MODELLING QUANTUM MECHANICS BY THE QUANTUMLIKE DESCRIPTION OF THE ELECTRIC SIGNAL PROPAGATION IN TRANSMISSION LINES

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Abstract

It is shown that the transmission line technology can be suitably used for simulating quantum mechanics. Using manageable and at the same time non-expensive technology, several quantum mechanical problems can be simulated for significant tutorial purposes. The electric signal envelope propagation through the line is governed by a Schrödinger-like equation for a complex function, representing the low-frequency component of the signal, In this preliminary analysis, we consider two classical examples, i.e. the Frank-Condon principle and the Ramsauer effect.

1 Introduction

There exist several purely classical systems whose behaviour can be described with models which are fully similar to the formal apparatus of quantum mechanics, although their nature have nothing to do with the quantum one. These models are usually referred to as ”quantumlike models” [1, 2]. Typically they are governed by a Schrödinger-like equation where \( \hbar \) is replaced by another physical parameter of the particular classical system considered. In particular, quantumlike models have been successfully proposed in the transport of electromagnetic radiation beams in linear and nonlinear regimes [3, 4] (see, for instance, the theory of optical fibers [5, 6] and related subjects), in the sound wave theory [7], in plasma physics [8], in the theories of transport and dynamics of charged-particle beams [9] and in dynamics of the ocean waves [10]. To describe the analytic signal theory [11, 12], a quantumlike formalism has been
used; recently it was successfully employed to translate new results of quantum mechanics into the theory of analytic signal [13, 14]. Quantumlike models received also a big deal of attention in the literature to describe the nonlinear electric signal propagation in transmission lines in a number of papers. In particular, this subject received a very important development in literature in connection with some experimental investigations of modulational instability and soliton formation in nonlinear transmission lines [15].

Transmission line theory is also very important for a number of scientific and technological applications. In particular, it is widely used in accelerator physics to model several parts of a given accelerating machine [16]. Also one should point out that the transmission line can be considered as a continuous set of short circuits with capacitance $C$, inductance $L$, and resistance $R$. The separate short circuit and two interacting short circuits were considered in the quantum limit in [18]. The chain of circuits was studied in the quantum domain in [19].

The quantumlike treatment of the transmission line is the goal of our paper. For instance, in this modelling operation, electric signals, propagating through a transmission line, modelling a piece of accelerating machines, may be analyzed to study their characteristic impedances. It seems to be clear that in the accelerator theory two ”classical” quantumlike problems are combined, namely, the analytic signal theory and the propagation through the transmission lines.

The aim of this work is pedagogical and consists in the following steps: (i) to point out that the propagation of an electric signal through a trasmission line can be described with a quantumlike model in terms of a complex wavefunction whose evolution is governed by a Schrödinger-like equation; (ii) to point out that our results are relevant for simulating quantum effects with manageable and non-expensive transmission line technology.

The article is organized as follows. In the next section, we briefly introduce the concept of a transmission line and give the wave equation governing the propagation of a signal as a current perturbation in space and time. The effective refractive index of the line is also introduced. In section 3, taking the slowly-varying amplitude approximation, a Schrödinger-like equation for a complex function is derived from the wave equation. The quantumlike formalism obtained in this way allows us in section 4 to discuss the signal propagation in the cases of some simple space profiles of the effective refractive index that can simulate some tutorial problems of quantum mechanics. For instance, we consider the case a quadratic space-profile of the refrative index to show that the quantum harmonic oscillator problem and the related coherent states can be simulated by employing a transmission line. In section 5, the perspective of how to produce a simulation of more advanced quantum topics, such as the Frank-Condon principle and the Ramsauer effect, is drawn stressing the advantages that their simulation can be obtained for tutorial purposes by using a transmission line. Finally, conclusions and remarks are presented
2 The wave equation of a linear dispersive transmission line

Let us consider the usual representation of an electromagnetic (e.m.) transmission line as a series of R-L-C short circuits (series of moduli). Each modulus exhibits inductance, capacitance and resistance per unity length, \( L' \), \( C' \) and \( R' \), respectively. Let us denote with \( x \) and \( t \) the longitudinal space coordinate and the time, respectively. Thus, it is easy to see that the perturbation voltage and current signals, \( \delta v(x,t) \) and \( \delta i(x,t) \), appearing at the end of an arbitrary modulus of the line, located at the longitudinal position range \( x, x + dx \), obey to the following coupled short-circuit equations:

\[
\frac{\partial \delta v}{\partial x} = L' \frac{\partial \delta i}{\partial t} + R' \delta i \quad (1),
\]
\[
\frac{\partial \delta i}{\partial x} = C' \frac{\partial \delta v}{\partial t} \quad (2).
\]

Combining (1) and (2), and solve f.i. for \( \delta i \) we get the following usual wave equation for the transmission line:

\[
\frac{\partial^2 \delta i}{\partial t^2} - \frac{1}{L'C'} \frac{\partial^2 \delta i}{\partial x^2} + \frac{R'}{L'} \frac{\partial \delta i}{\partial t} = 0 \quad (3).
\]

This wave equation accounts for the dissipations along the line of ohmic nature.

For the sake of simplicity, let us assume that the ohmic dissipations are negligible. Consequently, Eq. (3) becomes:

\[
\frac{\partial^2 \delta i}{\partial t^2} - V^2 \frac{\partial^2 \delta i}{\partial x^2} = 0 \quad (4),
\]

where we have introduced the phase velocity

\[
V \equiv \sqrt{\frac{1}{L'C'}} \quad (5).
\]

When the parameters \( L' \) and \( C' \) of the line are homogeneous, the phase velocity does not depend on the coordinates, i.e.

\[
V_0 \equiv \sqrt{\frac{1}{L_0'C_0'}} \quad (6),
\]

where \( L_0' \) and \( C_0' \) are some unperturbed values.

In order to take into account some very slow space and time modulations of \( L' \) and \( C' \), let us assume:

\[
L' = L_0'f_1(x,t) \quad (7),
\]
\[
C' = C_0'f_2(x,t) \quad (8),
\]
where $f_1(x,t)$ and $f_2(x,t)$ are some specific functions that account for the inhomogeneity profile in space and time. Thus,

$$V^2 = \frac{1}{L_0 C_0 f_1(x,t) f_2(x,t)} \equiv \frac{V^2_0}{N^2(x,t)} \ ,$$

where it is clear that $N(x,t)$ accounts for the refractive index, say $n(x,t)$, of the medium.

Note that: $V^2/c^2 = V^2_0/c^2 N^2$ ($c$ being the light speed); thus

$$n(x,t) = n_0 N(x,t) \ ,$$

where $n_0$ is the unperturbed refractive index (namely, the one of the homogeneous case). Consequently,

$$N(x,t) = \frac{n(x,t)}{n_0} \ ,$$

is the relative refractive index.

By taking into account Eq. (11), Eq. (4) becomes:

$$n^2(\eta,t) \frac{\partial^2 \delta i}{\partial \eta^2} - \frac{\partial^2 \delta i}{\partial t^2} = 0 \ ,$$

where $\eta = x/c$.

### 3 A Schrödinger-like equation for weakly-dispersive transmission lines

#### 3.1 The Telegraphist’s equation

In case of a time-independent refractive index, i.e. $n = n(\eta)$, By taking a solution of (12) of the form

$$\delta i = (\delta i)_0 \exp(-i\omega_0 t) \ ,$$

we obtain the following Helmholtz-like equation (usually referred as to ”the telegraphist’s equation”):

$$\frac{\partial^2 \delta i}{\partial \eta^2} + c^2 K^2(\eta) \delta i = 0 \ ,$$

where $K^2(\eta) \equiv \omega^2_0 n^2(\eta)/c^2$. A number of problems in linear transmission lines have been described by means of eq. (13)[16].
3.2 Slowly-varying amplitude approximation in weakly-dispersive transmission lines

Now, in order to describe the propagation of electric signal envelopes, let us assume that \( n(\eta, t) \) is weakly perturbed, i.e.

\[
n(\eta, t) \approx n_0 + \delta n(\eta, t) ,
\]

where \(|\delta n| << n_0\). We look for a solution of (12) can be taken in the form:

\[
\delta i(\eta, t) = \Phi(\eta, t) \exp(-i\omega t) ,
\]

where \( \Phi(\eta, t) \) is a very slow function of \( t \) compared to the phase term variation, i.e.

\[
\left| \frac{\partial \Phi}{\partial s} \right| << \omega |\Phi| .
\]

The slow function \( \Phi \) accounts for an amplitude modulation of the signal, in such a way that \( \delta i \) plays the role of a wave envelope (electric signal envelope). Substituting (15) and (14) in (12) and taking into account the first-order quantities only, we get the following Schrödinger-like equation for the current perturbation envelope:

\[
i\omega \frac{\partial \Phi}{\partial t} = -\frac{1}{2n_0^2} \frac{\partial^2 \Phi}{\partial \eta^2} - \omega^2 \frac{\delta n}{n_0} \Phi - \frac{\omega^2}{2} \Phi .
\]

Putting

\[
\Phi(\eta, t) = \Psi(\eta, t) \exp(-i\omega t/2) ,
\]

we finally get the following equation

\[
i \frac{\partial \Psi}{\partial s} = -\frac{1}{2n_0^2} \frac{\partial^2 \Psi}{\partial \tau^2} - \frac{\delta n}{n_0} \Psi ,
\]

where the dimensionless variable \( s = \omega t \) and \( \tau = \omega \eta \) have been introduced.

Note that (19) corresponds to the usual one-dimensional Schrödinger equation with \( \hbar = 1 \), and where \( s \) and \( \tau \) replace the time \( t \) and the space-coordinate \( x \), respectively. Consequently, we realize that the relative refractive index perturbation \(-\delta n/n_0\) plays the role of an effective potential and \( n_0^2 \) plays the role of an effective mass. Thus, the (19) can be cast as

\[
i \frac{\partial \Psi}{\partial s} = -\frac{1}{2n_0^2} \frac{\partial^2 \Psi}{\partial \tau^2} + U(\tau, s) \Psi ,
\]

where

\[
U(x, s) = -\frac{\delta n}{n_0} .
\]

It is easy to see that (21) can be also cast as:

\[
U(\tau, s) = \sqrt{\frac{L_0 C_0'}{L'(\tau, s) C'_{\tau}(\tau, s)} - 1} .
\]

where
For the sake of simplicity, in the following we fix to the unity the constant $n_0$. Note that, as in Quantum Mechanics the wavefunction of an elementary particle is a solution of the Schrödinger equation, the complex function $\Psi(\tau, t)$ involved in the (20) represents the analog of the quantum wavefunction which is associated with the electric signal envelope propagating through the transmission line. In this paper we conventionally call this complex function the signal envelope wavefunction (SEW). Once the signal envelope wavefunction is normalized, i.e.

$$\int_{-\infty}^{\infty} |\Psi(\tau, s)|^2 \, d\tau = 1 ,$$

the quantity $|\Psi(\tau, s)|^2$ plays the role of probability density to find the electric signal envelope at the location $x = c\tau$ and at the time $t = s/c$. We may syntetically call $|\Psi|^2$ the probability density associated with the electric signal envelope.

4 An examples of tutorial interest: propagation through a line with a quadratic space-profile refractive index

In this section we consider a special case of effective refractive index that may be useful to simulate some of the typical quantum problems of tutorial interest. In fact, we examine the propagation of an electric pulse through a transmission line with an effective quadratic space-profile refractive index.

In case $\delta n$ is time-independent, an interesting case to be considered is the one in which $-\delta n/n_0$ is a parabolic function of $\tau$, namely ($n_0 = 1$)

$$i \frac{\partial}{\partial s} \Psi(\tau, s) = -\frac{1}{2} \frac{\partial^2}{\partial \tau^2} \Psi(\tau, s) + \frac{1}{2} k \tau^2 \Psi(\tau, s) ,$$

(24)

where we have assumed $-\delta n/n_0 \equiv k\tau^2/2$, with $k > 0$. It is easy to show that Eq. (24) admits the following orthonormal discrete modes for the SEW

$$\Psi_n(\tau, s) = \frac{1}{\sqrt{2\pi\sigma^2(s) \sigma^2(n)!^{1/4}}} H_n \left( \frac{\tau}{\sqrt{2}\sigma(s)} \right) \times \exp \left[ -\frac{\tau^2}{4\sigma^2(s)} + i \frac{\tau^2}{2\rho(s)} + i(1 + 2n)\phi(s) \right] \quad \text{with} \quad n = 0, 1, 2, .. ,$$

(25)

where $H_n$ are the Hermite polynomials, $\sigma(s)$ obeys to the following envelope equation

$$\sigma'' + k \sigma - \frac{1}{4\sigma^2} = 0 ,$$

(26)

and

$$\frac{1}{\rho} = \frac{\sigma'}{\sigma} ,$$

(27)
\[ \phi' = -\frac{1}{4\sigma^2} . \]  

(28)

where each prime denotes the derivative with respect to \( s \). It is easy to see that \( \sigma(s) \), appearing in (25)-(28), coincides with the r.m.s. of the fundamental mode \( \Psi_0(\tau, s) \); in general, an arbitrary SEW \( \Psi \) has a r.m.s. \( \sigma(s) \) defined by:

\[ \sigma(s) = \left[ \int_{-\infty}^{+\infty} \tau^2 |\Psi_0(\tau, s)|^2 \, d\tau \right]^{1/2} . \]  

(29)

In addition, we can also define the expectation value for the transverse linear momentum associated to an arbitrary SEW \( \Psi(\tau, s) \)

\[ \sigma_p(s) = \left[ \int_{-\infty}^{+\infty} \left| \frac{\partial \Psi_0(\tau, s)}{\partial \tau} \right|^2 \, d\tau \right]^{1/2} . \]  

(30)

where \( \hat{p} = -i \frac{\partial}{\partial \tau} \); in fact, \( \tau \) and \( p \) play the role of conjugate variables.

It is suitable to introduce the following matrix

\[ \hat{T}(s) \equiv \begin{pmatrix} \sigma_p^2(s) & -\sigma(s) \, \sigma'(s) \\ -\sigma(s) \, \sigma'(s) & \sigma^2(s) \end{pmatrix} , \]  

(31)

whose determinant is an invariant, namely

\[ \sigma_p^2 \sigma^2 - (\sigma \sigma')^2 = \frac{1}{4} = \text{const.} . \]  

(32)

It is easy to prove that

\[ \sigma(s) \sigma'(s) = \int_{-\infty}^{+\infty} \Psi_0^*(\tau, s) \left( \frac{\tau \hat{p} + \hat{p} \tau}{2} \right) \Psi_0(\tau, s) \, d\tau = \langle \frac{\tau \hat{p} + \hat{p} \tau}{2} \rangle . \]  

(33)

Consequently, from (32) follows that

\[ \langle \tau^2 \rangle \, \langle \hat{p}^2 \rangle - \langle \frac{\tau \hat{p} + \hat{p} \tau}{2} \rangle = \frac{1}{4} , \]  

(34)

which is formally identical to Robertson-Schrödinger uncertainty relation [20], [21] for partial cases when \( \langle \tau \rangle = \langle p \rangle = 0 \). Note that (32), or equivalently (34), gives the usual form of the Heisenberg-like uncertainty principle which is analogous to Heisenberg uncertainty relation in quantum mechanics (again for \( \langle \tau \rangle = \langle p \rangle = 0 \))

\[ \sigma_p \sigma \geq \frac{1}{2} . \]  

(35)

The equilibrium solution of (26) \( (d^2 \sigma(s)/ds^2 = 0) \), namely

\[ \sigma^2_0 = \frac{1}{2\sqrt{k}} . \]  

(36)
implies that the set of Hermite-Gauss modes (25) reduces to the hamiltonian eigenstates of the harmonic oscillator

\[
\Psi_0^n(\tau, s) = \frac{1}{[2\pi\sigma_0^2 2^n(n!)^2]^{1/4}} \exp \left( -\frac{\tau^2}{4\sigma_0^2} + i(1 + 2n)\phi_0(s) \right) H_n \left( \frac{\tau}{\sqrt{2}\sigma_0} \right), \tag{37}
\]

where \( n = 0, 1, 2, \ldots \),

\[
\phi_0(s) = -\sqrt{k} \frac{s}{2}, \tag{38}
\]

and the energy values \( E_0^0 \), given by averaging the Hamiltonian of the system with the wavefunction (37), are the analog of the hamiltonian eigenvalues of the quantum harmonic oscillator

\[
E_0^n = \left( n + \frac{1}{2} \right) \sqrt{k}. \tag{39}
\]

In particular, for \( n = 0 \) (37), (38) and (39) give the ground-like state

\[
\Psi_0^0(\tau, s) = \frac{1}{[2\pi\sigma_0^2]^{1/4}} \exp \left( -\frac{\tau^2}{4\sigma_0^2} + i\phi_0(s) \right), \tag{40}
\]

which is purely Gaussian and the lowest energy reachable by the electric signal envelope is \( E_0^0 = (1/2)\sqrt{k} \). The means \( \langle \tau \rangle \) and \( \langle p \rangle \) are equal to zero at this state of the electric signal envelope. In these conditions the uncertainty relation is minimized as

\[
\sigma_\tau \sigma_p = \frac{1}{2}. \tag{41}
\]

Eq. (41) holds also during the evolution of the electric signal, because, in addition to (36), we have \( \sigma_p(s) = \sigma_{\rho 0} = \text{const.} \). In summary, we conclude that if we initially prepare the SEW according to the matching conditions (36), its evolution is ruled by a quantum-like behaviour in terms of ground-like state which minimizes the uncertainty relation and corresponds to the lowest accessible energy \( (1/2)\sqrt{k} \) of the electric signal envelope.

As it well known, SEW (40) belongs to the infinite series of coherent state functions, labeled by a complex number \( \alpha = \alpha_1 + i\alpha_2 \), and widely used in quantum mechanics and quantum optics [22, 23].

5 Quantumlike Analogs of Frank–Condon and Ramsauer–Twonsend Effects for Transmission Lines

Let us discuss what physical consequences can be extracted from the observation that the electric signal in the transmission line can be associated with the Schrödinger-like equation. We got the result that transmission line can be considered as a quantumlike system; the distributed along the line conductance \( C \) and inductance \( L \), in principle can be considered as inhomogeneous
and time-dependent functions, i.e., $C = C(\tau, s)$ and $L = L(\tau, s)$. They account for the effective potential-energy function $U = U(\tau, s)$. In fact, in the quantumlike systems, such as the light ray in optical fiber or electron beam in accelerator in the framework of thermal-wave model, the refractive index profile plays a role of the effective potential-energy function.

Let us consider now the interesting situation in the presence of some filtering of the modes in the transmission line. To this end, let us consider a transmission line whose refractive index gives an effective potential well $U$ which has a rectangular structure in domain of nonhomogeneity of the transmission line. In this case, electrical signal in transmission line is propagating along the line in complete analogy with the electromagnetic wave in a waveguide. The solutions to the Schrödinger-like equations (modes of the signal; in transmission line) can be treated as wave functions which are reflected or transmitted by a potential barrier. There are effects involved in the above propagation that we discuss qualitatively in the following. We may consider this problem as an analog of two known quantum subjects: The Ramsauer effect and the Frank–Condon principle.

### 5.1 Ramsauer effect

In the case where there is non-homogeneity in the transmission line, the potential-energy term has a deformation, e.g., increasing (or decreasing) the depth of the potential well for some length $L$. In the wavelength of the signal function satisfies the condition that $2L/\lambda = n$ where $n$ is even, the potential well is transparent to the signal with wavelength $\lambda$. It is just an analog of Ramsauer effect in which the electron beam which scatters by atoms for some values of energy (with corresponding de Broglie wavelength) has smaller cross section for this process than for the other wavelength when the ratio is odd. In the model experiment, one can see that for some adapter a change in frequency corresponds to different reactions of the transmission line which just corresponds to the analog of the Ramsauer effect. By this method, one can measure the characteristics of the line (distributed inductance and capacitance) that is equivalent to the impedance measuring.

### 5.2 Frank-Condon principle

Another qualitative effect can be checked if one connects two different pieces of the transmission line which corresponds to connecting two potential barriers of different depths. The problem of penetrating the modes of the first piece and their transforming into the modes of the second piece of the transmission line is equivalent to calculating the Frank-Condon factor for electronic transitions in polyatomic molecules. This factor is overlap integral of the wave-like functions describing the two different modes in the two pieces. $|C_{nm}|^2$ describes the portion of the signal
energy of the $n$th mode in the first part of the transmission line which goes into $m$th mode of the second line. Again one can pose the problem of maximality, e.g., of the transformation of the fundamental mode energy into the fundamental mode of the second piece of the line, having in mind that distributed inductance and capacitance in the second piece of the line are different from the first one. In terms of introduced Wigner functions, the transformation coefficient $|C_{nm}|^2$ which give the probability of transforming $n$th mode into $m$th mode reads

$$|C_{nm}|^2 = \int W_n(q,p)W_m(q,p)\frac{dq dp}{2\pi}. \quad (42)$$

The coefficient $C_{nm}$ can be expressed in terms of overlap integral of electric current modes $\psi_n(\tau), \psi_m(\tau)$

$$C_{nm} = \int \psi_n^*(\tau)\psi_m(\tau)d\tau,$$

where the mode $\psi_n(\tau)$ is the mode in the first piece of the transmission line and the mode $\psi_m(\tau)$ corresponds to the second piece of the transmission line. If one can vary in time the inductance and capacitance of the transmission line, the Frank-Condon factor describes the parametric excitation of the modes in the transmission line. For example, if the rectangular profile of ‘refractive index’ is modeled by the parabolic profile with the varying frequency, the electric current propagation in the line is described by the oscillator with time-dependent frequency used for analysis of the thermal wave model [24]. We can adopt the results of the analysis and apply them to the transmission line. Thus, we have for the fundamental mode the Gaussian solution [24]

$$\psi_0(\tau,t) = \pi^{-1/4}\varepsilon^{-1/2}(t)\exp\left(\frac{i\dot{\varepsilon}(t)\tau^2}{2\varepsilon(t)}\right), \quad (43)$$

where the function of time $\varepsilon(t)$ satisfies the equation

$$\ddot{\varepsilon}(t) + \omega^2(t)\varepsilon(t) = 0 \quad (44)$$

and the initial conditions

$$\varepsilon(0) = 1 \quad \dot{\varepsilon}(0) = i$$

for units chosen in such a manner that $\omega(0) = 1$. The frequency $\omega(t)$ corresponds to parabolic shape of the ‘refractive index’ $\omega^2(t)\tau^2/2$ of the transmission line. In the case of instantaneous change of the line parameters in time, one has excitation of the line modes given by the Frank-Condon factor. If the change is not instantaneous one can get the modes in the form [24]

$$\psi_n(\tau,t) = \psi_0(\tau,t)\left(\frac{\varepsilon(t)}{\varepsilon^*(t)}\right)^{n/2}\frac{1}{\sqrt{2^n n!}}H_n\left(\frac{\tau}{\varepsilon(t)}\right), \quad (45)$$

where $H_n$ is Hermitte polynomial. the Frank-Condon factor can be calculated in the explicit form for the model under consideration. Thus, the coefficients $C_{nm}$ are expressed in terms of
two-dimensional Hermite polynomials. The Frank-Condon factors qualitatively can be evaluated by a geometric method. As it is known, the maximally excited is the mode for which refractive index curve after the change of the line parameters is intersecting the perpendicular taken from the rest point of the initial refractive index curve. Thus the modes are connected which have common rest points on the plot of two curves. One curve is the initial refractive index and the other one is the final refractive index.

6 Conclusions and perspectives

We have shown that the electrical current signal in the transmission line obeys to the Schrödinger-like equation. We have extracted some physical consequences from the fact that the transmission line is the quantumlike system. In our preliminary analysis, we have considered two physical situation of a transmission line that can be thought as the quantum analog of known quantum phenomena. The first physical situation involves the analog of the Ramsauer effect which explains the behaviour of the line signal if one considers two pieces of the line connected by some adapter. In this case, one can filter the modes in order to increase the transmission of some modes and suppress some others. The other physical situation involves the quantum analog of the parametric excitation of the modes in the line which corresponds to quantum transitions in polyatomic molecules described by the Frank-Condon factor. Remarkably, this approach seems to be very helpful and promising for providing a very simple method of simulating a number of quantum-mechanical effects, using both manageable and non-expensive technology.

References


