Introduction

In the analysis of correlation functions, we define the correlation functions of the type [1] 

\[ \frac{d\rho}{(\rho')^\lambda} \equiv \langle \rho \rangle - \langle \rho' \rangle \langle \rho' \rangle \]

where \( \rho \) is the associated differential and

\[ \rho' = \rho \]

Any correlated quantity is given by

\[ Q(\rho, \rho') \]

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presented in section III. In section IV we study the evolution of the system. Section V and VI is devoted for the results at the freeze-out for SPS and RHIC energies respectively. And finally in section VII we present the summary and conclusions.

II. EVOLUTION SCENARIOS

We consider the following three possible evolution scenarios.

1. QGP scenario: The system starts evolving from an initial QGP state formed at a time $\tau_i$ and temperature $T_i$. It then expands, hence cools and reaches the critical temperature $T_c$ at a time $\tau_f$. In the mixed phase the cooling due to expansion is compensated by the heating of the system due to the liberation of the latent heat in a first order phase transition. Hence the temperature remains constant at $T_c$ (super-cooling is neglected here). After the mixed phase ends at a time $\tau_i$, further expansion takes place and finally the system disassembles at a time $\tau_f$ and temperature $T_f$. At this stage, called the freeze-out stage, the mean free path of the particles are too large to have any further interactions, and those are detected experimentally.

2. Hadron gas scenario: The hot and dense system is formed in the hadronic state at a time $\tau_i$ and temperature $T_i$ and the system expands from the initial to freeze-out state (at time $\tau_f$ when the temperature is $T_f$) without a phase transition.

3. Hadron gas with mass variation: The other possibility is that the system may form in the hadronic state as in (2) above, but the spectral function of the hadrons are different from their vacuum counterparts. Among others, in the present case we will consider the shift of the pole of the hadronic spectral function according to the universal scaling law proposed by Brown and Rho [13]:

$$m_h^* = m_h \left( 1 - \frac{T^2}{T_c^2} \right)^\lambda,$$

where $m_h^*$ ($m_h$) is the in-medium (vacuum) mass of the hadrons (except pseudo-scalar) and the index $\lambda$ takes a value between 0 and 1. Here we choose $\lambda = 1/6$ according to the well known Brown-Rho scaling [14].

III. INITIAL CONDITIONS

The initial conditions in terms of initial temperature ($T_i$) can be set for the three scenarios from the following relation:

$$dS = \frac{2\pi^4}{45\zeta(3)} dN = \frac{4\pi^2}{90} g_{\text{eff}} T_i^3 \Delta V,$$  \hspace{1cm} (3)

where $dS(dN)$ is the entropy (number) contained within a volume element $\Delta V = \pi R^2 \tau_i d\eta$. $R$ as the radius of the colliding nuclei. $g_{\text{eff}}$ is the effective statistical degeneracy, $\zeta(3)$ denotes the Riemann zeta function and $\eta$ is the space-time rapidity. For massless bosons (fermions) the ratio of $dS$ to $dN$ is given by $2\pi^4/(45\zeta(3)) \sim 3.6$ (4.2), which is a crude approximation for heavy particles. For example the above ratio is 3.6 (7.5) for 140 MeV pions (938 MeV protons) at a temperature of 200 MeV.

First we consider the situation at SPS energies where $dN/d\eta \sim 700$ for Pb+Pb collisions. For the three scenarios taking $g_{\text{eff}}$ as given in Table I, we obtain the initial temperature by using Eqn. 3. The values of the initial temperatures are given in Table I.

The chemical potential at the initial state is fixed by constraining the specific entropy (entropy per baryon) to the value obtained from the analysis of experimental data. The specific entropy at SPS is about 40 for Pb + Pb collisions [15, 16, 17, 18]. The net baryon number can be calculated using the baryon (anti-baryon) number density, $n_b$ ($n_\bar{b}$) given by,

$$n_b = \frac{g}{(2\pi)^3} \int f(\vec{p}) d^3 p, \hspace{1cm} (4)$$

where $g$ is the baryonic degeneracy. We take $g = 4$ for proton and neutron, $f(\vec{p})$ is the well known Fermi-Dirac distribution,

$$f(\vec{p}) = \left[ \exp \left( \frac{(E \pm \mu)/T}{1} \right) + 1 \right]^{-1}, \hspace{1cm} (5)$$

where $\mu(-\mu)$ is the chemical potential for baryon (anti-baryon), $E = \sqrt{p^2 + m^2}$ and $n_\bar{b} \equiv n_b (\mu \rightarrow -\mu)$.

For the QGP scenario we take the mass of the quarks as, $m_q^* = m_q^0 + \frac{\gamma}{2} g_{\text{eff}}$, where $m_q^0$ is the thermal mass [19] and $m_q^* \equiv m_q$ is the current quark mass. We have taken vacuum mass and effective mass for the hadrons (given by Eqn. 2) for cases 2 and 3 respectively. We obtain the value of the chemical potential to be 132 MeV, 340 MeV and 105 MeV for cases (1), (2) and (3) respectively. These are also summarized in Table I.

Having fixed the initial temperature and chemical potential we now calculate the initial fluctuations.
TABLE I: Initial conditions and the initial values of fluctuation for the three scenarios.

<table>
<thead>
<tr>
<th>Initial Values/Scenarios</th>
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<td>$T_i$ (MeV)</td>
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<td>$\mu_i$ (MeV)</td>
<td>132</td>
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<tr>
<td>$(\Delta N_b(\tau_i))^2/S$</td>
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<td>$0.029$</td>
<td>$0.061$</td>
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<tr>
<td>$(\Delta N_b(\tau_i))^2/N_{b,y}$</td>
<td>0.56</td>
<td>1.16</td>
<td>2.46</td>
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</table>

for the three scenarios.

1. Quark Gluon Plasma: From the initial net baryon number, one can easily calculate the net baryon number fluctuations using Eqn. 1. The fluctuation in the net baryon number in the QGP phase at time $\tau_i$, temperature $T_i$ and chemical potential $\mu_i$, can be shown to be [10]:

$$ (\Delta N_b(\tau_i))^2_{QGP} = \frac{2V}{9} T_i^3 \left( 1 + \frac{1}{2} \left( \frac{\mu_i}{T_i} \right)^2 \right). $$ (6)

With the initial conditions as discussed above, the initial fluctuations for the QGP scenario turns out to be $(\Delta N_b(\tau_i))^2_{QGP} = 35$. It may be noted that the value of $g$ in Eqn. 4 is taken to be 12, for two flavor case.

The entropy density in the QGP phase is calculated from Eqn. 3 with $g_{eff} = 37$. The total entropy of the system is given by:

$$ S = V \frac{4\pi^2 g_{eff} T_i^3}{90} \sim 2500. $$ (7)

For fixed initial entropy of the system the initial temperature is different for the three cases because of the different values of the $g_{eff}$. In the present work we solve the evolution equation for ideal fluid neglecting entropy generations due to various viscous effects. The total entropy is kept constant for all the three scenarios as it is obtained from the number of particles per unit rapidity measured experimentally. So the fluctuation in net baryon number per entropy is $(\Delta N_b(\tau_i))^2_{QGP}/S = 0.014$, a value similar to that obtained in [10]. In this case the fluctuation per unit baryon, $(\Delta N_b(\tau_i))^2_{QGP}/N_{b,y} = 0.56$ where $N_{b,y}$ is the net baryon number per unit rapidity, $dN_b/dy \sim 62$ for SPS energies.

2. Hadron Gas: The net baryon number fluctuation in the hadronic gas can be calculated using Eqn. 1 and is given by:

$$ (\Delta N_b)^2_{HG} = \frac{gV}{4} \left( \frac{2mT_i}{\pi} \right)^{3/2} \exp(-m/T_i) \cosh(\mu/T_i). $$ (8)

Substituting the values of $T_i$, $\mu_i$ and $m = 938$ MeV for the nucleons, we obtain the fluctuation in net baryon number, $(\Delta N_b(\tau_i))^2_{HG} \sim 72$. The ratio of the fluctuation to total entropy is $(\Delta N_b(\tau_i))^2_{HG}/S = 0.029$. This value is also similar to the one obtained in [10]. In this case the value of $(\Delta N_b(\tau_i))^2_{HG}/N_{b,y} = 1.16$.

3. Hadron gas with mass variation medium: The initial fluctuation for this case can be obtained using the Eqn’s 4 and 5 together with Eqn. 2 used for the value of mass. The initial conditions of $T_i = 226$ MeV and $\mu_i = 105$ MeV are constrained to reproduce the hadronic multiplicity and the entropy per baryons. In this case we get $(\Delta N_b(\tau_i))^2_{HG,m} = 153$ and $(\Delta N_b(\tau_i))^2_{HG,m}/S = 0.061$. The value of $(\Delta N_b(\tau_i))^2_{HG,m}/N_{b,y} = 2.46$ here.

We observe that for the baryon number fluctuations corresponding to the above three scenarios:

$$ (\Delta N_b(\tau_i))^2_{HG} \sim 2 \quad \text{and} \quad (\Delta N_b(\tau_i))^2_{HG,m} \sim 4. $$ (9)

This clearly shows that in the initial stage, there is a clear distinction between the three cases. The initial values of fluctuations are summarized in Table I. It will be of interest now to see, if these fluctuations (and the differences given by Eqn. 9) survive till the freeze-out. Next we discuss the evolution of this initial fluctuation for the three scenarios.

IV. EVOLUTION OF THE INITIAL FLUCTUATION

We follow ref. [10] to study the proper time ($\tau$) evolution of the baryon fluctuation in the space-time rapidity interval $\Delta \eta$ for different EOS as well as initial states as mentioned above. At the freeze-out point the fluctuation measured experimentally contains the residue of the initial fluctuation which survived the space time evolution and the fluctuations generated due to the exchange of baryons with the adjacent sub-volumes. We discuss each of the above cases separately in the following sub-sections.

A. Dissipation of the initial fluctuation

The difference in baryon flux $(\Delta n \bar{n})$ originating from different densities inside and outside of the sub-volume $(A \tau \Delta \eta)$ leads to the following differential equation for $\Delta N_b(\Delta \bar{n} A \tau)$:

$$ \frac{d\Delta N_b}{d\tau} = -\frac{\bar{n}}{2\Delta \eta} \frac{\Delta N_b}{\tau}. $$ (10)
FIG. 1: Variation of average thermal velocity with temperature for three different scenarios. The results are fitted by nth-order polynomials. (1) For QGP scenario the fitting parameters are $a_0 = 0.93$, $a_1 = 0.11$, $a_2 = -0.56$, $a_3 = 1.32$ and $a_4 = 1.20$. (2) For hadronic gas scenario with fit parameters as $a_0 = 0.19$, $a_1 = 4.0$, $a_2 = -14.4$, $a_3 = 32.2$ and $a_4 = -31.2$. (3) For hadronic gas with mass variation in medium with fit parameters as $a_1 = -488.9$, $a_2 = 768.9$, $a_3 = -0.640E+06$, $a_4 = 0.297E+07$, $a_5 = -0.73E+07$ and $a_6 = 0.74E+07$.

where $\bar{v}$ is the average thermal velocity of the particles under consideration.

The solution to the differential equation 10 from initial time $\tau_i$ to final time $\tau_f$ is then given as:

$$\Delta N_b(\tau) = \Delta N_b(\tau_i) \exp \left( -\frac{1}{2 \Delta \eta} \int_{\tau_i}^{\tau_f} \frac{d\tau}{\tau} \bar{v}(\tau) \right).$$

(11)

The average thermal velocity at a given temperature can be calculated by using the following equation,

$$\bar{v} = \frac{\int_0^\infty d^3 p f(\vec{p})}{\int d^3 p f(\vec{p})}.$$

(12)

The values of $\bar{v}$ as a function of temperature for the three cases are shown in Fig. 1. The curves are fitted by the polynomials of the form $a_0 + a_1 T + a_2 T^2 + a_3 T^3 + a_4 T^4 + a_5 T^5 + a_6 T^6$. The values of the parameters are shown in the caption of Fig. 1. The average velocity in QGP is found to be substantially larger than the velocity in hadronic gas.

However, in case of the hadronic system where the effective masses of the baryons (neutron and proton here) approach zero at $T_c$, their velocities approach the velocity of light.

The equation of state plays a vital role in deciding how the fluctuations in the net baryon number evolve. The variation of energy density ($\epsilon$) as a function of temperature is obtained from the 3-flavor lattice QCD results [20] which is parametrized as follows:

$$\epsilon = T^4 A \tan h(B \frac{T}{T_c})^C,$$

(13)

where the values of the parameters, $A, B$ and $C$ are $12, 0.517$ and $10.04$ respectively. Note that the effect of net baryons in the EOS is neglected here. Fig. 2 shows the variation of the energy density with temperature. The increase in the effective degeneracy near $T_c$ can be obtained from the hadronic phase with effective masses varying with temperature as in Eqn. 2. The increase in the effective degeneracy originates from the heavier hadrons going to a massless situation (see also ref. [21, 22]).

The evolution of the system under the boost invariance along the longitudinal direction is governed by the equation [23],

$$\frac{d\epsilon}{d\tau} + \frac{\epsilon + P}{\tau} = 0; \quad P = c_s^2 \frac{\epsilon}{\tau}$$

(14)

where $c_s$ is the velocity of the sound in the medium. To perform the integration in Eqn. 11 we need $d\tau/\tau$, which can be obtained as

$$\frac{d\tau}{\tau} = \left[ \frac{\beta}{T} + \frac{\alpha}{T} \frac{dT}{d\tau} \right] dT = -f(T) dT$$

(15)

FIG. 2: The variation of energy density as a function of temperature. The result shown by solid line is obtained from Eqn. 13. Open circles show the lattice results with out the error bars.
where, \( \alpha = \frac{1}{1+c_e} \), \( \beta = 4\alpha \) and

\[
\chi = \left[ \tanh B(T/T_c)^C \right]^\alpha \tag{16}
\]

The expression for the second term within the square bracket of Eqn. 15 is given by

\[
\frac{1}{\chi} \frac{dT}{d\tau} = a \frac{BC(T/T_c)^C}{C} \left( \frac{1}{\cosh(B(T/T_c)^C)} - \frac{1}{\sinh(B(T/T_c)^C)} \right) \tag{17}
\]

Using the relation \( c_2^2 = \frac{S}{T} \frac{dS}{dT} = \left[ 3 + \frac{T}{\beta_{eff}} \frac{d\beta_{eff}}{dT} \right]^{-1} \), we get the velocity of sound corresponding to the lattice QCD calculations,

\[
c_2^2 = \left[ 3 + BC(T/T_c)^C - 1 \frac{1}{\cosh(B(T/T_c)^C)} \sinh(B(T/T_c)^C) \right]^{-1} \tag{18}
\]

Now the fluctuation at the freeze-out point can be written as,

\[
\Delta N_b(\tau_f) = \Delta N_b(\tau_i) \exp \left( -\frac{1}{2 \Delta \eta} \int_{\tau_i}^{\tau_f} f(T) dT \right) \tag{19}
\]

In order to study the effect of the equation of state, we will present results for three different values of

\[
c_v: (a) \ c_v^2 = 1/3 \text{ corresponds to the ideal gas case, (b) } c_v^2 = 0.18 \text{ corresponding to an EOS of hadronic gas where particles of mass up to 2.5 GeV has been taken into account from particle data book and (c) } c_v^2 \text{ obtained from Eqn. 18. It may be mentioned that for the ideal gas the rate of cooling is faster than the other two cases, (b) and (c).}
\]

Now we shall calculate the total dissipation of fluctuation at the freeze-out temperature. We shall consider the freeze-out temperature to be 120 MeV [18, 24] and a critical temperature for the QGP transition to be 170 MeV [20]. In all the three cases below we have taken the value of \( \Delta \eta = 1 \).

1. Quark Gluon Plasma: For the QGP initial state, the dissipation equation has three parts. In the first part, we calculate the dissipation from the initial temperature \( T_i \) to \( T_c \) (for the QGP phase). The second part is the dissipation during the mixed phase and in the final phase it is the dissipation from \( T_c \) (end of mixed phase) to the freeze-out temperature \( T_f \). Hence the complete dissipation equation becomes,

\[
\Delta N_b(T_f) = \Delta N_b(T_i) \exp \left( -\frac{1}{2 \Delta \eta} \left( \int_{T_i}^{T_f} \bar{\tau}_{\text{qgp}}(T) f(T) dT + \bar{\tau}_{\text{mix}}(T_c) \ln(r) + \int_{T_f}^{T_i} \bar{\tau}_b(T) f(T) dT \right) \right) \tag{20}
\]

where \( r \) is the ratio \( g_{eff}/g_{eff} \text{ at } \sim 2.5 \) and \( \bar{\tau}_{\text{mix}} \sim \bar{\tau}_{\text{qgp}}(T_c) \).

The fluctuation in the net baryon number is evaluated for: (a) \( c_v^2 = 1/3 \), for which we get \( \Delta N_b(\tau_f) \text{ }_{\text{QGP}}^2 \sim 6 \), (b) \( c_v^2 = 0.18 \), giving \( \Delta N_b(\tau_f) \text{ }_{\text{QGP}}^2 \sim 2.23 \) and (c) \( c_v^2 \) as given by Eqn. 18, resulting in \( \Delta N_b(\tau_f) \text{ }_{\text{QGP}}^2 \sim 0.54 \).

The initial and final fluctuation values along with the evolution process of the values for the QGP scenario is shown in Fig. 3. Only the results for \( c_v^2 = 1/3 \) and \( c_v^2 \) as given by Eqn. 18 are shown in the figures for the clarity of presentation. We have checked that the results for \( c_v^2 = 0.18 \) lies between the above two cases.

2. Hadron Gas: In case of hadronic initial state we have,

\[
\Delta N_b(T_f) = \Delta N_b(T_i) \exp \left( -\frac{1}{2 \Delta \eta} \int_{T_f}^{T_i} \bar{\tau}_b(T) f(T) dT \right) \tag{21}
\]

(a) For \( c_v^2 = 1/3 \), the contribution from the exponential is \( \sim 0.53 \). Hence the fluctuation in net baryon number is \( \Delta N_b(\tau_f) \text{ }_{\text{QGP}}^2 \sim 44 \), (b) for \( c_v^2 = 0.18 \), we get \( \Delta N_b(\tau_f) \text{ }_{\text{QGP}}^2 \sim 37 \) and (c) taking \( c_v^2 \) from Eqn. 18, the contribution from the exponential term is \( \sim 0.08 \) a factor of 7 lower than

3. Hadron gas with mass variation in medium: For the hadronic initial state with mass variation,

\[
\Delta N_b(T_f) = \Delta N_b(T_i) \exp \left( -\frac{1}{2 \Delta \eta} \int_{T_f}^{T_i} \bar{\tau}_b^m(T) f(T) dT \right) \tag{22}
\]

(a) For \( c_v^2 = 1/3 \), the contribution from the exponential is \( \sim 0.53 \). Hence the fluctuation in net baryon number is \( \Delta N_b(\tau_f) \text{ }_{\text{QGP}}^2 \sim 44 \), (b) for \( c_v^2 = 0.18 \), we get \( \Delta N_b(\tau_f) \text{ }_{\text{QGP}}^2 \sim 37 \) and (c) taking \( c_v^2 \) from Eqn. 18, the contribution from the exponential term is \( \sim 0.08 \) a factor of 7 lower than
The dissipation of net baryon number fluctuation for the different scenarios as a function of temperature. The dissipation is plotted in decreasing order to reflect the evolution in time. (1) Variation of $\langle (\Delta N) \rangle^2$ for QGP scenario, (2) Variation of $\langle (\Delta N) \rangle^2$ for hadronic gas scenario and (3) Variation of $\langle (\Delta N) \rangle^2$ for hadronic gas with quark variation scenario. The dashed lines correspond to results obtained for $c_s^2 = 1/3$, while the solid lines show the results obtained using the value of $c_s^2$ given in Eqn. 18.

the case (a). So the fluctuation in net baryon number is $\langle (\Delta N_b) \rangle^2_{\text{fig.,m.}}$ is 0.92. The evolution of fluctuation for this case is shown in Fig. 3.

**B. Generation of fluctuation with time**

The baryon fluxes exchanged with neighboring sub-volumes can lead to generation of fluctuations, and is the main source that is to be detected experimentally at the freeze-out point. The total number of baryons leaving or entering $\langle N_b^{\text{gen}} \rangle$ the sub-volume generation during the mixed phase and in the final phase it is the generation from $T_c$ (end of mixed phase) to the freeze-out temperature $T_f$. Hence the complete evolution equation becomes,

$$\langle (\Delta N_b(T_f)) \rangle^2 = N_b(T_i) \left( \frac{1}{2\Delta \eta} \left( \int_{T_c}^{T_f} \overline{\tau}_{\text{QGP}}(T) f(T) dT + \tau_{\text{mix}}(T_i) \ln(r) + \int_{T_f}^{T_i} \overline{\tau}_b(T) f(T) dT \right) \right)$$

The fluctuation in the net baryon number is evaluated for: (a) $c_s^2 = 1/3$, for which we get $\langle (\Delta N_b(T_f)) \rangle^2_{\text{QGP}}$ is 56.5, (b) $c_s^2 = 0.18$, resulting in

$$(A\tau)$$ between time $\tau_i$ and $\tau_f$ is given by [10]

$$N_b^{\text{gen}}(T_f) = \frac{N_b(T_i)}{2\Delta \eta} \left( \int_{T_i}^{T_f} \overline{\tau}(T) f(T) dT \right) \quad (23)$$

**FIG. 3:** The dissipation of net baryon number fluctuation for the different scenarios as a function of temperature. Notations are same as that of Fig. 3.

**FIG. 4:** The generation of net baryon number fluctuation for the different scenarios as a function of temperature. Notations are same as that of Fig. 3.
\((\Delta N_b(T_f))^2_{QGP}\) is 60.4 and (c) \(c_2^2\) from Eqn. 18 giving \((\Delta N_b(T_f))^2_{HG}\) is 129.

The initial and final values of the fluctuations along with its time evolution for the QGP scenario are shown in Fig. 4.

2. Hadron Gas: In case of hadronic initial state we have,

\[
(\Delta N_b(T_f))^2 = N_b(T_0) \left( \frac{1}{2\Delta \eta} \int_{T_f}^{T_i} \bar{\tau}_b(T) f(T) dT \right) \tag{25}
\]

(a) For \(c_2^2 = 1/3\), the fluctuation in net baryon number is \((\Delta N_b(T_f))^2_{HG}\) is 44. (b) For \(c_2^2 = 0.18\), we get \((\Delta N_b(T_f))^2_{HG}\) is 50. (c) If we take \(c_2^2\) from Eqn. 18, the fluctuation in net baryon number is \((\Delta N_b(T_f))^2_{HG}\) is 179. The evolution of fluctuation for this case is displayed in Fig. 4.

3. Hadron gas with mass variation in medium: For the hadronic initial state with mass variation,

\[
(\Delta N_b(T_f))^2 = N_b(T_0) \left( \frac{1}{2\Delta \eta} \int_{T_f}^{T_i} \bar{\tau}_b(T) f(T) dT \right) \tag{26}
\]

(a) For \(c_2^2 = 1/3\), the fluctuation in net baryon number is \((\Delta N_b(T_f))^2_{HG}\) is 38.9. (b) For \(c_2^2 = 0.18\), we get \((\Delta N_b(T_f))^2_{HG}\) is 44. (c) Taking \(c_2^2\) from Eqn. 18, the fluctuation in net baryon number is \((\Delta N_b(T_f))^2_{HG}\) is 158. The evolution of fluctuation for this case is shown in Fig. 4.

V. FLUCTUATIONS AT THE FREEZE-OUT

The net baryon fluctuation at the freeze-out \((T_f = 120\text{ MeV})\) is a combination of the dissipation and the generation effect as presented in the previous section. The resultant fluctuations is the sum of the variances \((\Delta N_b(T_f))^2\) obtained for each of the two processes.

We present results in terms of net baryon fluctuation per unit baryon, \((\Delta N_b(T_f))^2/N_{b,y}\). For Poissonian noise this value should be close to unity. Deviation from this numerical values will indicate the presence of dynamical fluctuations.

1. Quark Gluon Plasma: The final fluctuation in the net baryon number per unit baryon in the QGP scenario at the freeze-out point for the three EOS mentioned above are given by, (a) for \(c_2^2 = 1/3\), \((\Delta N_b(T_f))^2_{QGP}/N_{b,y}\) is 1.0, (b) for \(c_2^2 = 0.18\), \((\Delta N_b(T_f))^2_{QGP}/N_{b,y}\) is 1.0 and (c) taking \(c_2^2\) from Eqn. 18, \((\Delta N_b(T_f))^2_{QGP}/N_{b,y}\) is 2.0.

2. Hadron Gas: The net baryon number fluctuation per unit baryon in the hadronic gas at freeze-out for different values of of \(c_2\) are, (a) for \(c_2^2 = 1/3\), \((\Delta N_b(T_f))^2_{HG}/N_{b,y}\) is 1.0, (b) for \(c_2^2 = 0.18\), \((\Delta N_b(T_f))^2_{HG}/N_{b,y}\) is 1.0 and (c) taking \(c_2^2\) from Eqn. 18, \((\Delta N_b(T_f))^2_{HG}/N_{b,y}\) is 2.0.

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</tbody>
</table>

3. Hadron gas with mass variation in medium: The net baryon number fluctuation per unit baryon for the hadronic initial state with mass variation at freeze-out for the three EOS are, (a) for \(c_2^2 = 1/3\), \((\Delta N_b(T_f))^2_{HG}/N_{b,y}\) is 1.3, (b) for \(c_2^2 = 0.18\), \((\Delta N_b(T_f))^2_{HG}/N_{b,y}\) is 1.3 and (c) taking \(c_2^2\) from Eqn. 18, \((\Delta N_b(T_f))^2_{HG}/N_{b,y}\) is 2.8.

The net baryon number fluctuations per unit baryon is summarized in Table II. For all the scenarios the values of the quantity \((\Delta N_b(T_f))^2/N_{b,y}\) is larger than Poissonian noise for EOS taken from lattice QCD. This indicates presence of dynamical fluctuations. However, it should be mentioned here that the errors in lattice QCD calculations is large for temperature below the critical temperature. Furthermore \((\Delta N_b(T_f))^2_{QGP}/(\Delta N_b(T_f))^2_{HG}\) is 1.4 and \((\Delta N_b(T_f))^2_{HG}/(\Delta N_b(T_f))^2_{QGP}\) is 1.25. This means, although scenario (1) may be distinguishable from the scenarios (2) and (3) it is very difficult to differentiate (2) and (3) from fluctuation measurements. However, for EOS with \(c_2^2 = 1/3\) and 0.18 they are close to Poisson value of 1. This indicates that for these EOS; it is difficult to distinguish among the three scenarios.

VI. RESULTS AT RHIC ENERGIES

The net baryon number fluctuation has been evaluated for RHIC energies \(\sqrt{s} = 200\text{ GeV}\) of \(Au+Au\) collisions. We have taken the total (charged and neutral) \(dN/dy \approx 1100\) [25]. The initial temperatures obtained from the above multiplicity are quite large, 370 MeV for hadronic gas and 290 MeV.
for the case of hadronic gas with mass variation. As these temperatures are well above the critical temperature predicted by lattice QCD, we have considered only the QGP scenario for RHIC energies. We have taken the initial time to be 0.6 fm/c [26] and specific entropy to be 150 for Au + Au collisions. For QGP we take \( g_{\text{eff}} = 47.5 \) which gives \( T_q \approx 251 \) MeV and the constraint on the specific entropy gives an initial chemical potential of 73 MeV. With these initial conditions for QGP scenario at RHIC, the initial fluctuations turn out to be: \( (\Delta N_b(\tau_i))^2_{\text{QGP}} = 42 \). The entropy density in the QGP phase is calculated from Eqn. 7 with \( g_{\text{eff}} = 47.5 \) to be \( \sim 3900 \). So the fluctuation in net baryon number per entropy is \( (\Delta N_b(\tau_i))^2_{\text{QGP}}/S = 0.011 \) and \( (\Delta N_b(\tau_i))^2_{\text{QGP}}/N_{b,y} = 1.6 \). The value of \( dN_b/dy \) for RHIC energies a factor of about 2.4 lower than that for SPS energies.

As before, here the values of \( T_c \) and \( T_f \) are 170 and 120 MeV respectively. (a) For \( c^2_1 = 1/3 \), the fluctuation in net baryon number due to dissipation at freeze-out, \( (\Delta N_b(\tau_f))^2_{\text{QGP}} \) turns out to be 2.7. Whereas the generation mechanism gives \( (\Delta N_b(\tau_f))^2_{\text{QGP}} \sim 35.5 \). So the resultant fluctuation in net baryon number per unit baryon is \( (\Delta N_b(\tau_f))^2_{\text{QGP}}/N_{b,y} \sim 1.46 \). (b) For \( c^2_2 = 0.18 \), the fluctuation in net baryon number due to dissipation, \( (\Delta N_b(\tau_f))^2_{\text{QGP}} \) turns out to be 2.3. Whereas the generation mechanism gives \( (\Delta N_b(\tau_f))^2_{\text{QGP}} \sim 48.7 \). The evolution of the dissipation and the generation of fluctuation for the QGP scenario at RHIC energies is shown in the Figs. 5 and 6. So the resultant fluctuation in net baryon number per unit baryon is \( (\Delta N_b(\tau_f))^2_{\text{QGP}}/N_{b,y} \sim 1.96 \). (c) If we take \( c^2_3 \) as given by Eqn. 18, the fluctuation in net baryon number due to dissipation, \( (\Delta N_b(\tau_f))^2_{\text{QGP}} \) is 0.25, the generation mechanism gives \( (\Delta N_b(\tau_f))^2_{\text{QGP}} \sim 82.25 \). So the resultant fluctuation in net baryon number per unit baryon is \( (\Delta N_b(\tau_f))^2_{\text{QGP}}/N_{b,y} \sim 3.17 \).

Note that though the absolute value of the fluctuations in case of RHIC is smaller than SPS the fluctuation per baryon at RHIC is larger at the freeze-out point.

**VII. SUMMARY**

We have discussed the evolution of the fluctuation in net baryon number from the initial state to the final freeze-out state for three different scenarios, viz., (1) formation of QGP (2) hadronic gas and (3) hadronic gas with modified mass in the medium. In case of QGP formation we have assumed a first order phase transition. We find that the fluctuations at the initial stage agree with previously obtained values [10] where there are clear distinctions among the three cases. The fluctuations at the freeze-out point depend crucially on the equation of state (value of \( c^2 \)). Fluctuations with ideal EOS is seen to dissipate at a slower rate compared to EOS from lattice calculation. At SPS energies the values of the variance depend crucially on the EOS. For EOS from lattice QCD parametrization the fluctuation at the freeze-out is larger than the Poissonian noise. However, for ideal gas EOS and EOS with \( c^2 = 0.18 \), we do not observe fluctuations of dynamical origin. At RHIC energies the value of the fluctuations are larger than the Poissonian noise for all the three EOS under consideration here, indicating the fluctuations of dynamical origin. The effects of the finite acceptance on the fluctuation enters our calculations through

FIG. 5: The dissipation of net baryon number fluctuation for the QGP scenarios as a function of temperature calculated for Au + Au collisions at RHIC energies. The dashed lines corresponds to results obtained for \( c^2 = 1/3 \), while the solid lines corresponds to results obtained using the value of \( c^2 \) given in Eqn. 18.

FIG. 6: The generation of net baryon number fluctuation for the QGP scenarios as a function of temperature calculated at RHIC energies of 200 AgEv Au + Au Collisions. Notations are same as Fig. 5.
\( \Delta \eta \) and according to Eqn. 11 and 22 it is same for all the three scenarios (1), (2) and (3) discussed above. For general discussions on the effects of acceptance on the fluctuations we refer to Ref. [9, 27]. The dependence of the fluctuation on the centrality of the collisions (impact parameter) get canceled to a large extent in the ratio, \( \Delta N_{\ell}(\tau)_{\text{lab}} / N_{\ell} \). It is shown in [2] that it is possible to control the impact parameter dependence of the fluctuation by measuring \( E_T \) and analyzing data in narrow bins of \( E_T \). A full (3+1) dimensional expansion will lead to faster cooling, and hence it is interesting to see the survivability of fluctuations in such a scenario [28].

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