen, with \( F(X_2) = 1 \). For a spatially uniform scalar field and tachyon,}

\[
\dot{\phi} + (3H + \Gamma_\phi) \phi = -F(X)[V(|T|, \phi)]_2, 
\]

\[
\ddot{T} + 3HA(X) = -\frac{V_T}{2V}B(X) - \frac{V}{V_2}A(X)\dot{T} \phi, 
\]

where \( A(X) = F_{XX}/(F_X + 2XF_{XX}) \) and \( B(X) = (F - 2XF_{XX})/(F_X + 2XF_{XX}); \) \( \phi \) is a phenomenological term that models the decay of \( \phi \) to ordinary particles and radiation, whose density is \( \rho_{\text{matter}} \). If the inflaton decays primarily to relativistic particles (“radiation”), then

\[
\rho_{\text{matter}} + 4H\rho_{\text{matter}} = \Gamma_\phi \phi^2. 
\]

The total energy density in this situation is

\[
\rho = V(|T|, \phi)(F(X) + 2T^2F_{XX}) + \frac{\dot{\phi}^2}{2} + \rho_{\text{matter}}. 
\]

Here, we are not interested in the entire process of brane inflation, but rather what happens at the end, when the tachyons start to roll. We assume that there is a prior period of slow rolling (\( \phi \) decreasing slowly), during which almost all of the inflation occurs, and density fluctuations are generated [7–9]. During the inflationary period, when \(|T|\phi \) is small and (let \( \lambda = 1 \) \( \phi > 2^{-1/2} \)), the evolution of \( T(t) \) resembles a massive field, with a mass \( \sim \phi \), and we therefore expect \( T(t) \) to remain small, and dominated by quantum fluctuations. When the tachyon becomes unstable, it begins with an amplitude \( T \sim H_I/2\pi \sqrt{v_0} \sim M_W^{-1} \) and \( \dot{T} \sim H_I T \), where \( H_I = (8\pi G U_0/3)^{1/2} \) is the expansion rate during slow roll. We follow the evolution implied by Eq. (15) and (16) after \( \phi = 2^{-1/2} \).

As an illustration, numerical results are shown in Fig. 1, assuming (in terms of string scale) \( 1/M_P = 10^{-3} \), \( U_0 = 1, 1 - v_0 = 0.1, d = 4, \phi_0 = 1, \) and \( \Gamma_\phi = 0 \) or 0.1. In fact, because \( H_I \approx 0.003 \), cosmological expansion is relatively unimportant. The simulations start with \( \phi(t) = 1/\sqrt{2} \) and \( \dot{\phi} = 0 \), at \( t = 0 \) and \( a(t) = 1 \). The initial data can be varied, but the behavior shown in Fig. 1 is generic. At first, \( \phi(t) \) decreases, and \( T \) increases. Ultimately, as \( T \to 1, T \approx t \) grows, and the product \( V(T, \phi)F(X) \) decreases rapidly for \( \phi^2 < 1/2 \).

In the non-dissipative case, \( \Gamma_\phi = 0 \), \( \phi \to \phi_0 \) constant, and since \( a(t) \) is only changing slowly, \( \phi \) increases or decreases linearly with time. However, since the total energy is conserved, and \( V(T, \phi) \) would grow exponentially large for \( \phi^2 > 1/2 \), the growth of \( |\phi| \) cannot continue indefinitely, so \( \phi(t) \) bounces near \( \phi^2 = 1/2 \). Associated with these
bounces are short-lived dips in $\dot{T}$, which are evident but not completely resolved in Fig. 1. Eventually, cosmological expansion alone would cause $\dot{\phi}$ to decay, and $\phi$ would settle to some constant, nonzero value. Dissipation accelerates the decay of the nonlinear oscillations. The bottom panel in Fig. 1 shows what happens for $\Gamma_\phi = 0.1$, which may be unrealistically large, but illustrates how dissipation suppresses the oscillations, and causes $\phi$ to settle to a nonzero value asymptotically. For either $\Gamma_\phi$, $\rho_T a^3 \rightarrow$ constant $\sim 1$ eventually. For $\Gamma_\phi = 0$, the Universe does not become radiation dominated.

In summary, in a realistic early universe scenario, the tachyon energy must dissipate via some way for it to reduce to an acceptable level by the present. Natural ways for this to happen would be a combination of (1) production of cosmic strings, which intersect, intercommute, and decay, as proposed above and in Ref. [9–11] and (2) heating the universe via the tachyon coupling to the inflaton and other fields.

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**FIG. 1** Cosmological evolution of $\phi(t)$ (red), $\dot{T}$ (black), $\ln a(t)$ (green), $\ln \rho a^3$ (blue) and $\ln \rho_T a^3$ (magenta) for $\Gamma_\phi = 0$ (top) and $\Gamma_\phi = 0.1$ (bottom). $M_{Pl} = 10^{18}$, $v_0 = 0.9$, $U_0 = 1$.