FORMATION OF AN EVANESCENT PROTO-NEUTRON STAR BINARY AND THE ORIGIN OF PULSAR KICKS

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ABSTRACT

If core collapse leads to the formation of a rapidly rotating bar-unstable proto-neutron star surrounded by fall-back material, then we might expect it to cool and fragment to form a double (proto)-neutron star binary into a super-close orbit. The lighter star should survive for awhile, until tidal mass loss propels it toward the minimum stable mass of a (proto)-neutron star, whereupon it explodes. Imshennik & Popov have shown that the explosion of the unstable, cold star can result in a large recoil velocity of the remaining neutron star. Here, we consider several factors that mitigate the effect and broaden the range of final recoil speeds, in particular the finite velocity and gravitational deflection of the ejecta, a range of original masses for the low mass companion and its cooling history, rotational phase averaging of the momentum impulse from non-instantaneous mass loss, and the possibility of a common envelope phase. In spite of these mitigating factors, we argue that this mechanism can still lead to substantial neutron star recoil speeds, close to, or even above, 1000 km s$^{-1}$.

Subject headings: stars: neutron - supernovae: general - pulsars: general - stars: rotation

1. INTRODUCTION

It is recognized that radio pulsars have peculiar space velocities between $\approx 30$ km s$^{-1}$ and $\approx 1600$ km s$^{-1}$, significantly greater than those of their progenitor stars. The highest speed ever recorded is from the Galactic Nebula pulsar (Cordes et al. 1993; Cordes & Chernoff 1998), while the lowest has been recently measured for B2016+28 by Brisken et al. (2002) from very accurate VLBA pulsar parallaxes. Statistical studies, aimed at inferring the peculiar velocity at birth from the observed speed, give mean three-dimensional velocities of 100 – 500 km s$^{-1}$ for the isolated pulsars (Lyne & Lorimer 1994; Lorimer, Bailes, & Harrison 1997; Hansen & Phinney 1997; Cordes & Chernoff 1998; Arzoumanian, Chernoff, & Cordes 2002).

An early explanation for the large space velocities called for recoil in a close binary that becomes unbound at the time of (symmetric) supernova explosion (Blauw 1961; Iben & Tutukov 1996). Now, a number of observations hint at a natal origin of these high space velocities, or at a combination of orbital disruption and internal kick reaction (when the progenitor lives in a binary). Evidence for a kick at the time of neutron star birth is now found in a variety of systems: in runaway O/B associations (Leonard & Dewey 1992), in highly eccentric Be/NS binaries (van den Heuvel & Rappaport 1986; Portgies-Zwart & Verbunt 1996), in the binary pulsar J0045-7319 (Kaspi et al. 1996; to explain its current spin-orbit configuration), and in double neutron star binaries such as B1913+16 (Bailes 1988; Weisberg, Romani, & Taylor 1989; Cordes, Wasserman, & Blaskiewicz 1990; Kramer 1998; Wex, Kalogera & Kramer 2000) where misalignment between the spin and orbital angular momentum axes indicates velocity asymmetry in the last supernova. All these observations support the view that the formation of neutron stars is accompanied by anisotropic explosion. This notion is bolstered by evolutionary studies of binary populations (Dewey & Cordes 1987; Fryer & Kalogera 1997; Fryer, Burrows, & Benz 1998) and by studies (e.g. Cordes & Wasserman 1984) on the survival of binary systems into their late evolutionary stages after supernova explosion.

Among the physical processes that have been proposed to account for the kicks are large-scale density asymmetries seeded in the pre-supernova core (leading to anisotropic shock propagation), asymmetric neutrino emission in presence of ultra-strong magnetic fields (see Lai, Chernoff, & Cordes 2001 for a review), or off-centered electromagnetic dipole emission from the young pulsar (Harrison & Tademaru 1975). However, none of these mechanisms can explain kicks as large as $\approx 1600$ km s$^{-1}$.

In this paper we reconsider the idea first put forward by Imshennik & Popov (1998) that, in the collapse of a rotating core, one or more self-gravitating lumps of neutronized matter may form in close orbit around the central nascent neutron star, transfer mass in the short lived binary, and ultimately explode, causing the remaining, massive neutron star to acquire a substantial kick velocity, as high as the highest observed. The light member explodes as mass transfer drives it below the minimum stable mass for a neutron stars. In the light star, stability is lost upon decompression by the $\beta$-decaying neutrons and nuclear fissions by radio-active neutron-rich nuclei (Colpi, Shapiro & Teukolsky 1989, 1991; Blinnikov et al. 1990), that deposit energy driving matter into rapid expansion (Colpi, Shapiro & Teukolsky 1993; Sumiyoshi et al. 1998). The kick tries its origin from the orbital motion of this evanescent super-close binary, that forms in the collapse of a rapidly rotating (isolated) iron core.

We will study several effects that may modify the magnitude of the kick, such as gravitational bending of the exploding debris, rotational averaging of the momentum impulse, the presence of multiple lumps, orbit decay, and delayed neutron star cooling.

Formation of a proto-neutron star companion around the main neutron star has never been verified in numerical simulations due to computational limitations. For this reason we elaborate on a study of Bonnell (1994) on the formation of binary/multiple systems in collapsing gas cloud cores and its extension to the stellar core collapse in the aftermath of a super-

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nova explosion (Bonnell & Pringle 1995) to motivate our working hypothesis. The formation of such exotic binaries has been conjectured to occur in many works (Ruffini & Wheeler 1971, Clark & Eardley 1977; Blinnikov, Novikov, Perevodchikova, & Polnarev 1984; Nakamura & Fukugita 1987, Stella & Treves 1987).

2. LIGHT FRAGMENTS AROUND PROTO-NEUTRON STARS

2.1. The Scenario

Formation of a light companion around a main body implies breaking of spherical and axial symmetry during collapse and following core bounce. During dynamical collapse, unstable bar modes \((m = 2)\) can grow in a fluid (even non rotating) that may end with fragmentation. However this is known to occur only if the cloud core contracts almost isothermally, as in the case of star’s formation from unstable cold gas clouds (Bonnell 1994). Core collapse in type II supernovae is far from isothermal (it is described by a effective polytropic index \(\gamma \approx 1.3\)) so that instabilities of this type do not have time to grow (Lai 2000) \(^3\), and simulations of non-axisymmetric rotating core collapse confirm this trend (Rampp, Muller & Ruffert 1998). Can fragmentation/fission be excited after core bounce?

Rapid rotation in equilibrium bodies is known to excite non-axisymmetrical dynamical instabilities and these instabilities may grow in the proto-neutron star core. Interestingly, core collapse simulations of unstable rotating iron cores (Heger, Langer & Woosley 2000; Fryer & Heger 2000) or polytropes (Zwerger & Muller 1997) indicate that proto-neutron stars, soon after formation, can rotate differentially above the dynamical stability limit set when the rotational to gravitational potential energy ratio \(T_{\text{rot}}/|W|\) is larger than the value \(\beta_{\text{dyn}} = 0.26\) (Saijo et al. 2001). Strong non-linear growth of the dominant bar-like deformation \((m = 2)\) is seen in these cores (described as polytropes by Rampp, Muller, & Ruffert 1998). However there is no sign of fission into separate condensations. The bar evolves, producing two spiral arms that transport the core’s excess angular momentum outwards (see also Shibata, Baumgarte & Shapiro 2000). This reduces the bar’s angular momentum such that a single central body is formed. How can a body develop local condensations when bar-unstable?

According to Bonnell’s picture, the evolution of the bar instability is more complex, in reality. If the rapidly spinning proto-neutron star core goes bar unstable when surrounded by a fall-back disk, then matter present in the bar-driven spiral arms interacts with this material. The sweeping of a spiral arm into fall-back gas can gather sufficient matter to condense into a fragment of neutronized matter. This occurs because the \(m = 1\) mode grows during the development of the \(m = 2\) mode. The \(m = 1\) mode causes the displacement of the unpinned (free-to-move) core and creates an off-center spiral arm that sweeps up more material on one side than the other during "continuing" accretion. The condensation eventually collapses into a low mass neutron star.

Detailed simulations confirming or dismissing the occurrence of such instability are still lacking, so further considerations of the lump masses, temperatures and entropy contents are necessarily speculative.

2.2. Cooling scenarios and the minimum mass

\(^3\)Large-scale asymmetries imprinted in the iron core prior to collapse may lead to anisotropic explosions that produce kicks as indicated by Goldreich, Lai & Sahrling (1996). Whether they lead to fragmentation is unknown.
by gravitational deflection of the ejecta, and velocity phase averaging during the explosion. A further threat would be a rapid decay of the orbit separation to unstable mass transfer that may lead to final coalescence, and emission of gravitational waves.

In §3 we will describe the gravitational bending of the ejecta, while in §4 we will study initial conditions and, subsequently, orbit decay and mass exchange, for cases (B) and (C). We will then give an estimate of the kick speed including phase-velocity averaging.

3. GRAVITATIONAL BENDING OF THE EXPLODING DEBRIS AND THE FINAL KICK

Consider now the instant at which the light secondary of mass \( m \) explodes, having reached the dynamical instability point \((m \approx \text{m}_{\text{neu}})\), while orbiting the heavier, primary neutron star of mass \( M \).

Obtaining the final kick speed imparted to the heavy neutron star is complicated by several factors. First, some of the material ejected from the exploding star can remain bound to the system and eventually accretes onto the remaining star. Second, material that escapes may not be moving at extremely large speeds, even right at ejection, and therefore emerges with a different momentum than it has at the point of explosion. Third, the mass of ejected material may be of order 10% of the total mass of the system, and self gravity could play a role. Fourth, the ejecta are a fluid, and, depending on how much time elapses between neutron star formation and the explosion of the low mass star, moves through ambient gas left over from the original supernova explosion. If the ejecta are slowed significantly by the surrounding gas, some material that would be judged unaltered supernova explosion. If the ejecta are slowed significantly by the surrounding gas, some material that would be judged unaltered.

Suppose that relative to the remaining heavy neutron star, the orbital velocity of the exploding star is \( V \) at the instant of explosion \((t = 0)\), and the position of the center of mass of the exploding star is at \( r \). Let us write the final velocity of the remaining neutron star as

\[
V_{\text{kick}} = -\eta \frac{mV}{(M + m)},
\]

where \( V = |V| \). If the ejecta are expelled isotropically relative to \( r \), and at speeds large compared with \( V \), then we expect \( \eta = |\eta| \approx 1 \), in which case the remaining neutron star recoils with the maximum possible kick speed

\[
V_{\text{kick,max}} \approx \frac{mV}{(m + M)}
\]

as implied by momentum conservation in the center of mass of the binary. To get a more realistic, but still approximate, idea of the size of \( \eta = V_{\text{kick}}/V_{\text{kick,max}} \), we work to zeroth order in the ejected mass, and also treat the trajectories of the ejecta ballistically; this is equivalent to ignoring the third and fourth complications listed above entirely. By working to zeroth order in the mass of the ejecta, we can also assume that all particles are ejected virtually from a single point, \( r \). Let the velocity of a particle relative to the exploding star be \( w \), and assume that \( w \) is distributed isotropically about the position of the exploding star with probability \( P(w) \). Since we work to zeroth order in the mass of the ejecta, we may take the heavy neutron star to remain at fixed position in the calculation.

The problem that remains is the orbital mechanics for each particle relative to the massive star. The orbits are characterized by the conserved quantities

\[
\begin{align*}
E &= \frac{1}{2} |V + w|^2 - \frac{GM}{r} \\
J &= r \times (V + w) \\
A &= \frac{GM(r + (V + w) \times J)}{r}
\end{align*}
\]

which are the orbital energy and angular momentum per unit mass, and the Runge-Lenz vector, respectively. We note that

\[
A = |A| = GMe \sqrt{(GM)^2 + 2EF^2};
\]

\( e < 1 \) for bound orbits, \( e > 1 \) for unbound orbits.

To calculate \( \eta \), we need to find the momentum per unit mass carried away by the ejecta that escape to infinite distance. First, we need to identify the initial velocities of particles that escape. For these, we must have \( E > 0 \), and from Eq. (3) we find the condition

\[
V \cdot w = Vw \mu \geq \frac{GM}{r} \left( \frac{(V^2 + w^2)}{2} \right)
\]

where \( V = |V| \) and \( w = |w| \), and \( \mu = V \cdot w/Vw \). Second, we need to find the mapping between initial and final velocity. To this aim, it is useful to define a coordinate system in which \( \hat{e}_3 = \hat{J}, \hat{e}_1 = \hat{A}, \) and \( \hat{e}_2 = \hat{J} \times \hat{A} \), where \( \hat{J} = J/J \) and \( \hat{A} = A/A \). The orbit is then confined to the \( r=2 \) plane, and we can read off the final velocity in this system from Eq. (15.12) in Landau and Lifshitz, Mechanics:

\[
v_\infty = [-A \sqrt{E} + (2E/GM)J \times A].
\]

We wish to express the final velocity \( v_\infty \) in a coordinate system fixed in the binary at the point of explosion. We thus choose, explicitly

\[
r = r\hat{e}_2 \quad \text{and} \quad V = V\hat{y}
\]

so that the orbital angular momentum of the light star relative to \( M \) just prior to explosion points along \( \hat{z} \). We have explicitly selected the point of explosion to occur at either peri- or apocenter of the orbit, and for simplicity, we specialize to circular orbits below. (Both tidal effects and gravitational radiation will tend to circularize the orbits, but, even for eccentric orbits, we expect tidal disruption to be likeliest at pericenter.) In this coordinate system,

\[
J = -\hat{y}(rw_z + \hat{z}r(V + w_y))
\]

\[
A = \hat{z}[-GM + r(V + w_y)^2 + rw_z^2] + \hat{y}[-rw_z(V + w_y) + \hat{z}(-rw_z w_y)]
\]

and

\[
E = \frac{GM}{r} + \frac{w_z^2 + (V + w_y)^2 + w_z^2}{2}
\]

The velocity of the ejecta can thus be expressed in terms of the known explosion parameters \((w, V, \text{and} r)\) by combining Eq. (6) with Eq. (7)-(10). The result is that the total momentum carried off per unit mass of ejecta is

\[
u = \int_{E[w]>0} d^3 \mathbf{w} \mathbf{P}(\mathbf{w})v_\infty(\mathbf{w}),
\]
so that $\eta = |u|/V$, to zeroth order in $m/M$. As $P(u) = P(w)$ is isotropic we can separate out the integrals

$$u = \int dw w^2 P(w) \int d^2 \hat{w} v_\infty (\hat{w} \hat{w}) \ ,$$

and consider different distributions of ejection speeds separately. The integration only extends over unbound orbits. For $w > \sqrt{2GM/r + V}$, all ejecta escape, and for $w < \sqrt{2GM/r - V}$, no ejecta can escape.

Because of reflection symmetry with respect to the binary orbital plane, $u_e$ vanishes identically. The kick is thus given in the orbital plane of the pre-explosion binary. It is easy to show that when the particles are ejected with equal velocity $w_o$ so that $P(w) = \delta(w-w_o)/(4\pi w_o)$, the integrals yield $u_e \sim -\sqrt{GM/rw_o^3}$ and $u_t \sim V$ for very large values of $w_o$ ($V > V$) which is consistent with the requirement that at large ejection speeds, the outflow reaches infinity with an average velocity comparable to that of the exploding star. Figure 1 shows $\eta$ as a function of the dimensionless explosion velocity $w_o/\sqrt{GM/r}$. Notice that there is no kick (i.e., $\eta$ vanishes) when $E = 0$ (see eq. [5]). This occurs at a critical value of the expansion speed $w_o/\sqrt{GM/r} = \sqrt{2 - V/\sqrt{GM/r}}$ which is $0.414$ for $V \sim \sqrt{GM/r}$ to zeroth order in $m/M$. When $E \leq 0$, the mass outflow vanishes identically and the ejecta fall back to the remaining neutron star transferring all their linear momentum to it.

4. Final kick speed from the short-lived binary

4.1. Masses and orbital separations

The final kick speed imparted to the remaining neutron star depends crucially on the evolutionary scenario leading to the formation and disruption of its low mass companion. The range of possible values of $m_{nm}$, the initial mass of the secondary (hereon), depends on the time the fragment forms and how fast it cools. Both of these complications distinguish the problem of “early” formation and decay of an evanescent proto-neutron star binary (case B) from the problem of “late” decay of a neutron star binary (case C) originally discussed by Clark & Eardley (1977), Bildsten & Popov (1998). Subsequently, the low mass star (which has a core-envelope structure) loses its extended halo evolving along a sequence of hydrostatic equilibria while the orbit widens (Blinnikov et al 1984); the star reaches the minimum stable neutron star mass, whereupon it explodes (Page 1982; Colpi, Shapiro & Teukolsky 1989, 1991, 1993; Blinnikov et al 1990; Sumiyoshi et al 1998). In the case of early formation of the binary (B) we show below that mass transfer is likely to be unstable, and we argue that because of this it may just drive the star to the point of exploding dynamically.

Consider a binary in which Roche lobe overflow is in effect; then, the radius of the low mass star $R(m)$ overfills the Roche lobe and this determines the typical orbital separation at the onset of mass transfer

$$r = 2.2R(m)(M + m)^{1/3}/m^{1/3}.$$ (13)

The dotted lines in Figure 2 give the tidal radius as a function of $m$ for the two different mass-radius relations describing a cold and warm neutron star, respectively. A ring/disk of material forms around the primary star at the moment of Roche filling, and accretion begins to the primary. As the disk drains angular momentum from the orbit there may not be sufficient time to return it to the orbit through disk-donor tidal torques (see for details Bildsten & Cutler 1992; Lubow & Shu 1975). If $1 - f$ is the fraction of orbital angular momentum stored in the disk, the orbital separation would vary with time (following Bildsten & Cutler 1992) as

$$\frac{\dot{r}}{r} = \frac{2m(fM^2-m^2)}{Mm(M+m)} \times \frac{64G^3mM(M+m)}{5r^4}.$$ (14)

Fitting the values of $f(m/M)$ computed by Hut & Paczynski (1984) for the Roche problem, Bildsten & Cutler (1992) found

$$f \sim (5/3)(m/M)^{1/3} - (3/2)(m/M)^{2/3};$$ (15)

during Roche spillover $m/R(m)^3 \propto (M + m)/r^3$ the rate of change of $r$ is

$$\frac{\dot{r}}{r} = \left(\frac{d \ln M}{d \ln m} - \frac{1}{3}\right) \frac{m}{M}.$$ (16)
where $\dot{m}$ is the mass transfer rate. Using this result in Eq. (14) for the orbital evolution, we have

$$\frac{1}{3} \frac{d \ln R(m)}{d \ln m} - 2 \frac{(fM^2 - m^2)}{M(M+m)} \dot{m} = \frac{64G^3mM(M+m)}{5r^4}. \quad (17)$$

Mass transfer is stable only if

$$\frac{1}{3} \frac{d \ln R(m)}{d \ln m} - 2 \frac{(fM^2 - m^2)}{M(M+m)} < 0 \quad (18)$$
given the fact that $\dot{m}/m < 0$.

In case (C), if we adopt the mass-radius relationship as in Jaranowski & Krolak (1992) valid for a cold star (and mass below $1M_\odot$), $R(m) = R_0 m^b/(m-m_0)^c$ where $R_0 = 7.5$ km, $m_0 = 0.09M_\odot$, $b = 0.79$, and $a = 0.83$, we find that the condition for stable mass transfer is

$$T = \frac{1}{3} + \frac{a+(a-b)(m_0/m_0)}{m_0/m-1} - \frac{2(fM^2 - m^2)}{M(M+m)} < 0. \quad (19)$$

In general there is only a finite interval for the mass $m$ for stability whose boundaries are determined by the two root of the Eq. $T = 0$: $m_{\text{stabl}}$ (the lower bound) and $m_{\text{star}}^+$ (the upper bound). When $m_{\text{stabl}} < m < m_{\text{star}}^+$ mass transfer is stable. Outside this interval stability is lost. There may be no roots in which case the fate of the light star depends on a number of details that we will discuss below.

Adopting a constant value for $f$, and setting $\beta \sim \gamma$, to simplify analysis, we note that one regime where the necessary inequality fails is at low masses, near $m \simeq m_0$,

$$m_{\text{stabl}} \simeq \frac{1}{3} + \frac{\beta}{2f-1/3} \quad (20)$$

(For $f = 0.4$ and $\beta = \gamma = 0.8$, $m_{\text{stabl}}/m_0 = 2.7$ while if $f = 1$, $m_{\text{star}}^+/m_0 \simeq 1.5$.) The other regime of instability is at masses well above $m_0$, in which case $d \ln R(m)/d \ln m$ is relatively small, and $m = m_{\text{stabl}}$ with

$$m_{\text{star}}^+ \frac{M}{m} \simeq \frac{1}{2} \sqrt{4f - \frac{23}{36}} \frac{1}{12}. \quad (21)$$

(For $f = 0.4$, $m_{\text{star}}^+/M \simeq 0.4$ while for $f = 1$ $m_{\text{star}}^+/M \simeq 0.8$.) Thus, if there is a disk that stores orbital angular momentum (i.e., $f < 1$), the range of masses for which there can be a stable, Roche filling transfer shrinks considerably relative to the case $f = 1$ explored in Blinnikov et al. (1984) and Imshennik & Popov (1998).

We have computed the roots of Eq. (19) numerically for different $R(m)$. For the Jaranowski & Krolak (1992) parametrization of $R(m)$ for cold neutron stars (C) we find the following intervals of $(m_{\text{stabl}}/m_{\text{star}}^+)$ in units of $M_\odot$: (0.14, 1.08) or (0.14, 1.33) for $f = 1$ and $M = 1.4M_\odot$ or $1.7M_\odot$; for $f$ given by Eq. (15), there are no roots for $M = 1.4M_\odot$, and the roots are (0.29, 0.50) for $M = 1.7M_\odot$; the stability interval $(m_{\text{stabl}}/m_{\text{star}}^+)$ depends sensitively on the logarithmic derivative of $R(m)$. Note that the root $m_{\text{star}}^+$ exists because of the special dependence of $R(m)$ on the parameter $m_0$. Numerical hydrodynamical models for the description of these semi-detached tight binaries are necessary to address the problem of mass transfer, and preliminary results points toward the existence of stable phases (Davies, private communication). Below, we will adopt the values of $m_{\text{stabl}}$ and $m_{\text{star}}^+$ for a 1.7$M_\odot$ neutron star.

What is the fate of the binary then? In case (C) if the two roots $m_{\text{stabl}}$ and $m_{\text{star}}^+$ exist, and if the initial mass of the low mass star is $m_{\text{stabl}} < m_{\text{min}} < m_{\text{star}}^+$, the binary may first go through a stage of evolution via gravitational radiation without mass exchange, then a phase of evolution with mass exchange at a rate determined by the rate of angular momentum loss due to gravitational radiation, and finally become unstable when $m_{\text{stabl}}$ is reached (Blinnikov et al. 1984; Imshennik & Popov 1998). If, on the other hand, $m_{\text{stabl}} \leq m_{\text{min}} \leq m_{\text{star}}^+$, then the binary tightens as a consequence of gravitational radiation without mass loss, until $m$ fills its Roche lobe, whereupon it becomes unstable. If we define $m_{\text{stabl}}$ to be the stellar mass at the onset of instability, then $m_{\text{stabl}} = m_{\text{stabl}}$ if $m_{\text{min}} > m_{\text{star}}^+$, and $m_{\text{stabl}} = m_{\text{star}}^+$ if $m_{\text{stabl}} \leq m_{\text{min}} \leq m_{\text{star}}^+$.

What happens once $m = m_{\text{stabl}}$ sets in has been described by Blinnikov et al. (1984). As already mentioned, they argue that although the equilibrium radius of a low mass neutron star may become larger than the Roche radius, the vast majority of its mass may still be enclosed within it (99% is contained inside the inner 38 km or so). The stripping of the stellar mass envelope occurs slowly enough that the companion evolves through a series of nearly equilibrium states until $m_{\text{stabl}}$ is attained, while the orbit widens.

More accurately, Eq. (14), in the absence of gravitational waves losses leads to a change in orbital radius $r$ as a function of mass loss equal to

$$\frac{d \ln r}{d \ln m} = -2f/2f + \frac{2fm}{M+m} + \frac{2m^2}{M(M+m)} \quad (22)$$

which can be integrated to yield when the mass is $m$,

$$L \sim \left( \frac{m_{\text{min}}}{m} \right)^{2f} \exp \left[ \frac{2(1-f)(m_{\text{min}}-m)}{M+m} \right], \quad (23)$$

neglecting changes in $M$. In this approximation the orbit widens by a factor $\sim (m_{\text{stabl}}/m_{\text{stabl}})^{2f}$ as the mass decreases from the value $m_{\text{stabl}}$ toward $m_{\text{stabl}}$.

The orbital radius at the point of explosion is somewhat larger than the radius of Roche spillover, and is accordingly

$$r_{m_{\text{stabl}}} \simeq \frac{2.2R(m_{\text{stabl}})M^{1/3}}{m_{\text{stabl}}^2 m_{\text{stabl}}^{2f/1/3}}. \quad (24)$$

The orbital speed of the low mass neutron star when it explodes is

$$V \simeq \sqrt{\frac{GM}{r_{m_{\text{stabl}}}}} \simeq 0.68G^{1/2}M^{1/3}m_{\text{stabl}} \left( \frac{f}{1-6} \right)^{1/2} \sqrt{R(m_{\text{stabl}})} \left( m_{\text{stabl}}/m_{\text{stabl}}^2 \right)^{1/2}, \quad (25)$$

implying a maximum kick

$$V_{\text{kick max}} \simeq \frac{m_{\text{stabl}}}{(m_{\text{stabl}} + M)} V \simeq \frac{0.68G^{1/2}m_{\text{stabl}}}{m_{\text{stabl}}^{f/1-6/1-6} \left( R(m_{\text{stabl}}) \right)^{1/2}} \quad (26)$$

that scales as $M^{-2/3}$, due to the dependence of $V$ on $r_{m_{\text{stabl}}}$. This is a direct consequence of the hypothesis of Roche lobe mass transfer; the system has lost memory of the initial orbital separation. Also note that in $V_{\text{kick max}}$ we have $m_{\text{stabl}}$ at numerator, even when $m_{\text{stabl}} = m_{\text{star}}^+$. When $m_{\text{stabl}} = m_{\text{star}}^+ < 0.29M_\odot$, (or when $m_{\text{stabl}} = m_{\text{stabl}} = 0.0925M_\odot$) the maximum kick is $V_{\text{kick max}} \sim 936\text{ km s}^{-1}$ (800 km s$^{-1}$), and $f \sim 0.4$.

Concerning case (B), warmer proto-neutron stars seem to satisfy a smoother mass-radius relation. A fit to Strobel et al.
(1999) data gives \( R(m) \propto m^{-6/5} \) which would lead to a stability interval \((\mu_{\text{mnc}}, \mu_{\text{ig}})\) only for \( f \) close to unity (note that there is only a single root in this case). Since disk-donor torquing may act on a timescale longer than the lifetime of the binary, it is likely that mass transfer is always unstable under these circumstances, and as soon as the light star overfills its Roche lobe at \( \mu_{\text{st}} \), unstable mass transfer begins and there might be time for a phase of common envelope evolution. In this context, we wish to argue that the star is driven almost instantly toward explosion. The cooling time \( \tau_{\text{cool}} \sim 50 \) sec (case B; see Figure 2) exceeds the typical time for mass transfer \( \tau_{\text{mass}} \sim m_{\text{mnc}}/\dot{m}_{\text{w}} \text{c}_{\text{sound}} \sim 0.02 \) sec \((\dot{\rho} \sim 10^{11} \text{g cm}^{-3} \text{s}^{-1}) \) denotes the density, close to neutron drip, of the mass losing star in its envelope, \( R \sim 50 \) km the size of the Roche lobe and \( c_{\text{sound}} \) is the sound speed in units of \( 10^{10} \text{km s}^{-1} \). In this case, the mass losing star encounters the stability limit \( m_{\text{mnc}}(T) \) before being stabilized by cooling. We speculate that the core of the light star explodes on its own dynamical time \((\sim 0.001 \) sec) before coalescence of the two stars in completed over a orbital period of \( P_{\text{orb}} = 2\pi (r_{\text{mnc}}^3/\text{GM})^{1/2} \sim 0.033 \) sec. The core of the light star may thus become unstable before coalescence is completed, because of the strong dependence of \( m_{\text{mnc}} \) on temperature. Common evolution is known to be accompanied by ejection of part of the envelope enshrouding the system, and this mass loss may provide an additional thrust to the merged object. Only hydrodynamical simulations can quantitatively credit to this scenario accurately, establishing a sort of continuity in the physical processes when moving from C (cold late type mode) to A (hot hydrodynamical mode) passing through B (warm early time mode). If we were to compute \( V_{\text{kick}} \) by assuming that the of the warm star explodes right at the time it fills the Roche lobe, we would obtain nominal speeds close to \( 10.000 \text{km s}^{-1} \). The actual value of the kick velocity is computed below combining gravitational bending with orbital phase averaging of the momentum impulse.

4.3. Velocity phase-averaging and kick speeds

Our estimate of \( V_{\text{kick}} \) for cases (C) and (B) using the expression for \( \eta \) would be not complete without considering that explosion does happen non instantaneously. The star explodes on a timescale comparable to the dynamical timescale, and this can be close to the time of revolution in the binary. In case (B) in particular this effect might be severe.

The final kick speed will be diminished if the explosion is not instantaneous to a good approximation. The orbital frequency when the companion explodes is

\[ \omega_{\text{mnc}} = \left( \frac{GM}{r_{\text{mnc}}^3} \right)^{1/2} \approx \frac{0.314G^{1/2}m_{\text{mnc}}^3}{m_{\text{stab}}^{3(1-f)/2}[R(m_{\text{stab}})]^{3/2}}, \quad (27) \]

which ranges from \( \omega_{\text{mnc}} \approx 17 \) s\(^{-1} \) for \( m_{\text{stab}} = m_{\text{mnc}} \) to \( \omega_{\text{mnc}} \approx 360 \) s\(^{-1} \) for \( m_{\text{stab}} = m_{\text{tid}} \approx 0.29M_\odot \) and \( M = 1.7M_\odot \) (case C), and 185 s\(^{-1} \) for case B with \( m_{\text{mnc}} = 0.3M_\odot \). At the time of explosion, the light star overfills its Roche radius \( R_{\text{R,exp}} \sim R(m_{\text{stab}}) \).

Matter ejected from the unstable star at a velocity \( w_0 \) crosses it in a time \( R_{\text{R,exp}}/w_0 \) and we can take \( R_{\text{R,exp}}/w_0 \) as an estimate of the duration of the explosion, \( \tau_{\text{exp}} \).

The explosion is almost instantaneous as long as the dimensionless combination \( P_{\text{stab}}/\tau_{\text{exp}} \ll 1 \), or equivalently

\[ \frac{\omega_{\text{mnc}}R_{\text{R,exp}}}{w_0} \ll 1. \quad (28) \]

For \( m_{\text{stab}} \sim 0.0952M_\odot \) the dimensionless ratio is \( 2.800 \text{km s}^{-1}/w_0 \sim 0.1 \) for \( m_{\text{stab}} \approx m_{\text{mnc}} \) (case C), and \( 9.000/w_0 \text{km s}^{-1} \sim 0.3 \) for \( m_{\text{stab}} = m_{\text{mnc}} = 0.3 \) in case B, taking \( w_0 \sim 30.000 \text{km s}^{-1} \) as an example.

To be more quantitative, suppose that the star loses mass at a rate \( \dot{m}(t) = \dot{m}_{\text{exp}}(t) \) once it becomes unstable, with

\[ \int_0^t dt \Gamma(t) = 1. \quad (29) \]

Consider circular orbits only, and assume that although mass loss may extend over many orbital periods, the orbit of the exploding star remains unaltered during the mass loss. (The latter approximation should be valid if mass loss is rapid enough, to zeroth order in the decreasing secondary mass.) Adopt a fixed coordinate system defined at \( t = 0 \), when mass loss begins, so that \( \dot{x} = r(0)/r \) and \( \dot{y} = \Gamma(0)/V \). The momentum per unit ejected mass is now

\[ u = \int_0^\infty dt \Gamma(t) \left[ u_x(\dot{x}\cos \omega t + \dot{y}\sin \omega t) + u_y(\dot{y}\sin \omega t + \dot{x}\cos \omega t) \right] \]

\[ \equiv u_x\dot{e}_x + u_y\dot{e}_y \equiv \Gamma \dot{x}, \quad (30) \]

where \( u_x = (u_x \pm iu_y)/\sqrt{2}, \dot{e}_x = (\dot{x} \pm i\dot{y})/\sqrt{2} \), and

\[ \Gamma \dot{x} = \int_0^\infty dt \Gamma(t)e^{i\omega t}. \quad (31) \]

Here, \( (u_x, u_y) \) are the same as were computed in § 3 for instantaneous mass loss (at \( t = 0 \)). When the finite duration of the mass loss is accounted for, the efficiency of the recoil is diminished from \( \eta \) as computed in § 3 for instantaneous mass loss, to \( \eta \Gamma \). For example, if mass is lost at a constant rate, then \( \Gamma(t) = \tau_{\text{exp}}^{-1}(\tau_{\text{exp}} - t) \), and we find

\[ \Gamma \dot{x} = \frac{\sin(\omega_{\text{mnc}}\tau_{\text{exp}}/2)}{\omega_{\text{mnc}}\tau_{\text{exp}}/2}; \quad (32) \]

if instead \( \Gamma(t) = \tau_{\text{exp}}^{-1}\exp(-t/\tau_{\text{exp}}) \) then

\[ \Gamma \dot{x} = \frac{1}{\sqrt{1 + (\omega_{\text{mnc}}\tau_{\text{exp}})^2}}. \quad (33) \]

The correction due to phase averaging, \( \Gamma \dot{x} \), is only modest (and somewhat model dependent) as long as \( \omega_{\text{mnc}}\tau_{\text{exp}} \ll 1 \), but for long explosions, the efficiency is diminished by a factor of order \( (\omega_{\text{mnc}}\tau_{\text{exp}})^{-1} \) in general.

Figure 3 shows the kick velocity \( V_{\text{kick}} \) of the neutron star (of mass \( 1.7M_\odot \); but for \( 1.4M_\odot \) scale the velocity with \( M^{-2/3} \)) that remains as a function of the speed of the ejecta \( w_0 \) for the two cases (C) and (B). The solid curves refer to scenario (C), when \( f \) is given by the Roche value (eq. [15]) and \( m_{\text{stab}} \) is equal to \( m_{\text{tid}} = 0.29M_\odot \) (lower curve before crossover occurring at \( w_0 \sim 40.000 \text{km s}^{-1} \)) and \( m_{\text{mnc}} \) (upper curve before crossover). The dotted curve is for with \( m_{\text{stab}} = m_{\text{tid}} \sim 0.14M_\odot \). The dash-dotted line, for case (B), is computed starting the explosion during common envelope, when \( m_{\text{mnc}} \sim 0.3M_\odot \) and when the orbit separation is \( P_{\text{orb}}/\tau_{\text{cross}} = r/V \) times less than the separation at the moment of filling the Roche lobe (eq. [14]), to mimic orbit decay during common envelopes. We have included the phase-average orbit correction as given in Eq. (33).

Very high kicks come from such complex hydrodynamical process that alternative models have not been able to produce.
5. DISCUSSION

In this paper, we have elaborated on suggestions that substantial neutron star recoil can result from the explosion of a low mass neutron star formed in the aftermath of rotating core collapse (Imshennik & Popov 1998). A noteworthy feature of this process is that the final kick speed is determined by nuclear physics. The details about the formation mechanism and hydro-dynamical effects produce in reality a widespread range in velocities, as the one we see.

We have extended earlier work by estimating the effect of the finite speed of the material ejected in the explosion, including both gravitational deflection, phase averaging, and indirectly taking into account cooling effects. We argue that a range of neutron star recoil speeds could arise from disruption scenarios with different companion masses. The recoil speed resulting from the dissolution of an evanescent binary system can be, within a factor of two, around 1000 km s\(^{-1}\), explaining the larger values deduced observationally. It is remarkable that nuclear physics implies a value in the observational range at all, once such a binary system is presumed to form, a concordance that lends some support to the idea.

The kicks that result from this mechanism are confined to the orbital plane of the evanescent neutron star binary. We expect the spin angular momentum vector of the remnant neutron star to be nearly, if not perfectly, aligned with the orbital angular momentum of the binary, and in turn aligned with the angular momentum vector of the collapsing iron core. Near alignment would be consistent with the requirements imposed by observations of geodetic precession of B1913+16, where the kick is constrained to lie very nearly in the plane of progenitor binary, which was most likely perpendicular to spins of the spun-up neutron star (i.e. B1913+16) and its pre-explosion companion star (Wex, Kalogera & Kramer 2000). In contrast, though, X-ray observations of the Vela pulsar have revealed a jet parallel to its proper motion (Pavlov et al. 2000; Helfand, Gotthelf & Halpern 2001), and it has been argued that the proper motions of both Vela and the Crab pulsar are closely aligned with their spin axes (Lai, Chernoff, & Cordes 2001). The kick mechanism studied here would not be able to account for parallel spin and velocity. However, we note that the proper motions of both Vela and the Crab correspond to transverse speeds of 70–141 km s\(^{-1}\) and 171 km s\(^{-1}\) respectively, using reasonable estimates of the distances to the pulsars (Lai, Chernoff, & Cordes 2001). For the alignment to be real, the space velocities of these two systems must lie in the plane of the sky. The inferred speeds of these two pulsars are then considerably smaller than the characteristic speed arising from explosion of a low mass, tidally disrupted companion. We propose that the formation of Vela and the Crab did not produce an evanescent binary, and therefore some other mechanism was responsible for their spin-aligned kicks (such as those explored by Lai, Chernoff, & Cordes 2001). We suggest that the larger kick component is due to formation and disruption of an evanescent binary, and is perpendicular to the spin axis, and that the smaller kick component is associated with other less vigorous kicks that tend to align with the rotation axis, perhaps because of phase averaging (e.g. Spruit & Phinney 1998; Lai, Chernoff, & Cordes 2001). A superposition of these two classes of kicks would also be consistent with the requirement of nearly but not precisely spin-perpendicular kicks to account for the observation of geodetic precession in B1913+16 (Wex, Kalogera & Kramer 2000). If this idea is correct, then one expects that lower velocity neutron stars should have their space velocities predominantly along their spin axes, and high velocity neutron stars should have space velocities predominantly perpendicular to their spins, giving origin to two independent (low and high velocity) distributions. A contamination of low velocity stars (belonging to the low velocity tail of the high velocity distribution) would come from those explosions in the evanescent binary where light bending and rotational averaging have been more important. This could give rise to a bimodal distribution of velocities as is inferred from the observations (Arzoumanian, Chernoff, & Cordes 2001).

This scenario predicts a clear signature in the neutrino emission, in the aftermath of core collapse: Two neutrino bursts occurring several seconds or minutes after core bounce, i.e., after the main neutrino burst, should signal (i) Kelvin contraction of the condensation gathered by the instability and (ii) explosion (on the dynamical time) of the light neutron star. Neutrino emission between the two bursts should also come form material that accretes onto the neutron star that remains. Heavy r-process elements, debris of the explosion of the light star, should also be ejected and found deep in the supernova expanding shells.

Finally, we note that the same phenomenon could occur if the formation of a light neutron star accompanies rotating core collapse to a black hole. In that case, we find that the recoil velocity of the black hole will be proportional to \(M_{\text{BH}}^{2/3}\), which is not the simple \(M_{\text{BH}}^{1}\) scaling one would expect for a kick mechanism that ejects the same amount of momentum irrespective of whether a supernova leaves behind a neutron star or a black hole. The higher kick implied by this scaling could be compensated by stronger relativistic effects of gravitational bending and unstable mass transfer that can lead to a much lower escape probability of the exploding debris.

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\(^4\)During revision of this manuscript we learned of a paper by Davies et al. (astro-ph/0204358) in which a connection between gamma-ray bursts and pulsar kicks is made, where kicks result from recoil by a short-lived binary.
Fig. 1.— The parameter \( \eta = \frac{V_{\text{kick}}}{V_{\text{kick, max}}} \) as a function of the speed of the ejecta \( w_0 \) expressed in units of \( \sqrt{\frac{GM}{r}} \).
Fig. 2.— Solid lines gives the orbital separation $r_{\text{orb}}$ (in km) versus $m_{\text{in}}$, the initial mass of the light component, in a binary of total mass $M_{\text{tot}} = 1.7 M_{\odot}$ and orbital angular momentum $J_{\beta \text{ dyn}} = 4 \times 10^{49}$ g cm$^2$ s$^{-1}$. The dashed lines are the loci in the $r_{\text{orb}}, m_{\text{in}}$ plane of constant time $\tau_{\text{GW}}$ for 1, 30, 100, 300 seconds. The filled dots denote the various values of $m_{\text{mme}}$ at the times $t_\ast = 1, 20, 100, 300$ seconds, according to the cooling calculation of Strobel et al. (1999). The dotted lines give the binary separation at the time of Roche spillover for a cold star with $R(m) = R_0 m^b / (m-m_0)^a$ as in Jaranowski & Krolak, and for $R(m)$ as given from the fit to Strobel’s data: $R(m) = R_\ast (m/m_\ast)^{-6/5}$ with $m_\ast = 0.3 M_{\odot}$ and $R_\ast = 48$ km.
Fig. 3.— Solid lines show the neutron star kick velocity $V_{\text{kick}}$ as a function of the speed of the ejecta $w_o$, for case (C) when $m_{\text{stab}} = m_{\text{tid}}$ (lower curve before crossover) and $m_{\text{stab}} = m_{\text{mmc}}$ (upper curve before crossover). The dotted line is for $m_{\text{stab}} = m_{\text{tid}} = 0.14M_\odot$ for $f = 1$. The dash-dotted line refers to case (B) for $m_{\text{mmc}} = 0.3M_\odot$; for this case velocity phase-averaging is an important correction.