Below I review results obtained recently in theoretical models, which are in line with the experimental results in recent years. There is no direct experimental data on the cross section of the reactions $\pi^+\pi^- \to \pi^\pm\pi^\mp$ at low energy. However, there is a wealth of information about the cross section of these reactions presented in this paper. There is only very limited information at large energies, above those of the pion-nucleon scattering threshold.

I. Introduction

PACS numbers: 11.30.Fs, 13.70.Hb, 14.20.Lm, 14.60.Pq, 14.70.Rp, 14.80.Tz

\[ \pi^+\pi^- \to \pi^\pm\pi^\mp \text{ data} \]

The cross section data and theoretical predictions are discussed in this section. Theoretical models are presented, and the cross section data are compared with the predictions of these models. The results of the comparison are shown in the figures. The models are compared with the data, and the agreement or disagreement is discussed. The results of the comparison are presented in the figures. The agreement between the models and the data is good, and the models provide a good description of the data. The models are in agreement with the experimental data, and the predictions are consistent with the data.

Pion-Nucleon Scattering Above Resonances

\[ \pi N \to \pi N \text{ interaction} \]

\[ \pi^+\pi^- \to \pi^\pm\pi^\mp \text{ reactions} \]
2. The pQCD two-gluon exchange

Let us start with evaluation of the pQCD two-gluon contribution to the elastic $\pi\pi$ scattering. We treat the pion as the quark-antiquark state. The relevant pQCD diagrams which contribute to the pion impact factor are shown in Fig. 1. We use both nonrelativistic and the light-cone description of the pion \([4]\).

The impact factor representation of the pion-pion scattering amplitude reads

\[
A(q) = i\sigma \cdot \frac{2\mu}{g^2} \cdot \frac{1}{(2\pi)^2} \cdot \int d^2\kappa \ g_3^2(q_1^2)g_3^2(q_2^2)\Phi^{2g}_{\pi\pi\pi}(q, \kappa) \Phi^{2g}_{\pi\pi\pi}(\bar{q}, \bar{\kappa}) \frac{1}{(q/2 + \kappa)^2} \frac{1}{(q/2 - \bar{\kappa})^2},
\]

where $\kappa_{1/2} = \frac{q}{2} \pm \bar{\kappa}$ are the exchanged-gluon momenta, which are purely transverse, $2\mu/g$ is the QCD color factor for the $\pi\pi$ scattering and $g_3$ is the QCD strong charge.

At $|t| \gtrsim 2$ GeV$^2$ the hard contribution takes over the soft contribution to be discussed in more detail below. The result obtained with only impulse approximation terms \((a)\) and \((b)\) (dashed line) and with only Glauber-Gribov-Landshoff [5, 6, 7] terms \((c)\) and \((d)\) (dotted line) are shown separately for illustration in Fig. 2. The impulse approximation components of the impact factors dominate at small to moderate values of $|t| < 0.5$ GeV$^2$, where the nonrelativistic (NR) and light-cone (LC) amplitudes are nearly identical. A comparison of the NR and LC cases shows clearly a substantial suppression of the GGL contribution by the $q - z$ correlations inherent to the LC case. The GGL mechanism dominates at $|t| \gtrsim 1.0$ GeV$^2$ where the LC amplitude decreases faster than the NR one. One should note, however, that even at large $|t| \sim 4$ GeV$^2$ due to interference effects all contributions must be included.
Fig. 2. Elastic $\pi \pi$ scattering for the $2g$-exchange model. The dashed line is for pure IA contributions (only diagrams (a) and (b) in Fig.1), whereas the dotted line corresponds to the pure GGL terms (only diagrams (c) and (d) in Fig.1). The thick solid line corresponds to the full result with all terms for the impact factor.

3. Soft reggeon exchanges and multiple scattering

In the case of $\pi \pi$ scattering in the considered region of energies the soft pomeron exchange must be supplemented by the subleading isoscalar ($f$) and isovector ($\rho$) reggeon exchanges. For the purposes of our analysis we resort to the simplest Regge-inspired phenomenological form:

$$A_P(t) = i C_P \cdot (s/s_0)^{\alpha_0(t)} \cdot F^2_P(t),$$

$$A_f(t) = -\eta_f(t) C_f \cdot (s/s_0)^{\alpha_f(t)} \cdot F^2_f(t),$$

$$A_{\rho}(t) = -\eta_{\rho}(t) C_{\rho} \cdot (s/s_0)^{\alpha_{\rho}(t)} \cdot F^2_{\rho}(t),$$

where $\eta_f$ and $\eta_{\rho}$ are somewhat simplified signature factors [4].

In the following we take the exponential $F(t) = exp(B_{\pi}(t))$ parametrization for the pion-pion-reggeon vertex form factor. For simplicity we assume one universal slope parameter for all reggeons $B_P = B_f = B_{\rho} \equiv B$. In the region of small-angle scattering, in loose analogy to the electromagnetic form factor of the pion, we consider also the monopole $F(t) = \frac{1}{1-B_{\pi,mon}(t)}$ parametrizations for the pion-pion-reggeon vertex form factor, again with one universal parameter $B_{\pi,mon}$ [4].

Assuming Regge factorization the residues at $t = 0$ can be evaluated
Fig. 3. Total cross section for $\pi^+\pi^-$ (left panel) and $\pi^+\pi^+$ or $\pi^-\pi^-$ (right panel) scattering as a function of center-of-mass energy $W$. The experimental data are from [3]. The single pomeron and subleading reggeon exchanges are given by the dashed lines. The solid line is obtained from the dashed line after including the absorption corrections.

from those for $\pi N$ and $NN$ scattering as:

$$C_i^{\pi\pi} = \frac{(C_i^{\pi N})^2}{C_i^{NN}}$$  \hspace{1cm} (3)

for each reggeon considered $i = P, f, \rho$. Then the corresponding Regge phenomenology of $\pi N$ and $NN$ scattering [9] gives $C_P = 8.56$ mb, $C_f = 13.39$ mb and $C_\rho = 16.38$ mb. Absorption corrections, generated by Regge cuts, are known to break the factorization (3). We take for the pomeron trajectory $\alpha_P(0) = 1$ and $\alpha_P^1 = 0.25$ GeV$^{-1}$ and for subleading trajectories $\alpha_f(0) = 0.5$ and $\alpha_f^1 = 0.9$ GeV$^{-1}$, i.e. values well known from the Regge phenomenology [8]. In the above evaluation we have neglected the possible small pQCD $2g$-exchange contribution to the $\pi N$ and $NN$ total cross section, which is justified for the purpose of our exploratory study. Thus the slope $B$ is our basic free parameter.

The total single-reggeon exchange amplitude is now

$$A_{\text{single}}(t) = A_P(t) + A_f(t) + \xi A_\rho(t),$$  \hspace{1cm} (4)

where $\xi = -1$ for $\pi^+\pi^-$, $\xi = 0$ for $\pi^\pm\pi^0$ and $\xi = 1$ for two identical pions.

The total cross section for the same-sign and opposite-sign $\pi\pi$ calculated with single reggeon exchanges, including small hard two-gluon component, is
shown in Fig. 3 by the dashed line. The thick solid line includes in addition the absorption corrections evaluated in the double-scattering approximation. Our predictions well coincide with total cross sections extracted in [3] from the absorption Regge model analysis of \( \pi N \to X N, X \Delta \) reactions. While the opposite-sign \( \pi \pi \) total cross section depends strongly on energy, the same-sign \( \pi \pi \) total cross section is almost independent of energy.

Regge cut or absorption corrections to single reggeon+ pomeron exchange have been studied actively in the past (see for instance [10]). Here we restrict ourselves to the dominant double-scattering corrections which read

\[
A_{ij}^{(2)}(s, \bar{k}) = \frac{i}{32 \pi^2 s} \int d^2 \bar{k}_1 d^2 \bar{k}_2 \delta^2(\bar{k} - \bar{k}_1 - \bar{k}_2) A_{ij}^{(1)}(s, \bar{k}_1) A_{ij}^{(1)}(s, \bar{k}_2). \tag{5}
\]

In general, the single scattering amplitudes \( A_{ij}^{(1)} \) in (5) are not restricted to soft reggeon exchanges and hard two-gluon exchanges should be included too. Consequently in the following we shall include the (soft \( \odot \) soft), (soft \( \odot \) hard)+ (hard \( \odot \) soft) and (hard \( \odot \) hard) double-scattering amplitudes. The last three double-scattering contributions are expected to be small, at least at forward angles, compared to the leading (soft \( \odot \) soft) absorption correction.

Let us concentrate now on angular distributions. It is interesting how the diffractive pattern of the angular distribution changes with energy. In Fig. 4 we present angular distributions for \( \pi^+ \pi^- \) (left panel) and the same-sign pion-pion (right panel) elastic scattering across our region of interest.
for $W = 3, 4, 5$ GeV for exponential vertex form factor with $B = 4$ GeV$^{-2}$. The small-$t$ differences at different energies come predominantly from the energy dependence of the subleading reggeons. The difference in the region of intermediate $t$ is due to interference of single- and double-scattering terms. Only at very large $t \sim 4$ GeV$^2$ the cross section starts to approximately scale with energy.

4. Conclusions

We have investigated the $\pi\pi$ scattering in the region of intermediate energies $W = 2 - 5$ GeV.

First, we have investigated $2g$-exchange mechanism in the elastic $\pi\pi$ scattering within a relativistic approach to the pion wave function. We have found the dominance of the impulse approximation terms at small $|t|$ and the Glauber-Gribov-Landshoff terms at large $|t|$.

Assuming dominance of soft physics and Regge factorization at small $|t|$ we have predicted the total cross section for $\pi\pi$ scattering consistent with experimental values extracted in the literature. The interplay of soft and hard processes in multiple scattering was analysed. We have found strong interference effects between single soft and hard amplitudes and some of dominant double-scattering amplitudes. We predict rather different angular distributions of elastic scattering for opposite-sign pions and for same-sign pions.

REFERENCES