Reheating from Tachyon Condensation

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We argue that it is possible to reheat the universe after inflation driven by D-brane annihilation, due to the coupling of massless fields to the time-dependent tachyon condensate which describes the annihilation process. This mechanism can work if the original branes annihilate to a stable brane containing the standard model. Given reasonable assumptions about the shape of the tachyon background configuration and the size of the relevant extra dimension, the reheating can be efficient enough to overcome the problem of the universe being perpetually dominated by cold dark tachyon matter. In particular, reheating is most efficient when the final brane codimension is large, and when the derivatives of the tachyon background are large.

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I. INTRODUCTION

Because of the difficulty of finding direct laboratory probes of string theory, it is interesting to look for possible evidence from cosmology. Most notably, inflation may be tested more sensitively in the near future by the MAP [1] and PLANCK [2] observations of the cosmic microwave background radiation. Recently there has been significant progress in constructing stringy inflation models which make use of naturally occurring potentials between D-branes to provide the false vacuum energy [3,4]. However, these attempts have not adequately addressed the question of how reheating occurs after inflation. In fact, there are reasons to fear that reheating may be generically difficult to achieve in D-brane inflation. For this reason, we aim to propose a generic mechanism for reheating in such models, which is qualitatively different from reheating in ordinary field-theory inflation models, and which has the hope of being fairly robust. It is based on particle production in a time-varying background [25], which will occur even if the background motion is not oscillatory.

Let us begin by describing the difficulty with reheating. In the simplest version of D-brane inflation, a parallel brane and antibrane begin with some separation between them in one of the extra dimensions required by string theory. Although parallel branes are supersymmetric and have no force between them, the brane-antibrane system breaks supersymmetry so that there is an attractive force and hence a nonvanishing potential energy. It is the latter which drives inflation. Once the branes have reached a critical separation, they become unstable to annihilation. The instability is described by condensation of a tachyonic mode [5]. Its low energy effective description is a field theory of a peculiar kind, whose Lagrangian has the form [6,7]

\[ \mathcal{L} = -T e^{-T^2/\alpha^2} F[-(\partial_\mu T)^2] \]  

where

\[ F(x) = \frac{4^2 x \Gamma(x)^2}{2 \Gamma(2x)} \]  

which is determined for the superstring from the boundary string field theory (BSFT). Here \( T \) is the sum of the original brane tensions and \( a \) is of order the string length, \( a \sim \sqrt{\alpha'} \) (the precise value to be used in the model we adopt will be discussed in section IV). The tachyon starts from the unstable maximum \( T = 0 \) and rolls to \( T \rightarrow \infty \). This process requires an infinite amount of time, during which the tachyon fluid has an equation of state identical to that of pressureless dust as \( \dot{T} \rightarrow 1 \) [8,9].

(There have been numerous recent attempts to make use of the tachyon fluid for cosmology [10], either as the inflaton or as quintessence. Although these ideas might work if one had the freedom to change the form of the tachyon potential, the action which arises from string theory is not suitable for either purpose. Since the late-time

\[^{1}\text{we use the metric signature } (1, -1, \ldots, -1)\]
The equation of state is $p = 0$, the string theory tachyon does not provide accelerated expansion in the recent universe. At early times, its potential is too steep to satisfy the requirements of inflation. Constraints on tachyon cosmology and its shortcomings have been discussed in [11] and [12].

Returning to our description of the endpoint of D-brane inflation, the situation is similar to that of hybrid inflation [13], where the tachyon plays the role of the unstable direction in field space which allows for inflation to quickly end. The important difference is that in a normal hybrid inflation model, $T$ would have a minimum at some finite value, e.g., due to a potential like $\lambda (|T|^2 - a^2)^2$, and the oscillations of $T$ around its minimum could give rise to reheating in the usual way, or the more efficient tachyonic preheating [14]. But with the exponential potential in (1) there can be no such oscillations. It thus appears that the universe will become immediately dominated by the cold tachyon fluid, and never resemble the big bang [12].

On the other hand, it is known that the tachyon couples to massless gauge fields; one form that has been suggested for the low energy theory is the Dirac-Born-Infeld action [15,16],

$$\mathcal{L} = -Te^{-T^2/a^2} \sqrt{|\det(g_{\mu \nu} + F_{\mu \nu} - \partial_\mu T \partial_\nu T)|}$$  \hspace{1cm} (3)

Because of its time dependence, we expect that some radiation will be produced by the rolling of the tachyon. It then becomes a quantitative question: can this effect be efficient enough to strongly deplete the energy density of the tachyon fluid, so that the universe starts out being dominated by radiation rather than cold dark matter? There is one immediate problem with this idea, however; the fact that the entire action is multiplied by the factor $e^{-T^2/a}$ means that the massless particles which are produced will not act like ordinary radiation [17]. Recent work has shown that these excitations have the same equation of state as the tachyon itself [18]. This is related to the fact that the system is annihilating to the closed string vacuum, which does not support any open string states like the gauge fields.

An obvious way to circumvent the above difficulty is to assume that the branes annihilate not to the vacuum, but rather to a brane of lower dimension [19]. This is a natural possibility since the final outcome is determined by a conserved Ramond-Ramond charge. The initial state could have a nonvanishing charge if, for example, the original $Dp$-brane had a $D(p-2)$ brane “dissolved” within it. In the effective description, the lower dimensional brane is represented by a solitonic configuration of the tachyon field. A $D(p-2)$ brane is a vortex of the complex tachyon field, whereas a $D(p-1)$ brane is a kink [6,7,20]. These possibilities are described by the descent relations of Sen [19]. The stable descendant brane is able to support open string excitations of gauge fields, including those of the standard model. They will no longer decouple as a consequence of the rolling tachyon because the topological defect which prevents pins $T = 0$ at the origin.

In this paper we will set up a simplified model of particle production by tachyon condensation, which we hope captures the essential features of a more realistic Lagrangian. The computational method developed here should carry over straightforwardly to more complicated situations. In section II we motivate an ansatz for the space- and time-dependent tachyon background which describes condensation to a brane of one dimension lower than the initial configuration. In section III we present the solutions for a gauge field in this classical tachyon background. Section IV describes how we compute the spectrum of particles produced during the early phase of the tachyon motion, and presents numerical results. We conclude in section V with the interpretation of these results and a discussion of how the calculation can be improved in future work.

II. TACHYON BACKGROUND

There are two kinds of tachyonic solutions which have been described in the literature: (1) static solutions which are topological defects and represent lower dimensional branes, and (2) dynamical solutions which are spatially homogeneous and describe a cosmological fluid with vanishing pressure at late times. The mechanism we have in mind combines these two pictures by supposing that at $t = 0$ the tachyon starts from (or very near to) the unstable

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This form is not the same as what one derives from BSFT. However it has the right qualitative features, besides being simpler to work with. Ignoring for a moment the gauge field, it can be shown that $\sqrt{|\det(g_{\mu \nu} - \partial_\mu T \partial_\nu T)|} = 1 - T^2$ for time-dependent configurations, so that the action vanishes as $T \to 1$. The same is true for the BSFT-derived function $F(-T^2)$ in (2). Moreover both functions behave like $\sqrt{\left(\partial_t T\right)^2}$ for spatially dependent profiles in the limit that $\partial_t T \to \infty$, whose relevance will be explained in the next section.
equilibrium configuration $T = 0$ throughout some number $d$ of extra dimensions which will be transverse to the final descendant brane. We denote the spatial coordinates of these extra dimensions by $x_1, \ldots, x_d$. Starting at $t = 0$, the tachyon starts rolling towards $T \to \infty$ for $|x_i|$ sufficiently far from the center, whereas it is pinned to $T = 0$ at $x_i = 0$. At any given time, the extra dimensions will contain an interior region where the defect resides, and an exterior one where the tachyon is still rolling. We might expect the tachyon profile to evolve with time similarly to fig. 1(a).

![Figure 1](image)

**Figure 1.** (a) Possible time evolution of the tachyon field $T$ during condensation. (b) Our ansatz for the tachyon evolution in spacetime: region III contains a linear kink, II contains the homogeneous rolling field, and I is the unstable vacuum configuration.

To find the real behavior of $T(t, x_i)$ we should solve the equation of motion for $T$ which follows from eq. (1). This is already a difficult numerical task in itself, and its results would surely complicate our next step, which will be to solve the gauge field equation of motion in the background $T(t, x_i)$. Therefore we are going to satisfy ourselves with a simplified ansatz for the background and defer more detailed investigations to future work. Namely, we will consider a linear profile for the defect, and a purely time-dependent one in the exterior:

$$T(x) = qx, \quad |x| < t/q \quad \text{(region III)}$$
$$T(t, x) = a \ln(\cosh(t/a))\text{sgn}(x), \quad |x| > t/q \quad \text{(region II)}$$
$$T = 0, \quad t < 0 \quad \text{(region I)}$$

We have specialized to the case of a single codimension for the descendant brane ($p = 1$) and this dimension has been compactified by identifying the points $x = L$ and $x = -L$. A parameter $q$ describing the steepness of the tachyon kink profile has been introduced. One might expect that $q = 1$ if the news of the kink formation propagates into the bulk at the speed of light. The spacetime regions I-III are illustrated in figure 1(b).

One may wonder to what extent the ansatz (4-6) reproduces a more realistic tachyon background. Let us first consider the rolling tachyon region (II). Eq. (5) is in fact an exact solution not for the action (1) but rather for a mutilated version in which the argument of the exponential is linear:

$$L = -Te^{-|T|}/a \sqrt{|\det (g_{\mu\nu} - \partial_\mu T \partial_\nu T)|}$$

This closely resembles the BSFT result for the effective action from the bosonic string, except for having $|T|$ rather than $T$ in the exponent. The bosonic string has an unphysical instability as $T \to -\infty$ which we artificially remove by taking the absolute value. The behavior of the solutions to (7) is similar to that coming from a potential with a quadratic argument, which is the appropriate one for the superstring. For example, the late-time behavior corresponding to the Lagrangian $L = Te^{-T^2/a^2}(1 - T^2)^b$ is

$$T(t) \equiv t + \frac{a^2(1 - b)}{4t}e^{-t^2/(a^2(1-b))} + O(e^{-2t^2/(a^2(1-b))})$$

assuming that $b < 1$. (This can most easily be derived by computing the corresponding Hamiltonian density and using energy conservation to get a first integral of the motion.) Similarly, if we expand (5) for large times, we get
\[ T(t) \cong t + ae^{-2t/a} + O(e^{-4t/a}) \]  

which resembles (8). Moreover if we compute the relevant factor \( \sqrt{1 - T^2} \) in the two cases (now taking \( b = 1/2 \)),

\[
\sqrt{1 - T^2} = e^{-t^2/a^2} + O(e^{-2t^2/a^2}), \quad e^{-t/a} + O(e^{-2t/a})
\]

respectively. In other words \( \sqrt{1 - T^2} \) is just equal to the other prefactor \( e^{-t^2/a^2} \) or \( e^{-t/a} \), as the case may be. This can easily be understood from the fact that the conserved Hamiltonian is \( V(T)(1 - T^2)^{-1/2} \), hence \( V(T) \) and \( \sqrt{1 - T^2} \) must be proportional to each other. It will turn out that the simplified form \( e^{-t/a} \) makes it easier to solve for the gauge field in the next section, which is one of our motivations for adopting the action (7).

Next we discuss the ansatz for the kink profile, \( T = qx \), in region III. This static profile is not a solution to the equations of motion\(^3\) except in the limit that \( q \to \infty \). If we consider a generalized action of the form \( \mathcal{L} = V(T)(1 - T^2)^b \), then the linear profile is a solution only if \( q^2 = 1/(2b - 1) \). The limit \( q \to \infty \) has been discussed in [6,7], where it was noted that the descendant brane resulting from tachyon condensation has the right tension to agree with string theory when \( q \to \infty \). In this limit the descendant brane looks like a genuinely lower dimensional object, whereas it would have a nonvanishing thickness (revealed by the energy density of the profile) for finite values of \( q \). However a different method, that of level-truncation [21,22], leads to tachyon defect profiles which do have a nonzero width. In any case, there is no reason to believe that the kink thickness goes to zero immediately; it seems quite reasonable to assume that it will have some nonzero value initially, and possibly tend toward zero only as \( t \to \infty \). We will assume that \( q \) remains approximately constant for \( 0 < t < qL \) since this is the time during which particles are produced. If \( q \to \infty \) at later times, it will not significantly change our conclusions.

A further issue is the shape of the spacetime boundary separating the static kink from the time-dependent solution. We have taken it to be linear, \( t = q|x| \), but this implies that \( T \) is not even continuous at the interface except for \( t \gg a \). It might seem more reasonable to deform the boundary such that \( e^{ix/a} = e^{t/a} + e^{-t/a} \). We could do so, but it would needlessly complicate the ansatz without even solving the problem of the tachyon Lagrangian being discontinuous at the interface, because the factor \( \sqrt{\det(g_{\mu\nu} - T_{\mu\nu})} = \sqrt{1 - (\partial_\mu T)^2} \) is still not smooth: \( (\partial_\mu T)^2 \) changes sign as well as magnitude across the interface:

\[
(\partial_\mu T)^2 \cong \begin{cases} 
1 - e^{-2t/a}, & \text{region II} \\
-q^2, & \text{region III} 
\end{cases}
\]

This is an indication that our ansatz (4-6) is defective; the true solution should have a smooth Lagrangian density at the transition. Nevertheless, it would greatly complicate the solution for the gauge fields in the next section if we tried to smooth out this behavior. We will instead compensate for the difficulties which arise from this oversimplification in another way, as will be explained.

We found it simpler to consider condensation to a kink instead of a vortex configuration of the tachyon. The latter would be more realistic because of the fact that, assuming the parent branes are supersymmetric BPS states, a brane of only one dimension less is not, and consequently not stable, whereas a brane whose dimensionality differs by an even number \textit{is} BPS. To reduce the dimensionality by two, the tachyon should be a complex field which condenses to a codimension-two defect, a vortex. Although we originally wanted to treat this case, it is not clear how to write the action of the complex tachyon in a way which extends to the time-dependent configurations of region II. BSFT calculations show that for a winding configuration of the form

\[ T = q_1x_1 + iq_2x_2 \]

the Lagrangian factorizes as [7]

\[ \mathcal{L} \propto -F(q_1^2)F(q_2^2) \]

\[ \text{an exact solution can be obtained from that of region II by analytically continuing } t \to ix, \text{ giving} \]

\[ T = a \text{sgn}(x) \ln \left( \frac{\cos((L - x)/a)}{\cos(L/a)} \right), \]

where we have imposed continuity of \( T' \) at \( x = \pm L \) and assumed that \( L/a \leq \pi/2 \). The solutions for the gauge field are complicated in this background, so we prefer to use the linear kink profile for this paper.
where $F(x) = \frac{4\epsilon x \Gamma(y)^2}{2(2x)^3}$. It is not immediately obvious how to rewrite (14) in a Lorentz and gauge invariant way which would allow us to deduce the equations of motion for time-dependent homogeneous configurations. Explicit constructions involve the use of independent tensors like $\partial_\mu T^\nu \partial_\rho T$ and $\partial_\mu T \partial_\rho T$ in matching powers of derivatives, so that a compact expression like (7) does not seem to emerge. We leave the consideration of these complications for future work.

That being said, the kink configuration may still be physically relevant because of the possibility of descending from the original $D_0 = T_0 \rho$ pair in two steps, with the unstable $D(p - 1)$ brane being a resonance through which the system passes on its way to the stable $D(p - 2)$ endpoint [23]. The transition from $D(p - 1)$ to $D(p - 2)$ would be described by the formation of a kink.

Finally, let us consider the fact that the rolling phase of the tachyon field ends in a finite time within our ansatz. Fig. 1(b) shows that at late times we have eliminated the homogeneous condensate by fiat since at $t = qL$ the entire bulk has been replaced by the static kink. We don’t know whether this is the actual behavior, or if the homogeneous region persists, which could be the case if $q$ grows with time:

$$T(t, x) = q(t)|x|, \quad |x| < t/q(t) \quad \text{(region III)}$$

$$T(t, x) = a \ln(\cosh(t/a)) \text{sgn}(x), \quad |x| > t/q(t) \quad \text{(region II)}$$

If $q(t)$ grows with $t$ faster than linearly, then region II survives and the tachyon fluid coexists with the final state brane at arbitrarily late times. We believe that the present calculation could give a reasonable approximation to the efficiency of particle production even in this case. The compactification length $L$ will be replaced by the size of region III at the characteristic time scale when the fast roll phase ends (i.e. when $T \equiv 1$ in the bulk). On the other hand, if $q(t)/t \to 0$ as $t \to \infty$, then as long as the bulk is compact, region II disappears completely. In this case it is still important to consider particle production on the brane since otherwise the energy that was stored in the rolling tachyon might go into invisible closed string modes in the bulk, namely gravitons, which would not be an acceptable form of reheating.

### III. GAUGE FIELD SOLUTIONS

Our aim is to find out whether the energy stored in the homogeneous tachyon fluid can be efficiently converted into radiation, so that the universe at least has a long period of radiation domination before possibly giving way to the cold dark matter of the rolling tachyon condensate. We will do this by quantizing the gauge field in the tachyon background and computing the Bogoliubov coefficients that quantify the mismatch between the vacuum states of regions I and III (see fig. 1(b)); see for example [24,25]. That is, if we start in the vacuum state appropriate for region I, we find that it is no longer the vacuum in region III, and therefore radiation must be produced.

The first step is to find the action for the gauge fields to quadratic order in the fields. Expanding (7) in the tachyon background described in the previous section, we obtain

$$S = \frac{1}{2} T \int dt \, dx \, d^3 y \left\{ \begin{array}{ll}
(\partial_\mu \tilde{A})^2 - (\nabla_\mu \tilde{A})^2 - (\partial_\mu \tilde{A})^2, & \text{region I} \\
(\partial_\mu \tilde{A})^2 - e^{-2t/a} (\nabla_\mu \tilde{A})^2 (\partial_\mu \tilde{A})^2, & \text{region II} \\
\sqrt{1 + q^2 e^{-g|x|/a}} (\partial_\mu \tilde{A})^2 - (\nabla_\mu \tilde{A})^2 - \frac{1}{1 + q^2} (\partial_\mu \tilde{A})^2, & \text{region III} 
\end{array} \right.$$

where $y_i$ are the coordinates of the large 3 dimensions, and we have absorbed the volume of any other compact dimensions which are merely spectators into the brane tension $T$. We employed radiation gauge ($A_0 = \nabla_\mu \cdot \tilde{A} = 0$) and projected out the extra polarization by setting $A_x = 0$, which is consistent since this state turns out to have a mass gap in region III, unlike the massless components among the large three dimensions, $\tilde{A}$. (The factor $e^{-2t/a}$ in region II should really be $\cosh(t/a)^{-2}$, but this not an important difference, in the spirit of the other approximations we have made.) In the following we will drop the polarization indices of the gauge field and write simply $\tilde{A}$ instead of $\tilde{A}$.

To make our analysis more tractable, we have ignored the time dependence of the metric due to the expansion of the universe in the above action. This neglect can be justified if the initial fast-roll regime of the tachyon, during which most of the particle production occurs, does not take more than approximately one Hubble time. We expect that this will be true if the string scale is somewhat below the Planck scale and strings are weakly coupled, since then $H \sim \sqrt{2LT}/M_p^2 \sim g_s M_s/(2\pi M_p)$ [using the relation $g_s^2 M_p^2 = M_s^4 V/(2\pi)^9$ [4] and eq. (34)], whereas the time scale for the tachyon roll is of order the string scale.
In the previous section we gave detailed motivations for our choice of the ansatz for the classical tachyon background. It is worth emphasizing one further criterion: by choosing $T(t,x)$ to depend only on $t$ or $x$ in each region, we ensure that the gauge field equations of motion can be solved using separation of variables, which greatly simplifies the task.

A. Solutions in each region

The solutions in region I are trivial since the background tachyon configuration is simply $T = 0$:

$$A_I = \frac{1}{\sqrt{4L}} \sum_m \frac{1}{\sqrt{2\omega_m}} \left[ (a_m e^{-i\omega_m t} + a_m^* e^{i\omega_m t}) \cos(k_m x) + (a_m e^{-i\omega_m t} + a_m^* e^{i\omega_m t}) \sin(k_m x) \right]$$  \hspace{1cm} (18)

where $k_m = m\pi/L$, $\omega_m^2 = \bar{p}^2 + k_m^2$, and $\bar{p}$ is the momentum in the three large dimensions. We have split the modes according to their parity in the extra dimension for later convenience. In region II, the equation of motion for $A$ is

$$\ddot{A} + \omega_m^2 e^{-2t/a} A = 0,$$  \hspace{1cm} (19)

which has the solutions

$$A_{II} = \frac{1}{\sqrt{4L}} \sum_m \left[ (b_m J_0(a\omega_m e^{-t/a}) + c_m Y_0(a\omega_m e^{-t/a})) \cos(k_m x) + \text{same with } b_m \rightarrow \bar{b}_m, c_m \rightarrow \bar{c}_m, \cos(k_m x) \rightarrow \sin(k_m x) \right]$$  \hspace{1cm} (20)

Near $t = 0$, these oscillate just like the region I solutions, but at large $t$ the oscillations freeze and the solutions grow linearly with time.

Region III is the important one at late times, since this is where the descendant brane and the standard model are supposed to reside. Here the equation of motion is

$$(1 + q^2) \left( -\dddot{A} + \nabla_y^2 A \right) + A'' - \frac{q}{a} \text{sgn}(x) A' = 0$$  \hspace{1cm} (21)

using primes to denote $\partial_x$. The solutions can be written as

$$A_{III} = \sum_n \frac{1}{\sqrt{2\omega_n}} \left[ (d_n e^{-i\bar{\omega}_n t} + d_n^* e^{i\bar{\omega}_n t}) f_n(x) + (\tilde{d}_n e^{-i\bar{\omega}_n t} + \tilde{d}_n^* e^{i\bar{\omega}_n t}) \tilde{f}_n(x) \right]$$  \hspace{1cm} (22)

with

$$\bar{\omega}_n = \begin{cases} \sqrt{\bar{p}^2 + \frac{1}{a} q^2 (\frac{q}{a^2} + k_n^2)} & n = 0 \\ \sqrt{\bar{p}^2 + \frac{1}{a} q^2 (\frac{q}{a^2} + k_n^2)} & n \geq 1 \end{cases}$$

$$f_n(x) = \begin{cases} N_0, & n = 0 \\ N_n e^{q|x|/2a} \left( \cos(k_n x) - \frac{q}{2k_n a} \sin(k_n x) \right), & n \geq 1 \end{cases}$$

$$\tilde{f}_n(x) = \begin{cases} \sqrt{\frac{2}{4L}} \left( 1 + \frac{q}{2k_n a} \right)^{-1/2} \left( 1 + q^2 \right)^{-1/4}, & n = 0 \\ \sqrt{\frac{2}{4L}} \left( 1 + \frac{q}{2k_n a} \right)^{-1/2} \left( 1 + q^2 \right)^{-1/4}, & n \geq 1 \end{cases}$$

and $k_n = n\pi/L$.

These solutions have the desirable property, from the point of view of string theory, that there is a zero mode accompanied by a tower of heavy states [20,23]. This is qualitatively similar to the spectrum of excited states of the open string, though we have sacrificed some of the similarity by taking the tachyon potential $e^{-|T|/a}$ instead of $e^{-T^2/a^2}$. With the latter choice one gets a more realistic spectrum of the form $\bar{\omega}_n^2 \sim \bar{p}^2 + n/a^2$, which has the correct $n$-dependence to match string theory. The disadvantage is that the solutions in region II cannot be found analytically since $\dot{A} + \omega_m^2 e^{-2t/a^2} A = 0$ is not a standard differential equation. We have therefore given up some of the quantitative similarities with the real theory for the sake of being able to go as far as possible analytically.

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To complete our task, we must relate the solutions in each neighboring region to each other at the interfaces $t = 0$ and $t = q|x|$. This will impose relations between $\alpha_n, d_n^\parallel$ and $b_n, c_m$ and between $b_n, c_m$ and $d_n, d_n^\parallel$. At $t = 0$ the procedure is straightforward; $A$ and $\partial_t A$ must be continuous, leading to the relations

$$\begin{pmatrix} b_m \\ c_m \end{pmatrix} = \frac{\pi a \sqrt{\omega_m}}{2 \sqrt{2}} \begin{pmatrix} -Y_1 - i Y_0 & -Y_1 + i Y_0 \\ J_1 + i J_0 & J_1 - i J_0 \end{pmatrix} \begin{pmatrix} a_m \\ a_m^\dagger \end{pmatrix}$$  \hspace{1cm} (24)

where the Bessel functions are all evaluated at $a \omega_m$. We used the Wronskian of $J_0$ and $Y_0$ to obtain (24). Notice that there is no mixing between different $m$ values at this point.

Matching at the $t = q|x|$ interface is more difficult. In this case we also demand continuity of $A$, but the fact that the prefactor $\sqrt{1 - (\partial_t A)^2}$ in the action is discontinuous means that derivatives of $A$ are discontinuous as well. By integrating the partial differential equation for $A$ along an infinitesimal path which crosses the interface perpendicularly, we can show that the following linear combination of derivatives must be continuous at the interface:

$$\Delta(F_t \dot{A} + q F_x A') = 0 \text{ from II to III}$$ \hspace{1cm} (25)

where $F_t$ is the coefficient of $\dot{A}^2$ and $F_x$ is that of $A^2$ in the action. Namely, $F_t = 1$ in region II and $\sqrt{1 + q^2 e^{-q|x|/a}}$ in III, while $F_x = e^{-2|x|/a}$ in II and $e^{-q|x|/a}/\sqrt{1 + q^2}$ in III. This gives the rather cumbersome matching conditions

$$\frac{1}{\sqrt{4L}} \sum_m \left[ b_m J_0(a \omega_m e^{-q|x|/a}) + c_m Y_0(a \omega_m e^{-q|x|/a}) \right] \cos(k_m x) = \sum_n \frac{1}{\sqrt{2 \omega_n}} \left[ d_n e^{-i \omega_n q |x|} + d_n^\dagger e^{i \omega_n q |x|} \right] f_n(x)$$ \hspace{1cm} (26)

for $A$ itself and

$$\frac{1}{\sqrt{4L}} \sum_m \left[ \omega_m \left( b_m J_1(a \omega_m e^{-q|x|/a}) + c_m Y_1(a \omega_m e^{-q|x|/a}) \right) \cos(k_m x) - q k_m e^{-q|x|/a} \left( b_m J_0(a \omega_m e^{-q|x|/a}) + c_m Y_0(a \omega_m e^{-q|x|/a}) \right) \sin(k_m x) \right]$$

$$= \sum_n \frac{1}{\sqrt{2 \omega_n}} \left[ -i \omega_n \sqrt{1 + q^2} \left( d_n e^{-i \omega_n q |x|} - d_n^\dagger e^{i \omega_n q |x|} \right) f_n(x) + \frac{q}{\sqrt{1 + q^2}} \left( d_n e^{-i \omega_n q |x|} + d_n^\dagger e^{i \omega_n q |x|} \right) f'_n(x) \right]$$ \hspace{1cm} (27)

for the derivatives of $A$. Similar conditions hold among the odd parity modes $[a, d \rightarrow \bar{a}, \bar{d}, f_n \rightarrow \bar{f}_n, \cos(k_m x) \rightarrow \sin(k_m x)]$, which do not mix with those of even parity.

C. Solution of matching conditions

The technical challenge is now to solve for $d_n, d_n^\parallel$ in terms of $b_n, c_m$. Normally this would be done by taking the inner product of each equation (26, 27) with some function which is orthogonal all but one of the functions multiplying $d_n$ or $d_n^\parallel$. But because of the diagonal nature of the II/III interface, the latter functions are products of orthogonal functions, which are no longer orthogonal. This makes it impossible to solve for $d_n, d_n^\parallel$ analytically.

We should therefore choose some set of basis functions $g_i(x)$ and integrate them against the matching conditions (26-27) to transform the latter into the discrete form

$$\sum_n L_{in} \begin{pmatrix} d_n \\ d_n^\parallel \end{pmatrix} = \sum_m R_{im} \begin{pmatrix} b_m \\ c_m \end{pmatrix}$$ \hspace{1cm} (28)

We were not able to identify any set of $g_i(x)$ that makes the matrix $L_{in}$ even approximately diagonal (which would facilitate an analytic approximation), making a numerical solution necessary. After some experimentation, one realizes that the most efficient way to do this is to discretize the system on a spatial lattice at positions $x_i$, so that $g_i(x) = \delta(x - x_i)$. This allows us to compute the matrices $L_{in}$ and $R_{im}$ without having to perform any integrals. An ultraviolet cutoff is thus introduced. Let $i = 0, \ldots, N$ so that $x_i = iL/N$. It makes sense to let the mode number of the spatial eigenfunctions $f_n(x)$ also range from 0 to $N$; then (28) gives exactly as many equations as unknowns. The system can be solved by numerically inverting the matrix $L_{in}$ [26]. The results are presented in the next section.
Our solutions for the gauge field in regions I and III are normalized so that $a_m$, $a_m^\dagger$, $d_m$, $d_m^\dagger$ are the correct creation and annihilation operators for particles in the distant past and future once the gauge field is quantized. We assume the universe starts in the vacuum state $a_n |0\rangle = 0$. But this state is not annihilated by $d_n$, since the latter is a superposition of $a$, $a^\dagger$ determined by the Bogoliubov coefficients $\alpha$ and $\beta$,

$$
\begin{pmatrix}
    d_n \\
    d_n^\dagger
\end{pmatrix} = \sum_m \begin{pmatrix}
    \alpha_{nm} & \beta_{nm}^* \\
    \beta_{nm} & \alpha_{nm}^*
\end{pmatrix} \begin{pmatrix}
    a_m \\
    a_m^\dagger
\end{pmatrix}
$$

(29)

Therefore observers in the future will see a spectrum of particles in the final state given by

$$
\mathcal{N}_n = \langle 0 | d_n^\dagger d_n | 0 \rangle = \sum_m |\beta_{nm}|^2
$$

(30)

In this section the numerical results for $\mathcal{N}_n$ and the total energy density of produced radiation will be presented.

A. UV sensitivity

Ideally one should obtain convergent results in the limit that $N \to \infty$, where we recall that $N$ is the number of sites of the spatial lattice in the extra dimension, introduced in the previous section. However we do not observe this from our numerical results; rather there is a steady growth with $N$. Our conjecture is that this is related to the discontinuity in the action which arises from the sign change in $(\partial_{\mu} T)^2$ across the II/III interface. In a simpler situation where particles are produced due to a (spatially constant) time-varying background, the spectrum is proportional to the square of the Fourier transform of the background. If the latter has sharp features, the spectrum falls only as a power of energy, whereas a smooth background leads to exponential suppression of high energies. For example, the Fourier transform of $e^{-t\Lambda}$ times a step function of time is $(\Lambda + i\omega)^{-1}$ whereas the Fourier transform of $(1 + t^2 \Lambda^2)$ is proportional to $e^{-|\omega|/\Lambda}$. In the former case, summing over high frequency modes could lead to nonconvergent results. We expect the high frequency contributions to be suppressed by a factor like $e^{-|\omega|/\Lambda}$ if we had a more realistic tachyon profile whose derivative changed smoothly over a time $1/\Lambda$. We have not yet found a way of altering $T(t,x)$ to incorporate this behavior while still allowing us to solve for $A(t,x)$ analytically.

To remove sensitivity to the lattice spacing, we therefore try to model the expected effects of smoothing out the background tachyon solution by inserting a convergence factor by hand, so that (28) becomes

$$
\sum_n L_{\text{in}} \begin{pmatrix}
    d_n \\
    d_n^\dagger
\end{pmatrix} = \sum_m \sum_{\beta_n} e^{-\omega_m/\Lambda} \begin{pmatrix}
    b_m \\
    c_m
\end{pmatrix}
$$

(31)

Once this is done, the limit $N \to \infty$ is well-behaved. We have thus essentially traded the original ultraviolet cutoff $N/L$ for a new one, $\Lambda$, whose physical meaning is more transparent. Figure 2(a) shows the dependence on $N$ of the low-momentum zero-mode production, $\mathcal{N}_0(p_0)$, at $p_0 = 0.05/a$, for several values of $\Lambda$ and $q$ (recall that the latter parameter determines the slope of the interface between spacetime regions II and III). The convergence with $N$ is faster for smaller values of $q$; numerical limitations therefore prevent us from accurately studying large values of $q$. Fig. 2(b) shows the dependence of zero-mode production on moderate values of $q$; large values of $q$ at fixed $N$ give exponentially increasing results due to the term $Y_1(a\omega_m e^{-qx/a})$ in (27), but these nevertheless become well-behaved again as $N$ is increased to sufficiently large values. On physical grounds one might anyway expect $q \leq 1$ due to the finite speed of propagation of the kink.

B. Spectra

We note that each state in region III is distinguished not only by its mode number $n$, but also its momentum $\vec{p}$ in the large dimensions, which we have until now suppressed except for its appearance in the energies $\tilde{\omega}_n$, eq. (23). In figure 3 we show the dependence of $\mathcal{N}_n(p)$ on these two variables. The spectrum of zero modes falls monotonically with $p$ as expected. Although the nonzero mode spectra temporarily rise with $p$, their contributions to the total energy density are smaller than that of the zero mode. We have taken $q = 1$ and $\Lambda = 1/a$ for the parameters controlling the
steepness the kink in spacetime and the suddenness with which it forms. The dependence on the size of the bulk \( L \) can be seen in fig. 3(b):

Here and in the remainder we have made the simplifying approximation of ignoring the odd-parity modes and keeping only the even ones, which include the dominant zero mode. We expect that this gives a slight underestimate of the actual efficiency of particle production, by no more than a factor of order unity.

Figure 2. (a) Dependence of \( N_0(p_0) \) on \( N \) for \( p_0 = 0.05/a, L = 2a, \Lambda = 0.5/a, 1/a \) and \( q = 0.5, 1, 2 \). b) Dependence of \( N_0(p_0) \) on \( q \) for \( N = 100, 200 \), at \( p_0 = 0.01/a, L = 2a \) and \( \Lambda = 1/a \). Physically meaningful results correspond to the \( N \to \infty \) limit.

To find the total energy density of produced radiation, we should sum over both \( n \) and \( p \):

\[
\rho_r = \int dp \frac{d\rho_r}{dp} \equiv \sum_n \int \frac{d^3p}{(2\pi)^3} N_n(p)
\]

The heavier modes are counted because they will presumably decay very quickly into massless standard model particles. In figure 4 we show the differential energy density, \( \frac{d\rho_r}{dp} \), as a function of momentum, for several values of the other parameters.

Figure 3. (a) Momentum dependence of the spectrum of produced particles, \( N_n(p) \) for the string excitation quantum numbers \( n = 0, \ldots, 5 \). (b) \( N_n(p) \) as a function of \( n \) for \( L = 8a, 4a, 2a, a \) and \( p = p_0 \equiv 0.01/a \). Both graphs are for \( q = 1, \Lambda = 1/a \).
We turn now to the main results, the total energy density $\rho_r$ of produced radiation, obtained from integrating eq. (32). In figures 5(a-c) we graph the dimensionless combination $\ln(a^4\rho_r)$ as a function of the main unknown parameters, $q$, $\Lambda$ and $L$. It is clearly an increasing function of all three parameters. The “critical value” shown in these figures, which we would like $\rho_r$ to exceed, is derived in the next subsection. We will argue that this is the value at which reheating starts to significantly deplete the energy stored in the rolling tachyon fluid.

Figure 4. Differential energy density of produced radiation as function of $p$. Fiducial values of parameters are $L = a$, $\Lambda = 1/a$ and $q = 1$.

Figure 5(a) $\ln(a^4\rho_r)$ versus $a\Lambda$ for $L = a, 4a, 8a$. (b) $\ln(a^4\rho_r)$ versus $q$ for several values of $L$ and $\Lambda$.

Figure 5(c). $\ln(a^4\rho_r)$ versus $L/a$ for $\Lambda = 2/a$, $1/a$ and $0.5/a$, with $q = 1$. 
Our goal is to see if the produced $\rho_r$ is a large enough fraction of the energy density which is available from the homogeneous rolling tachyon fluid, $\rho_T$. We thus need to specify the value of $\rho_T$ which is predicted by string theory. In terms of the effective tension of the original brane and the size of the extra dimension,

$$\rho_T = 2LT$$

(33)

By the effective tension, $T$, we mean the energy density in the $(4 + 1)$-dimensional spacetime which includes $x$. If we started from a non-BPS $Dp$-brane, then $T = \sqrt{2T_p V}$ where $V$ is the volume of compact dimensions within the brane excluding $x$, and $T_p$ is the tension of a BPS $Dp$-brane,

$$T_p = \frac{1}{gs}(2\pi)^{-p}\sqrt{\alpha'}^{-(p+1)}$$

(34)

Therefore $\rho_T$ depends on $g_s$, $V$, $L$ and $\alpha'$. However not all of these are independent; they are related to the fine structure constant of the gauge coupling evaluated at the string scale $M_s = (\alpha')^{-1/2}$ [4):

$$\alpha(M_s) = \frac{g_s(2\pi\sqrt{\alpha'})^{p-3}}{2(2LV)}$$

(35)

Interestingly, if we use this to eliminate the $V$ dependence from (33) we find that the dependence on the string coupling and on $L$ is also gone:

$$\rho_T = \frac{M_s^4}{\sqrt{2(2\pi)^3}\alpha(M_s)}$$

(36)

To find out how much energy is available for reheating, we must also consider the descendant brane’s 4-D energy density, $\rho_f$. This is obtained by integrating over the extra dimension in (7) for the kink profile $T = qx$ and taking the limit $q \to \infty$. The result is simply $\rho_f = 2aT$. Therefore the excess energy density which can be used for reheating is

$$\Delta\rho = \rho_T - \rho_f = \rho_T \left(1 - \frac{a}{L}\right)$$

(37)

Hence we should demand that $L > a$ to get any reheating at all.

In order to compare $\Delta\rho$ to the energy density of produced radiation, we need to know the parameter $a$ in terms of the string length. In our simplified model of tachyon condensation, this can be determined by demanding that the tension of the descendant $(p-1)$-brane match the string theoretic value for a BPS state, given by (34) with $p \to p - 1$. In the limit $q \to \infty$, the ratio of the initial and final brane tensions in our field theory model is $\sqrt{1 + q^2 \int_{-\infty}^{\infty} e^{-q|z|/a}\frac{dz}{a}}^{-1} = 1/2a$, whereas the string theoretic value is $\sqrt{2}M_s/(2\pi)$. We therefore obtain

$$a = \pi\sqrt{\alpha'/2} = \frac{\pi}{\sqrt{2}M_s}$$

(38)

Now we can quantify the efficiency of reheating. Our results for $\rho_r$ are expressed in units of $a^{-4}$. Using (38) we can convert the available energy density $\Delta\rho$ into the same units to find the critical value of $\rho_r$, call it $\rho_c$, for which the conversion into radiation would be 100% efficient:

$$\rho_c = \frac{\pi}{32\sqrt{2}\alpha(M_s)a^{-4}} \approx 1.7a^{-4}$$

(39)

We take the fine structure constant to be $1/25$, and we omit the factor $(1 - a/L)$ since it would require fine tuning of $L$ to make it very relevant. The figure of merit for reheating is therefore $\rho_r/\rho_c$. The critical value $\rho_c$ appears as a dotted line in figures 5(a-c). We see that for large enough values of $\Lambda$, $q$, or $L$, reheating can be efficient.
V. CONCLUSION

We have made a case for reheating the universe after brane-antibrane inflation by production of massless gauge particles, due to their coupling to the fast-rolling tachyon field which describes the instability of the initial state. Our results are encouraging, indicating that if the extra dimensions within the original branes but transverse to the final one are large enough compared to the string length scale $\sqrt{\alpha'}$, reheating can be efficient enough so that radiation dominates over cold dark tachyon matter. Depending on details of the tachyon background and the compactification, an extra dimension of size $\sim 10\sqrt{\alpha'}$ could be sufficient. (For example, with $L = 4a$ in an orbifolded compactification, the size of the extra dimension would be $4\pi/\sqrt{2}M_p^{-1} \approx 9\sqrt{\alpha'}$. In the present analysis we counted only a single polarization of one $U(1)$ gauge boson and ignored the odd parity modes in the extra dimension, so the real situation could be less constrained.

The calculation is complicated, and we have made it tractable by invoking a number of simplifying assumptions. We considered formation of a kink in the tachyon field rather than a vortex. We used a simplified version of the tachyon action rather than the one which has been derived in BSFT. We used an ansatz for the background tachyon field which is a good approximation to the actual solutions in the regions where it depends only on space (the kink) or on time (the homogeneous roll), but we do not know how suddenly it makes the transition between these two regimes, hence we parametrized this uncertainty by introducing a cutoff $\Lambda$ on the tachyon field’s derivative. We have also left the slope $q$ of the tachyon kink profile $T = qx$, during kink formation, as a free parameter. Moreover we have ignored the expansion of the universe, which is only correct if the brunt of the tachyon roll completes in a time not much exceeding the Hubble time. We have also ignored the back-reaction of the produced particles on the tachyon background, so our criterion of requiring the produced energy density to meet or exceed that which is initially available is a crude one.

To do a better job, more numerical computations will probably be necessary since it is hard to solve the gauge field equations of motion in a tachyon background that depends on both space and time. One check that might be carried out relatively easily is to numerically obtain the time- and space-dependent solution $T(x, t)$ for the more realistic action and compare its features to those we have assumed. This is in progress. Very recent work on how to generate time-dependent tachyon solutions in ref. [27] might prove to be helpful here.

As we were finishing this paper, a related proposal appeared [28]. The latter points out that the inflaton, which is related to the distance between the two original branes, may in fact continue to oscillate rather than halting at the end of inflation, leading to a more conventional kind of reheating or tachyonic preheating. The idea is in a similar stage of development to ours, requiring a more detailed calculation to establish that reheating is efficient enough. It will be interesting to compare the relative efficacy of the two mechanisms in a more complete investigation.

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