Weak radiative hyperon decays

P. Żenczykowski  

aIFJ Institute of Nuclear Physics
Radzikowskiego 152, 31-342 Kraków, Poland

a[This work was supported in part by the Polish State Committee for Scientific Research grant 5 P03B 050 21.]

The problem of weak radiative hyperon decays (WRHD) is reviewed. With the recent measurement of the $\Xi^0 \rightarrow \Lambda \gamma$ asymmetry confirming Hara’s theorem, implications from its violation in low-energy theoretical approaches are discussed. It is shown how an underlying symmetry link should be formulated for a successful description of both nonleptonic and radiative weak hyperon decays. The sign of the $\Xi^0 \rightarrow \Lambda \gamma$ asymmetry and the overall size of parity-violating WRHD amplitudes together lead to the resolution of the old S:P problem in nonleptonic decays.

1. Introduction

The puzzle of weak radiative hyperon decays dates back to the 60’s, when experiment [1] suggested large asymmetry in the $\Sigma^+ \rightarrow p \gamma$ decay, contrary to the expectations based on Hara’s theorem [2]. The size of the asymmetry was subsequently confirmed, the PDG average now being $-0.76 \pm 0.08$. Recently new information has been supplied by the NA48 measurement of the asymmetry of the related $\Xi^0 \rightarrow \Lambda \gamma$ decay [3], which is crucial for theoretical considerations.

Hara’s theorem states that the parity-violating (p.v.) amplitude of the $\Sigma^+ \rightarrow p \gamma$ decay must vanish in the limit of exact SU(3). If the $\Sigma^+ \rightarrow p \gamma$ parity-conserving (p.c.) amplitude is not small as the size of the branching ratio suggests, the asymmetry should deviate from zero due to SU(3) breaking. Since the latter is usually not large, a small asymmetry of $\pm 0.2$ was expected. The problem was confounded by the violation of Hara’s theorem in the quark model and other calculations. Since the only explicit assumptions of the theorem are CP- and gauge-invariance, the origin of the latter results proved hard to understand at hadron level. With the quark model suggesting violation of the theorem, the question seemed to be whether large $\Sigma^+ \rightarrow p \gamma$ asymmetry is a sign of a strong SU(3) breaking or that of a true violation of Hara’s theorem.

2. Approaches with built-in Hara’s theorem

The standard approach to WRHD was proposed by Gavela et al [4], who presented a pole model analogous to their model of nonleptonic hyperon decays (NLHD) [5]. In the latter paper the p.v. NLHD amplitudes were given in terms of the current algebra (CA) commutator plus a correction term originating from the excited negative parity $1/2^-$ baryons in intermediate states (parallelling the dominance of the ground-state baryons in the p.c. amplitudes, the $1/2^-$ baryons were considered here the most important contribution). In [4] WRHD amplitudes were described in an analogous way (the p.c. amplitudes specified by the pole model with intermediate ground-state baryons, while the p.v. amplitudes calculated from the model with intermediate negative parity ($1/2^-$) baryons). Parity-violating transitions between the ground-state and the $1/2^-$
baryons were driven by W-exchange only. With SU(3) broken, ref.[4] obtained a large asymmetry
\[ \alpha(\Sigma^+ \to p\gamma) = -0.80^{+0.32}_{-0.19}. \]  
(1)

Among other results of [4], the most important one is the prediction of the \( \Xi^0 \to \Lambda\gamma \) asymmetry:
\[ \alpha(\Xi^0 \to \Lambda\gamma) = -0.78 \]  
(2)

Calculations using the chiral perturbation theory proved to have little predictive power [6]. This conclusion was corroborated recently by Borasoay and Holstein, who had to discard their ChPT approach and adopt a pole model similar to ref.[4]. Their approach (see eg. [7]) yields
\[ \alpha(\Sigma^+ \to p\gamma) = -0.49 \]
\[ \alpha(\Xi^0 \to \Lambda\gamma) = -1.0 \]  
(3)

when the singlet assignment of the \( \Lambda(1405) \) intermediate state is used. Note the size and the negative sign of the \( \Xi^0 \to \Lambda\gamma \) asymmetry.

Results of the application of the QCD sum rules [8,9] were not conclusive (see Table 1), with no prediction of [9] for the \( \Xi^0 \to \Lambda\gamma \) asymmetry.

Table 1

<table>
<thead>
<tr>
<th>Predictions of QCD sum rules</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>( \alpha(\Sigma^+ \to p\gamma) )</td>
</tr>
<tr>
<td>( \alpha(\Xi^0 \to \Lambda\gamma) )</td>
</tr>
</tbody>
</table>

3. Quark model and vector-meson dominance

In 1983 Kamal and Riazuddin (KR) found that in the constituent quark model (CQM) the W-exchange diagrams produce a p.v. \( \Sigma^+ \to p\gamma \) amplitude which does not vanish in the limit of exact SU(3) [10]. The KR calculation has been often dismissed as erroneous, possibly violating gauge invariance. Independent checks have confirmed its technical correctness, however. The question that remained unanswered was what conclusions should be drawn from the KR result.

An experimental way of deciding whether the CQM result constitutes an artefact of the model or a real effect was pointed out in ref.[11]. It was observed that in approaches violating Hara's theorem a large and positive \( \Xi^0 \to \Lambda\gamma \) asymmetry is predicted, in marked contrast to the Hara's-theorem-satisfying case. In particular, in the phenomenological extension of the KR paper, Verma and Sharma [12] obtained:
\[ \alpha(\Sigma^+ \to p\gamma) = -0.56 \]
\[ \alpha(\Xi^0 \to \Lambda\gamma) = +0.68 \]  
(4)

In an attempt to avoid the concepts of the CQM, ref.[13] proposed a hadron-level approach in which spin-flavour symmetries are combined with the idea of vector meson dominance (VMD). The approach was based on the SU(6)-based extension of NLHD amplitudes to describe vector-meson p.v. couplings to nucleons, as proposed by Desplanques, Donoghue, and Holstein (DDH) [14] for the description of nuclear parity violation. It turned out that the approach of [13] (here dubbed DDH+VMD) leads to the violation of Hara's theorem as well. The origin of this result is readily traced: the p.v. coupling of vector meson to nucleon in DDH is of the form \( \bar{u}\gamma_{\mu}\gamma_5uV^\mu \). When combined with VMD, the coupling \( \bar{u}\gamma_{\mu}\gamma_5uA^\mu \) is generated. As the latter coupling is not gauge-invariant by itself, violation of Hara's theorem in the DDH+VMD approach was blamed on gauge noninvariance of VMD. The issue of gauge invariance of VMD is more subtle, however. In 1967 Kroll, Lee, and Zumino proposed that the VMD prescription constitutes an approximation to a fully gauge-invariant quark-level contribution (KLZ) [15]. Following their ideas, ref.[13] accepted that the quark model and VMD are essentially equivalent gauge-invariant approaches, and that the CQM result of [10] should be viewed as complementary to that of DDH+VMD. The latter approach predicted [11]:
\[ \alpha(\Sigma^+ \to p\gamma) = -0.95 \]
\[ \alpha(\Xi^0 \to \Lambda\gamma) = +0.8 \]  
(5)

Similarity of Eqs. (4,5) is striking. Nonetheless, violation of Hara's theorem in CQM/DDH+VMD approaches was still often blamed on gauge noninvariance.

In order to exhibit gauge invariance of the CQM calculations, and to clarify the possible physical origin of the violation of Hara's theorem, CQM calculations were recently performed
[16] in a different way, keeping gauge-invariance manifest till the very end. Moreover, CQM descriptions of NLHD and WRHD were discussed alongside current algebra and VMD. The calculation was deliberately formulated in a way fully analogous to the standard CQM calculations of baryon magnetic moments. Its first step consisted in the evaluation of the modifications of the three-quark states by the perturbing p.v. hamiltonian $H^{p.v.}$, which admixes $q\bar{q}$ pairs into $qqq$ baryon states:

$$H^{p.v.}|qqq\rangle = |qqq\bar{q}\rangle (6)$$

so that the $qqq\bar{q}$ admixture has parity $P = -1$.

Thus, evaluation of photon (vector-meson) interaction with quarks in a baryon in the presence of weak interaction proceeds according to the standard CQM rules by sandwiching the interaction with photon ($A_\mu \rightarrow V_\mu$ for vector meson) between appropriate quark states (Fig. 1):

$$\langle qqq|\bar{q}\gamma_\mu A_\mu|qqq\rangle$$

$$\langle qqq\bar{q}|\bar{q}\gamma_\mu A_\mu|qqq\rangle (7)$$

The above prescription is manifestly gauge-invariant and shows that photon and vector-meson couplings to baryons should be proportional to each other. Calculation performed along these lines demonstrates once again that in the CQM the p.v. $\Sigma^+ \rightarrow p\gamma$ amplitude is nonzero in the SU(3) limit. At the same time, the symmetries of the CA commutator are reproduced with standard pseudoscalar interaction $\bar{q}\gamma_5 q P$.

4. $\Xi^0 \rightarrow \Lambda\gamma$

With the old value of $\alpha(\Xi^0 \rightarrow \Lambda\gamma)$ being $+0.43 \pm 0.44$ as measured by [17], and the models predicting this asymmetry to be $-0.8 \pm 0.2$ for Hara’s theorem satisfied and $+0.8 \pm 0.2$ for Hara’s theorem violated, the latter possibility was discussed at length by the author. Since violation of Hara’s theorem in the CQM cannot be blamed on the lack of gauge invariance [16], its origin must be different. The latter was identified as genuine nonlocality of the p.v. photon-baryon coupling in the CQM [16]. In essence, violation of Hara’s theorem appears directly connected to the old quark model question of why magnetic moments of quarks can be added as if the quarks were free and hence completely independent particles which might be infinitely far apart from each other.

Fig. 1 Photon (meson) emission in CQM with $W$-exchange-induced admixtures

The p.v. amplitudes of the $\Xi^0 \rightarrow \Lambda\gamma$ and $\Xi^0 \rightarrow \Sigma^0\gamma$ decays each receive nonzero contributions from a different type of diagram (see Table 2, where relevant SU(6) factors $b_1(2)$ are shown). Thus, the relative sign between the contributions from (b1) and (b2) in Fig. 1 can be measured by comparing the asymmetries of these two decays.

This relative sign determines whether in $\Sigma^+ \rightarrow p\gamma$ the contributions from diagrams (b1) and (b2) add, or completely cancel (Table 2). Detailed considerations show that Hara’s theorem is satisfied (violated), if $\alpha_{exp}(\Xi^0 \rightarrow \Lambda\gamma)$ is negative (positive).

The new NA48 result [3] of

$$\alpha(\Xi^0 \rightarrow \Lambda\gamma) = -0.65 \pm 0.19 (8)$$

permits the following conclusions:
1. Hara’s theorem is satisfied
2. theoretical arguments against Hara’s theorem are invalid.

Table 2
Theoretical SU(6) factors $b_{1[2]}$ for p.v. amplitudes (b1) and (b2) and experimental asymmetries for selected WRHD (using ref.[3])

<table>
<thead>
<tr>
<th>decay</th>
<th>$\Sigma^+ \rightarrow p\gamma$</th>
<th>$\Sigma^0 \rightarrow \Lambda\gamma$</th>
<th>$\Xi^0 \rightarrow \Sigma^0\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>asym.</td>
<td>$-0.76\pm0.08$</td>
<td>$-0.65\pm0.19$</td>
<td>$-0.63\pm0.09$</td>
</tr>
<tr>
<td>$+b_1+b_2$</td>
<td>$-\frac{1}{4\sqrt{3}}$</td>
<td>$-\frac{1}{3\sqrt{2}}$</td>
<td>$+\frac{1}{2} + 0$</td>
</tr>
<tr>
<td>$-b_1+b_2$</td>
<td>$+\frac{1}{3\sqrt{2}}$</td>
<td>$-\frac{1}{3\sqrt{2}}$</td>
<td>$-\frac{1}{2} + 0$</td>
</tr>
</tbody>
</table>

Consequently, CQM calculations are unphysical: while hadron-level spin-flavour symmetries of the CQM are correct for individual amplitudes (b1) and (b2), the CQM connection between the latter two is not. Moreover, since VMD+DDH also leads to the violation of Hara’s theorem, either VMD is not universal or the DDH approach is not fully correct.

5. Proposed resolution

The p.v. NLHD amplitudes can be expressed either as sums of contributions from the CA commutator and a correction term proportional to pion momentum $q$, or as sums of amplitudes corresponding to single-quark and two-quark diagrams (c- and b-type respectively):

$$A_{NL} = \text{comm.} + R_\mu q^\mu = A_{NL}(b) + A_{NL}(c)$$

Symmetry considerations require that the contribution from the commutator be proportional to the sum of SU(6) factors $b_1$ and $b_2$, while the correction term be proportional to their difference:

$$A_{NL}(b) = (b_1 + b_2)b_{\text{com}} + (-b_1 + b_2)b_{\text{cor}} = b_2(b_{\text{com}} + b_{\text{cor}}) \equiv 2b_{NL}$$

The second line follows since for NLHD all $b_1$’s are zero. The NLHD data fix the two parameters defining the size of b- and c- amplitudes as:

$$b_{NL} = -5 \cdot 10^{-7}$$
$$c_{NL} = +12 \cdot 10^{-7}$$

$(A_{NL}(c)$ is proportional to $c_{NL}$), or equivalently,

$$d_S = b_{NL}$$
$$f_S = -b_{NL} + 2c_{NL}/3$$

corresponding to the $f_S/d_S = -2.6$ ratio for the S-waves. However, precisely because all $b_1$’s are zero, these data do not allow one to fix the size of $b_{\text{com}}$ and $b_{\text{cor}}$ separately.

For WRHD the contribution from single-quark diagrams is negligible. Furthermore, there can be no $b_1 + b_2$ terms (Hara’s theorem is satisfied). Thus, the WRHD amplitudes are proportional to the differences of SU(6) factors $b_1$ and $b_2$:

$$A_{WR} = (-b_1 + b_2)b_{WR} \cdot e/g$$

where factor $e/g$ converts between strong ($g$) and electromagnetic ($e$) couplings, so that the spin-flavor symmetry link for the $-b_1 + b_2$ terms in NLHD and WRHD has the simple form:

$$b_{\text{cor}} = b_{WR}$$

Previous successful description of the branching ratios of $\Xi^0 \rightarrow \Lambda\gamma$ and $\Xi^0 \rightarrow \Sigma^0\gamma$ decays (see [11]) indicates that, numerically,

$$|b_{WR}| \approx 5 \cdot 10^{-7} = |b_{NL}|$$

If $b_{WR} \approx b_{NL}$, Eqs. (11,15) imply that $b_{\text{com}} = 0$, which follows from CQM if Hara’s theorem is to be satisfied [16]. Since in this case positive $\Xi^0 \rightarrow \Sigma^0(\Lambda)\gamma$ asymmetries are predicted in disagreement with experiment, this possibility has to be rejected. The only other possibility is $b_{WR} \approx -b_{NL}$. This leads to correct negative $\Xi^0 \rightarrow \Sigma^0(\Lambda)\gamma$ asymmetries. One concludes that $b_{\text{com}} = 2b_{NL}$.

Single-quark contributions to the NLHD amplitudes obtained by symmetry from WRHD should be negligible, i.e. $c_{NL} = c_{\text{com}}$. The $f/d$ ratios for the S- and P-waves of NLHD amplitudes appear therefore different, since for the P-waves one obtains

$$d_P = b_{\text{com}} \approx 2b_{NL}$$
$$f_P = -b_{\text{com}} + 2c_{\text{com}}/3 \approx -2b_{NL} + 2c_{NL}/3$$

leading to a resolution of the S:P problem in NLHD:

$$d_P/d_S \approx 2$$
\[ f_P/f_S \approx 1.4 \]
\[ f_P/d_P \approx -1 + c_{NL}/(3b_{NL}) = -1.8 \] (18)

In Table 3 we compare the WRHD data with the predictions of Gavela et al [4]. The last column gives the branching ratios and asymmetries for the decays of \( \Xi^0 \) in an SU(6) symmetric approach just discussed. The relevant entries are obtained from those given in [11] by just reversing the sign of the \( \Xi^0 \to \Lambda \gamma \) amplitude. (Description of \( \Sigma^+ \) and \( \Lambda \) decays requires inclusion of SU(3) breaking and more modifications than a simple sign reversion).

Table 3
Comparison of branching ratios and asymmetries

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Sigma^+ \to p\gamma )</td>
<td>1.23 ± 0.06</td>
<td>0.921±0.049</td>
<td>-0.14</td>
<td>not simple</td>
</tr>
<tr>
<td>( \Lambda \to n\gamma )</td>
<td>1.75 ± 0.15</td>
<td>0.62</td>
<td>not simple</td>
<td></td>
</tr>
<tr>
<td>( \Xi^0 \to \Lambda\gamma )</td>
<td>1.06 ± 0.16</td>
<td>3.0</td>
<td>0.9 – 1.0</td>
<td></td>
</tr>
<tr>
<td>( \Xi^0 \to \Sigma^0\gamma )</td>
<td>3.56 ± 0.43</td>
<td>7.2</td>
<td>4.0 – 4.1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Sigma^+ \to p\gamma )</td>
<td>-0.76 ± 0.08</td>
<td>-0.801±0.032</td>
<td>-0.19</td>
<td>not simple</td>
</tr>
<tr>
<td>( \Lambda \to n\gamma )</td>
<td>-0.49</td>
<td>not simple</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Xi^0 \to \Lambda\gamma )</td>
<td>-0.65 ± 0.19</td>
<td>-0.78</td>
<td>-0.8</td>
<td></td>
</tr>
<tr>
<td>( \Xi^0 \to \Sigma^0\gamma )</td>
<td>-0.65 ± 0.13</td>
<td>-0.96</td>
<td>-0.45</td>
<td></td>
</tr>
</tbody>
</table>

6. Conclusions

The new NA48 result confirms Hara’s theorem. Consequently, large SU(3) breaking is needed to describe the \( \Sigma^+ \to p\gamma \) asymmetry. Furthermore, the CQM result appears as an artefact of the model. Thus, the CQM constitutes an abstraction from spin-flavor symmetries of hadronic amplitudes that goes too far. Since Hara’s theorem violation was also predicted in a symmetry-based framework which combined current-field identity with an approach describing p.v. couplings of vector mesons to nucleons (used in explanations of nuclear parity violation [19]), it follows that either current-field identity is not universal, or our present understanding of nuclear parity violation is not fully correct.

The proposed resolution of the problem of NLHD-WRHD symmetry connection implies that symmetry should be imposed at the level of ax-