Investigation of the large-scale neutral hydrogen near the supernova remnant W28

P. F. Velázquez\textsuperscript{1}

\textit{Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Apdo. Postal 70-543, C.P.: 04510, México D.F., México}
e-mail: pablo@nuclecu.unam.mx

G.M. Dubner\textsuperscript{2}

\textit{Instituto de Astronomía y Física del Espacio (CONICET, UBA), C.C.67, 1428 Buenos Aires, Argentina.}
e-mail: gdubner@iafe.uba.ar

W. M. Goss

\textit{National Radio Astronomy Observatory, P.O.Box 0, Socorro, New Mexico 87801, USA.}
e-mail: mgoss@nrao.edu

and

A. J. Green

\textit{School of Physics, University of Sydney, NSW 2006, Australia.}
e-mail: agreen@physics.usyd.edu.au

\textbf{ABSTRACT}

The distribution and kinematics of neutral hydrogen have been studied in a wide area around the supernova remnant W28. A 2\textdegree{}5 × 2\textdegree{}5 field centered at $l = 6\textdegree{}5$, $b = 0\textdegree{}$ was surveyed using the Parkes 64-m radio telescope (HPBW 14\textdegree{}7

\textsuperscript{1}Fellow of CONICET, Argentina

\textsuperscript{2}Member of the Carrera del Investigador Científico, CONICET, Argentina
at $\lambda$ 21 cm). Even though W28 is located in a complex zone of the Galactic plane, we have found different H\textsubscript{i} features which are evidence of the interaction between W28 and its surrounding gas. An extended cold cloud with about 70 M$_{\odot}$ of neutral hydrogen was detected at the location of W28 as a self-absorption feature, near the LSR velocity $+7$ km s$^{-1}$. This H\textsubscript{i} feature is the atomic counterpart of the molecular cloud shown by previous studies to be associated with W28. From this detection, we can independently confirm a kinematical distance of about 1.9 kpc for W28. In addition, the neutral hydrogen observed in emission around the SNR displays a ring-like morphology in several channel maps over the velocity interval $[-25.0, +38.0]$ km s$^{-1}$. We propose that these features are part of an interstellar HI shell that has been swept-up by the SN shock front. Emission from this shell is confused with unrelated gas. Hence, we derive an upper limit for the shell mass of $1200 - 1600$ M$_{\odot}$, a maximum radius of the order of 20 pc, an expansion velocity of $\sim 30$ km s$^{-1}$, an initial energy of about $1.4 - 1.8 \times 10^{50}$ ergs and an age of $\sim 3.3 \times 10^4$ yrs. The pre-existing ambient medium has a volume density of the order of $1.5 - 2$ cm$^{-3}$. W28 is probably in the radiative evolutionary phase, although it is not possible to identify the recombined thin neutral shell expected to form behind the shock front with the angular resolution of the present survey.

Subject headings: supernova remnants — ISM: individual (W28) — ISM: H\textsubscript{i} — ISM: structure

1. Introduction

Each supernova remnant (SNR) is the unique product of its own history (the progenitor and the explosion mechanism) and the characteristics of the environs in which it evolves. The study of the interstellar medium around SNRs can be used to understand the appearance of a remnant in different spectral regimes (distorted shapes, local brightness enhancements, filamentary emission, etc.). Such studies also allow the analysis of the temporal evolution of SNRs. In addition, the investigation of the gaseous matter around SNRs can lead to an understanding of the Galactic interstellar medium. These studies are important in understanding the response of the interstellar gas to the large injection of energy and momentum that a supernova (SN) explosion represents.

and b). These investigations show the manner in which the expansion of a SN shock front modifies the surrounding environment and the effect that the surrounding gas has, in turn, on the shape and dynamics of the SNR. In the present study, we report the results of an HI study around the SNR W28 (G 6.4–0.1).

The SNR W28 is located in a very complex region of the Galaxy, near the large HII regions M8 and M20, and the young clusters NGC 6530, NGC 6514, and Bo 14. It has a number of prominent morphological characteristics. In the radio continuum, there is diffuse emission together with thin filaments and small bright regions, as seen in Figure 1. This image is the result of combining 50 VLA pointings into a 20 cm mosaic (Dubner et al. 2000). In X-rays, diffuse thermal emission fills the interior of W28, although ear-shaped segments of a limb-brightened shell can also be observed toward the NE and NW (Rho et al. 1996). In the optical, there are bright narrow filaments strongly correlated with radio features and diffuse Hα nebulosities, apparently anti-correlated with the radio synchrotron emission (Long et al. 1991, Dubner et al. 2000).

A number of observations support the existence of a physical interaction between W28 and an adjacent molecular cloud: (1) the existence of shocked CO and other molecular species (Wootten 1981, Frail & Mitchell 1998, Arikawa et al. 1999) (2) the detection of over forty 1720 MHz OH masers distributed along the brightest synchrotron features (Claussen et al. 1997 and 1999), and (3) the coincidence of the molecular gas with the brightest synchrotron filaments which are the features with the flattest spectral index in the SNR (as expected for high Mach number shocks; from Dubner et al. 2000). All these indicators point to the existence of an interaction between the SNR and the molecular cloud.

In what follows, we analyze the distribution of the neutral hydrogen around W28, based on a survey of the λ21 cm HI line carried out for a 2.5° × 2.5° field with the Parkes 64-m radio telescope.

2. Observations and Data Reduction

An area of 6.25 square degrees, centered at $l = 60^\circ, b = -05^\circ$ was observed using the Parkes 64m telescope on June 23 and 24, 1995. The wide band (1.2–1.8 GHz) receiver was used, with orthogonal, linearly polarized feeds. The half-power beam-width of the telescope at the frequency of the HI line is 14.7 and the pointing accuracy is $\leq 20''$. The system noise temperature is 28 K, measured against cold sky. Each polarization was recorded with an instantaneous bandwidth of 4 MHz over 2048 channels, giving a channel separation of 1.95 kHz (0.4 km s$^{-1}$ at 1.4 GHz). The total velocity coverage is $\pm 400$ km s$^{-1}$ centered at 0
km s\(^{-1}\) with respect to the Local Standard of Rest (LSR). After Hanning smoothing, the velocity resolution per channel is 3.91 kHz (or 0.82 km s\(^{-1}\)).

In total, 289 positions were observed using constant Galactic latitude scans with an integration time of 50 sec per spectrum. A reference spectrum for band-pass calibration was taken using frequency switching to \(-400\) km s\(^{-1}\) at 25 minute intervals. Spectra were measured at the Nyquist sampling interval, on a 7.5 grid.

Flux density calibration was made using scans across Hydra A, for which the flux density was assumed to be 43.5 Jy at 1.4 GHz. The brightness temperature scale was calibrated against the IAU standard position S8, taken to have an integrated value of 897 ± 66 Kkm s\(^{-1}\) (Williams 1973). The conversion between flux density in Jy beam\(^{-1}\) and brightness temperature in K is 0.78 K/Jy beam\(^{-1}\). The rms uncertainty in the observations is 0.15 Jy, equivalent to 0.12 K. Initial processing was carried out using the SLAP and SDcube software. Final data analysis was performed using the AIPS software package. All velocities used in this paper are referred to the LSR.

3. The distance to W28 and the systemic velocity

Previous distance estimates for W28 range between 1.3 and 3.6 kpc. Milne (1970) estimated the distance to W28 to be 1.3–1.5 kpc. Lozinskaya (1974) derived a distance of 3.6 kpc based on H\(\alpha\) measurements (assuming an LSR velocity near +18 km s\(^{-1}\) for W28). Goudis (1976) estimated a distance of 1.8 kpc; Clark & Caswell (1976) suggested 2.3 kpc; a further estimate by Milne (1979) produced a distance of 2.4 kpc, and Venger et al. (1982) obtained a distance of 3 kpc based on H\(\text{i}\) absorption measurements carried out with the RATAN-600 telescope.

On the other hand, OH (1720 MHz) maser emission associated with W28 was detected by Frail, Goss & Slysh (1994) at LSR velocities between +5 and +15 km s\(^{-1}\), with most of the OH lines having velocities between +6 and +8 km s\(^{-1}\). Strong OH absorption lines at 1612, 1665 and 1667 MHz were reported at a radial velocity of +7.3 km s\(^{-1}\) (Goss 1968) while various molecular species in the dense gas interacting with W28 have a central velocity around +7 km s\(^{-1}\) (Pastchenko & Slysh 1974, Wootten 1981, Arikawa et al. 1999). Arikawa et al. (1999) have shown the existence of two different components in the CO emission at +7 km s\(^{-1}\), a narrow line corresponding to unshocked, quiescent molecular gas, and a broad line, most likely arising from gas overtaken by the SNR shock. An additional narrow CO component is detected at +21 km s\(^{-1}\), but this line probably originates in an unrelated cloud along the line of sight. From these studies, we will adopt \(\sim +7\) km s\(^{-1}\) as the
systemic velocity of W28. For this LSR velocity, circular rotation models provide near and far kinematic distances of 1.9 and 15 kpc. Because independent estimates favor the lower value, we adopt a distance of 1.9±0.3 kpc for W28.

4. The H\textsc{i} around the SNR W28

Figure 2 shows an H\textsc{i} profile obtained after averaging spectra from the entire observed region. The lower panel depicts the Galactic rotation curve toward $l = 6^\circ.5 \ b = 0^\circ$ based on the model of Fich et al. (1989), where $R_0 = 8.5$ kpc is assumed. The Galactic emission in this direction is mostly concentrated between -50 and +50 km s$^{-1}$ with a narrow absorption dip near +7 km s$^{-1}$. This is a very strong self-absorption feature produced by an unusually cold cloud that extends over a large region (covering at least 20° of longitude in the direction of the Galactic center), and including the direction of W28 (Riegel & Jennings, 1969).

Figures 3 and 4 show the distribution (in greyscale and white contours) of the H\textsc{i} emission within the velocity interval where significant H\textsc{i} emission is observed. In order to compare radio continuum and H\textsc{i} structures, the black contours represent the boundaries of the radio continuum emission associated with W28 (smoothed to the resolution of the H\textsc{i} data). The other bright continuum sources plotted in the field are the Trifid Nebula (M20; G07.00–0.3) to the left of W28, and the compact HII region W28 A–2 (G05.89–0.4) to the lower right corner of the Figures. The average H\textsc{i} field emission has been subtracted from all the images for presentation purposes. The greyscale plotted along the upper edge of Figures 3 and 4 is kept constant in all images in order to emphasize changes in structures for different velocities. With the exception of the first image of Figure 3 and the two last images of Figure 4, where the integration intervals were chosen to be 35 and 30 km s$^{-1}$, respectively, the remaining images result from the average over 5 km s$^{-1}$ (6 consecutive spectral channels). The central velocity of each integration interval is indicated in the top right corner of each panel.

The analysis of the H\textsc{i} distribution around W28 is quite complex, because of its location close to both the Galactic center and the Galactic plane. To identify structures that may be associated with the SNR, we look for features that may reveal the impact of the SNR expansion on the surrounding interstellar medium (ring-shaped H\textsc{i} structures, expanding H\textsc{i} caps, etc.). Also, we have attempted to locate H\textsc{i} concentrations that appear to be associated with prominent features observed in the radio continuum emission of W28.

Bright H\textsc{i} emission is observed at negative velocities. Based on Galactic circular rotation models, negative velocities should arise from gas at distances > 17 kpc. However, it is unlikely that the large bright structures observed between $V_{\text{LSR}} \sim -30$ and $-2$ km s$^{-1}$ correspond
entirely to this distant gas. We have analyzed the HI features in this velocity range in an attempt to find associations with W28. The negative velocities could result from kinematic perturbations arising from the SNR.

Between $\sim -32$ and $\sim -2$ km s$^{-1}$ (Fig. 3), the brightest HI emission regions are preferentially distributed around W28, encircling the source along different sides. Particularly, at $V_{\text{LSR}} = -27.5$ km s$^{-1}$ and $V_{\text{LSR}} = -22.5$ km s$^{-1}$ HI concentrations can be observed, forming a clumpy incomplete shell. It is possible that part of this gas had been swept-up by the expanding SN shock, although a “cap”-like feature would be the expected morphology for associated HI concentrations at these high negative velocities. Thus the association is uncertain. At $V_{\text{LSR}} = -7.5$ and $-2.5$ km s$^{-1}$ a good coincidence is observed between HI concentrations and some distortions observed in the outer envelope of W28 along the E and N sides. These structures are compatible with the hypothesis that the expanding shock wave of the SNR is pushing the interstellar neutral gas outwards.

At $V_{\text{LSR}} = +2.5$ km s$^{-1}$, the HI surrounds W28 with an almost complete shell-like structure. A striking morphological correspondence is observed between the two HI maxima to the N and NE of W28, at (6°50′, +0°15′) and (6°50′, −0°20′) respectively, and the two sites where the radio continuum shell is indented. The HI concentration near (6°50′, −0°20′) coincides with the CO concentration reported by Arikawa et al. (1999) to be quiescent molecular gas associated with W28. This HI component is also present in the following channel image at $V_{\text{LSR}} = +7.5$ km s$^{-1}$ (top left image of Figure 4). However, at $V_{\text{LSR}} = +7.5$ km s$^{-1}$ the HI feature in emission is masked by the strong absorption toward the SNR.

As mentioned before, in the image centered at $V_{\text{LSR}} = +7.5$ km s$^{-1}$ (Figure 4) the most remarkable feature is the central depression observed in HI emission. This HI depression results from self-absorption produced by an extended, cold cloud centered near +7 km s$^{-1}$, which is part of the complex of cold clouds reported by Riegel & Jennings (1969) in this direction of the Galaxy. To analyze the absorption features, in Figure 5 we display the negative contours (white lines) overlapping W28 (greyscale) as obtained from an integration of HI between +4 and +9 km s$^{-1}$. Based on Figure 5 we can conclude: (1) that the cold cloud is much larger than the SNR, and (2) the location of the deepest absorption hole does not coincide with the brightest synchrotron feature to the E of W28, but it approximately overlaps the thin radio filament that crosses W28 in the E-W direction (see Figure 1). This synchrotron filament has been shown by Dubner et al. (2000) to have the flattest spectral index of all parts of the SNR. Associated with this filament, Arikawa et al. (1999) have shown the existence of shocked CO gas (with broad wings, between $V_{\text{LSR}}$ -40 and +40 km s$^{-1}$) and unshocked CO gas (between $V_{\text{LSR}}$ +4 and +9 km s$^{-1}$). The shocked CO is associated with numerous OH (1720 MHz) masers (Claussen et al. 1997 and 1999). For
the unshocked gas, Arikawa et al. (1999) estimate a kinetic temperature $\leq$ 20 K, a density $\leq$ 10$^3$ cm$^{-3}$ and a total H$_2$ mass of 4000 M$_\odot$. For the shocked gas the physical parameters are: kinetic temperature $\geq$ 20 K, density $\geq$ 10$^4$ cm$^{-3}$ and a total H$_2$ mass of 2000 M$_\odot$. We conclude that we have detected the atomic hydrogen counterpart of the unshocked molecular cloud associated with W28, thus confirming on the basis of Hi data the systemic velocity and the distance of 1.9 $\pm$ 0.3 kpc for W28. From the present data we estimate that the mass of the cold Hi responsible for the self-absorption is 70 M$_\odot$. Therefore, only a small fraction of the total molecular gas mass is detected in atomic form.

At $V_{\text{LSR}}$= +17.5 km s$^{-1}$ a conspicuous Hi shell centered near ($l \sim 6^\circ40', b \sim 0^\circ12'$, with radius $\sim 0^\circ.6$) is observed surrounding the SNR. The presence of these concentrations distributed in a ring-like shape, together with the other features previously described at negative LSR velocities, suggest that part of the surrounding Hi gas may have been swept-up by the expanding SNR shock wave, forming a thick Hi interstellar shell. The existence of diffuse thermal X-ray emission filling the interior of W28 (Rho et al. 1996), supports the hypothesis that the center has been evacuated.

At higher positive velocities the most noticeable feature which may be associated with W28 is the central concentration present at $V_{\text{LSR}}$= +32.5 and +37.5 km s$^{-1}$. This concentration appears to be projected onto the interior of the SNR and can be interpreted as the “cap” of the Hi shell expanding around W28.

Based on these results we can propose a model where the explosion took place near ($l, b, V) = (6^\circ30', -0^\circ12', +7$ km s$^{-1}$). The present shell radius is $\sim$ 0$^\circ.6$, or 20 pc at a distance of 1.9 kpc. On the basis of the adopted systemic velocity of +7 km s$^{-1}$ and the presence of a “cap”-like feature near +37 km s$^{-1}$, the expansion velocity of this structure is estimated to be about 30$\pm$3 km s$^{-1}$.

The total associated Hi mass can be estimated by integrating the contributions of all the structures which are considered to be part of the atomic gas shell. In other words, the Hi emission between approximately $V_{\text{LSR}} \sim -25$ km s$^{-1}$ and $V_{\text{LSR}} \sim +37$ km s$^{-1}$, which appear encircling W28 in the different channel maps, plus the features projected onto the center of the SNR at the high positive velocities. After the subtraction of an appropriate background contribution (assumed to be 3$-$$\sigma$ below the isocontour which outlines the associated features), an Hi mass of 1600$\pm$240 M$_\odot$ is obtained. The quoted error takes into account the uncertainties in the selection of the boundaries for the integration. This is an upper limit for the associated mass, since it is impossible to separate the contributions from unrelated Hi. In this total mass estimate, about $\sim$ 400 M$_\odot$ correspond to the features at velocities more negative than $-7.5$ km s$^{-1}$, whose association with W28 may be questionable. Thus the total swept-up Hi mass can vary between $\sim$ 1200 and 1600 M$_\odot$. 
Assuming that the mass is uniformly distributed over a sphere of radius of 20 pc, we obtain an upper limit for the ambient interstellar medium (ISM) density of $\sim 1.5 - 2.0 \text{ cm}^{-3}$ (depending on the value used for the total mass). The kinetic energy would be $E_{\text{kin}} \leq 1 \times 10^{49} \text{ ergs}$ and the initial energy of the explosion about $1.4 - 1.8 \times 10^{50} \text{ ergs}$ (based on Chevalier’s 1974 model). Given these values for the initial explosion energy and the unperturbed ISM density, and by considering the following expression (Rohlfs & Wilson 1996):

$$\tau_{\text{rad}} = \left[ \frac{4.56 \times 10^7}{(1 + x_H)} \frac{T_4}{n_0} \right]^{5/6} \text{ yr},$$

we can estimate the time for the onset of the radiative phase of SNR evolution to be $\tau_{\text{rad}} \sim 2.1 - 2.4 \times 10^4 \text{ yr}$, while $R_{\text{rad}} \sim 10 \text{ pc}$ is the radius of the SNR at that stage. In Eq.(1) $T_4$ is the temperature just behind the SNR shock wave (in units of $10^4 \text{ K}$) which was set to 100 (radiative losses start to be an important process at about $10^6 \text{ K}$), $E_{51}$ is the initial SN energy in units of $10^{51} \text{ erg}$, $x_H$ is the ionization fraction (assumed to be 0) and $n_0$ is the unperturbed ISM density in $\text{ cm}^{-3}$.

By assuming a radius of about 13 pc for W28 (from an angular size of $\sim 23.5 \text{ arcmin}$ and a distance of $\sim 1.9 \text{ kpc}$), we can conclude that this remnant is well in the radiative phase of evolution and has a current age of $3.3 \times 10^4 \text{ yr}$, which is in good agreement with previous age estimates (Frail et al. 1993, Velázquez 1999). In this evolutionary stage, it is expected that a thin $\text{H}_i$ shell forms by recombination behind the shock front with a width of about 10% of the radius.

5. Conclusions

We have carried out a study of the neutral hydrogen in the environs of the SNR W28. Our analysis of the kinematics and distribution of the $\text{H}_i$ has revealed several $\text{H}_i$ features that are most probably with W28, revealing signatures of the interaction of this SNR with the interstellar medium. We have detected, as a self-absorption feature around $\sim 7 \text{ km s}^{-1}$, the neutral gas counterpart of the molecular cloud detected by Arikawa et al. (1999) in unshocked CO gas. Based on the presence of this cold cloud, we can independently confirm a kinematical distance of $1.9 \pm 0.3 \text{ kpc}$ for W28.

Portions of an incomplete $\text{H}_i$ shell are also observed in emission at different positive and negative LSR velocities, with a maximum angular size ($\sim 0.6$) at $V_{\text{LSR}} = +17.5 \text{ km s}^{-1}$. An $\text{H}_i$ cloud is detected near $V_{\text{LSR}} = +37 \text{ km s}^{-1}$ overlapping the center of W28. We interpret
this last feature as the “cap” of the irregular expanding interstellar shell swept-up by the W28 shock front. The mass of this shell has been estimated to be between 1200 and 1600 $M_{\odot}$.

Based on the present results, the following scenario for W28 can be proposed:

(a) A SN explosion of energy $\sim 1.6 \times 10^{50}$ ergs occurred about $3.3 \times 10^4$ yr ago, at the position $(l, b) = (6^\circ30', -0^\circ12')$ and at distance of $\sim 1.9$ kpc. At this location, the ambient density of the ISM was $\sim 1.5 - 2 \, \text{cm}^{-3}$.

(b) The expanding shock wave has collided with a cold gas concentration, observed as an absorption $\text{H}i$ feature and as molecular clouds around the LSR velocity of 7 km s$^{-1}$. The mass of this cold cloud ($\sim 70 \, M_{\odot}$), is only a small fraction of the total mass estimated for molecular hydrogen. Most of the atomic hydrogen is detected in emission as an $\text{H}i$ shell, as mentioned below.

(c) The interaction of the SN shock front with the surrounding $\text{H}i$ gas has swept-up a thick interstellar HI shell, presently expanding at $\sim 30$ km s$^{-1}$. W28 has entered into the radiative stage of evolution about $2 \times 10^4$ yrs ago. However, the thin neutral shell expected to form by recombination behind the shock front could not be identified because of the relatively low angular resolution of the present study.

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6. Figure captions

Fig. 1.— Radio continuum image of the SNR W28 and the HII regions M20 and W28A–2, and the SNR G7.06–0.12. The greyscale range is [0, 0.5] Jy beam$^{-1}$. This image was obtained with the VLA at 1415 MHz by Dubner et al. (2000). The resolution is $88'' \times 48''$, P.A. = 8°. The 1σ noise level is 5 mJy beam$^{-1}$.

Fig. 2.— (a) Average HI emission profile at $l \simeq 6^\circ.5$ and $b \simeq 0^\circ$ from the current data; (b) Galactic rotation curve at $l = 6^\circ.5, b = -0^\circ.1$ (Fich et al. 1989).

Fig. 3.— Images of HI emission (in grey and white contours) between $-55$ and $+3$ km s$^{-1}$. The first image is an integration from $-70$ to $-35$ km s$^{-1}$. In the subsequent images the integration is carried out over an interval of 5 km s$^{-1}$. The greyscale range is [-5, 15] Jy beam$^{-1}$ km s$^{-1}$, while the white contours correspond to -2, 2, 6, 10, 14, 18, 22 and 26 Jy beam$^{-1}$ km s$^{-1}$. The black contours show the 2, 2.5 and 3 Jy beam$^{-1}$ levels of the radio continuum of W28 at 1415 MHz (from Dubner et al. 2000), with an angular resolution of 14′.7. Each panel is labeled with the central velocity of the particular interval of integration. The arrows on the left top corner of the first panel show the N and E directions in the J2000 Equatorial Coordinate system, to facilitate comparison with Figure 1.

Fig. 4.— As in Fig. 3, HI images in the velocity range $[+7.5, +85]$ km s$^{-1}$. The last two images are obtained from integration over an interval of 30 km s$^{-1}$.

Fig. 5.— Overlay of the HI depression integrated between 4 and 9 km s$^{-1}$ (white contours), and the W28 radio continuum (greyscale). The contours correspond to the $-26$, $-22$, $-18$, $-14$, $-10$, $-6$, $-2$ and 2 Jy beam$^{-1}$ km s$^{-1}$ levels. The radio continuum of W28 (with a resolution of $88'' \times 48''$) is shown with a greyscale range of $[-0.06, 0.75]$ Jy beam$^{-1}$. 