The dark energy, such as the cosmological constant, has now turned out to be a necessary element to understand our universe. There are many indirect suggestions for the dark energy, including the age of the universe, the formation of the large-scale structure, the number count of the galaxies, and so on. More striking evidences stem from the combination of acoustic peaks of temperature fluctuations in cosmic microwave background radiation (CMB) and the Hubble diagram of the type Ia supernova. Natural expectation for the dark energy is arisen from the vacuum fluctuations of quantum fields, although the smallness of its observed value is extremely unnatural. Since the energy density of the cosmological constant is constant in time by definition, one needs an extremely suspicious fine-tuning of 120 digits to set a correct value at an initial stage (probably Planck time) so that the energy density of the cosmological constant should be comparable with the matter density today. Alternatively, such stringent fine-tuning should be moderated if the dark energy component consists of some dynamical field such as “quintessence.” In general, such dynamical dark energy affects cosmological observations through a time-dependent EOS, which is characterized by a parameter $w(t) = p/\rho$. If $w$ is found to be different from $-1$, the dark energy is proven to be different from the simple cosmological constant. Current observational data are consistent with the cosmological constant, although they are still not enough to impose sufficient constraints on the EOS.

Since the dark energy has profound implications on the nature of the universe, it is of great importance to explore the origin of dark energy. Near-future observations will enable us to clarify whether the dark energy is a mere cosmological constant or something else. Cosmological observations, such as the Type Ia supernovae, CMB fluctuations, cluster mass function, weak lensing field, and so on, are useful tools to probe the nature of the dark energy.

It is shown that an application of the Alcock-Paczynski (AP) test to the redshift-space correlation function of the high-redshift objects can be a useful probe of the cosmological constant. In literatures, this method is mainly applied to the Lyman-$\alpha$ forest, the Lyman-break galaxies, and the quasars, etc.

A drawback of the high-redshift objects is their sparseness. Relatively high shot noise prevents to accurately determine the correlation function. On the other hand, a typical sample of normal galaxies is dense enough to have sufficiently small shot noise, while it is too shallow to apply the AP test. Instead, a sample of intermediate-galaxies, such as the LRGs in the Sloan Digital Sky Survey (SDSS) has a right balance between density and depth.

Therefore, a natural expectation is that the clustering of intermediate-galaxies can distinguish various dark energy models quite well. In fact, the AP test around redshift 0.5 is suggested to be promising. The purpose of this Letter is to quantitatively investigate what constraints can be imposed on the parameter space of dark energy models from intermediate-redshift galaxies and to clarify possible degeneracies with other cosmological parameters.

The clustering of objects in redshift space is characterized by the correlation function. In linear regime, the full information on the clustering is contained in the two-point correlation function, if the non-Gaussianity of the density field is negligible. In a homogeneous, isotropic space, the two-point correlation function $\xi(r)$ is a function of only a separation $r$ of the two-points. However, the redshift-space clustering in reality is neither homogeneous nor isotropic. Peculiar velocities, evolutionary effects, and cosmological geometry introduce inhomogeneity and anisotropy in the observed space. Therefore, the two-point correlation function in apparent redshift space is generally expressed as $\xi(z_1, z_2, \theta)$, where $z_1, z_2$ are the redshifts of the two-points and $\theta$ is the apparent angle between two-points from the observer. The full analytic expression for the apparent correlation function in linear
It is not trivial how one can analyze the three-dimensional function $\xi(z_1, z_2, \theta)$. Instead of directly evaluating the two-point correlation function, we have been developing a likelihood method for the density field \cite{20, 23, 24}, which directly analyzes the probability of the observed density field given a cosmological model. A fast algorithm to compute the correlation matrix is crucial in this analysis. A detailed description of one of our algorithms is given in \cite{20}, in which distant-observer approximation is assumed. We generalize this algorithm to the one without this approximation, using the analytic expression of \cite{23}. Apart from this generalization, and from inclusion of the dark energy degrees of freedom, the algorithm we employ in this Letter is similar to that in \cite{23}, the details of which will be published elsewhere.

Here we briefly summarize major points of the generalization. In our likelihood analysis of the redshift–space clustering, first we place smoothing cells in redshift space and then count the number of object $n_i$ in each cell $i$. When the non-Gaussianity of the distribution is negligible, the correlation matrix $C_{ij} = \langle n_i n_j \rangle / \langle n_i \rangle \langle n_j \rangle - 1$ fully characterizes the statistical property of the clustering. The correlation matrix is given by numerical integrations of the analytic form of $\xi(z_1, z_2, \theta)$ which is generalized to take into account nontrivial the EOS for dark energy component. Both cosmological distortions and peculiar velocity distortions are taken into account in this analytic form. A spherical cell approximation \cite{20} greatly simplifies this integration without implementing the numerical integration cell by cell. Then shot noise term is added to the correlation matrix.

Most of the modification of the analytic form of the correlation function is originated from the non-standard time-dependence of the Hubble parameter. When the dark energy has a non-trivial EOS as a function of redshift, $w(z)$, the time-dependent Hubble parameter is given by

$$H(z) = H_0 \left[ (1 + z)^3 \Omega_{M0} - (1 + z)^2 \Omega_{K0} \right. $$

$$+ \exp \left( 3 \int_0^z \frac{1 + w(z)}{1 + z} dz \right) \Omega_{Q0} \right]^{1/2},$$

where $\Omega_{M0}$, $\Omega_{Q0}$ are the present density parameter of matter and dark energy components, respectively, and $\Omega_{K0} = \Omega_{M0} + \Omega_{Q0} - 1$ is the present curvature parameter. Evaluation of the analytic form of the apparent correlation function requires the comoving distance $\chi(z)$, the growth factor $D(z)$, and the logarithmic derivative of the growth factor $f(z) = d \ln D / d \ln n$ as functions of redshift. All these quantities are determined by the time-dependent Hubble parameter. The comoving distance is simply given by $\chi(z) = \int_0^z H^{-1}(z) dz$. The growth factor is the growing solution of the differential equation

$$D + 2HD - \frac{3}{2} H_0^2 \Omega_{M0} (1 + z)^3 D = 0,$$

where dot represents the differentiation with respect to the time $t$. It is useful to rewrite this equation into the following set of equations,

$$\frac{d \ln D}{d \ln a} = f,$$

$$\frac{df}{d \ln a} = -f^2 - \left( 1 - \frac{\Omega_M}{2} - \frac{1 + 3w}{2} \Omega_Q \right) f + \frac{3}{2} \Omega_M,$$

where $a = (1 + z)^{-1}$ is the scale factor of the universe, and $\Omega_M(z) = H_0^2 (1 + z)^3 \Omega_{M0}/H^2(z)$, $\Omega_Q(z) = H_0^2 \exp(3 \int_0^z \frac{1 + w}{1 + z} dz) \Omega_{Q0}/H^2(z)$ are the time-dependent density parameters of matter and dark energy, respectively. The Runge-Kutta integration of the set of equations (2), (3) simultaneously gives the growth factor and logarithmic derivative of the growth factor. Equations (2), (3) are valid even when $w$ evolves with time.

Once the correlation matrix is evaluated, it is straightforward to obtain expected bounds one can impose on a set of model parameters by a given data set, thanks to the Fisher information matrix. In linear regime, the distribution of the counts $n_i$ is multi-variate Gaussian. In this case, the Fisher information matrix has a simple form $F_{\alpha\beta} = \frac{1}{2} \text{Tr}(C^{-1}C_{,\alpha}C_{,\beta})$ \cite{20, 23, 25}, where $C$ is a theoretical model of the correlation matrix, $\alpha$ and $\beta$ are indices to indicate a kind of model parameter and $C_{,\alpha}$ is a derivative of the correlation matrix with respect to a model parameter. An inverse of a Fisher matrix gives an estimate of the minimum error variance of a given set of model parameters \cite{23}.

We consider the LRG sample of the SDSS as a specific example. The survey area is assumed to be $5$ steradian, the redshift range to be $0.2–0.4$. The number density of the LRGs is approximately homogeneous and is given by $10^{-4} h^{-3} \text{Mpc}^{-3}$ \cite{13}. To reduce the computational cost, we set a sub-region of a $10^2 \pi \approx 314$ square degree field in a redshift range $0.2–0.4$, and fill spherical cells of radius $15h^{-1} \text{Mpc}$ in this region. We assume nonlinear effects of velocity distortions are erased by this choice of smoothing radius. In actual applications, it is important to test the choice of smoothing radius to ensure nonlinear distortions do not bias the result. We use the cubic closed-packed structure which has the maximum spatial filling factor of 0.74 that can be filled with spherical cells without overlapping each other (there are other ways to achieve this maximum factor such as the hexagonal closed-packed structure, etc.). As a result, about 2500 cells are placed in this sub-region. The Fisher information matrix is scaled according to the ratio of the volume of the sub-region to the total volume.

The model parameters for the dark energy are $\Omega_{Q0}$ and $w(z)$. To characterize an evolutionary effect of EOS, we employ a parameterization $w(z) = w_0 + w_1 z$, which is a good approximation when the evolution of EOS is mild in our interested redshift range of $z < 0.4$. Therefore, we have three model parameters $\Omega_{Q0}$, $w_0$, and $w_1$ for the dark energy component. There are other cosmological
parameters to compute the model correlation matrix. We assume a cold dark matter (CDM) power spectrum which depends on the shape parameter $\Gamma = \Omega_{M0} h$, and the normalization $\sigma_8$ of mass. The apparent redshift-space correlation function depends on $\Omega_{M0}$ and bias parameter $b$, as well as dark-energy parameters and the power spectrum. In the following we choose the curvature parameter $\Omega_{K0}$ as an independent parameter instead of $\Omega_{M0}$. Thus we have seven model parameters. In the following analysis, we set fiducial values $\Omega_{Q0} = 0.7$, $w_0 = -1$ (or $-0.5$), $w_1 = 0$, $\Omega_{K0} = 0$, $h = 0.7$, $\sigma_8 = 1$, $b = 2$, and see how one can constrain the parameters around this models.

First, we consider a situation in which only two parameters of the dark energy are constrained, assuming all the other cosmological parameters are fixed. Fig. 1 shows the expected error bounds on two of dark energy parameters are varied. In $\Omega_{Q0} - w_0$ and $w_0 - w_1$ panels, two cases with fiducial values $w_0 = -1, -0.5$ are shown, and in $\Omega_{Q0} - w_1$ panel, only $w_0 = -1$ case is presented. Inner and outer ellipses correspond to expected $1\sigma$ bounds of one-parameter distributions, and $1\sigma$ bounds of two-parameter joint distributions, respectively. Solid ellipses represent the case in which bias parameter is fixed, as well as other cosmological parameters. Since the bias parameter is the most important uncertainty in analyses of the galaxy clustering, we also plot the expected bounds marginalized over bias parameter (dotted ellipses). This corresponds to the case when bias should be simultaneously constrained together with dark-energy parameters in a same data set.

One may suspect that the constraints on dark energy are imposed mainly from the evolution of the growth factor $D(z)$, rather than the AP effect. In which case, the uncertainty in a possible bias evolution could demolish the constraints. To see this is not the case, we run a modified code to calculate the Fisher matrix for an imaginary model in which the growth factor does not evolve at all. The resulting values of Fisher matrix are only $4\%$, $30\%$, and $35\%$ smaller than the original values for $\Omega_{Q0}, w_0 - w_0$, and $w_1 - w_1$ components, respectively. Therefore, the AP effect does mainly constrain the dark energy parameters.

In the $\Omega_{Q0} - w_0$ panel, non-evolved EOS models can be constrained surprisingly well. The bias uncertainty does not significantly demolish the promise of this aspect. In the $w_0 - w_1$ panel, the two EOS parameters $w_0$ and $w_1$ are strongly correlated. This suggests that the redshift range $0.2 - 0.4$ is not large enough to distinguish evolved model and non-evolved model, and that the constraints on $w$ mainly depend on an effective value of $w$. In fact, the degeneracy is represented by $w_0 + 0.13 w_1 = \text{const.}$, which means the effective EOS is given by $w_{\text{eff}} = w(z = 0.13)$. Since the EOS affects the apparent clustering through Eq. (10), it is reasonable that the effective redshift is halfway to the sample volume.

To see how the uncertainty of the EOS evolution affects the constraints on $\Omega_{Q0}, w_{\text{eff}}$, we re-parameterize the EOS by $w(z) = w_{\text{eff}} + (z - 0.13) w_1$. In Fig. 2, expected bounds are plotted for the case when one of the dark-energy parameters $\Omega_{Q0}, w_{\text{eff}}, w_1$ are marginalized over. The error bounds on $\Omega_{Q0}$ and $w_{\text{eff}}$ are barely affected by $w_1$ marginalization. This means that our method is not sensitive to the uncertainty of the EOS evolution, and is only sensitive to the effective value of the EOS.

In the above analyses, the cosmological parameters other than that of the dark-energy component and bias parameter are fixed. These other parameters can be determined from various cosmological observations. In order to estimate the effects of the uncertainties from these other parameters, error bounds between dark-energy parameters and other parameters are plotted in Fig. 3, without any other marginalization. Each panel in this figure indicates a correlation between each pair of parameters. Correlations with $\sigma_8$ are not plotted, but are quite similar to those with bias $b$, as naturally expected. The parameter $\Omega_{Q0}$ is positively correlated with $\Omega_{K0}$ and $h$. This is because the galaxy clustering accurately constrains $\Omega_{M0} h = (1 + \Omega_{K0} - \Omega_{Q0}) h$. If we do not use prior knowledge of either the curvature or the Hubble’s constant, the error bound on $\Omega_{Q0}$ is enlarged by a factor of $4 - 5$. There are negative correlations between each dark-energy parameter and the bias parameter. This can be understood by noticing that both parameters increase the amplitude of fluctuations when the present amplitude.

![Figure 1](image1.png)
is fixed. The EOS parameters are quite independent on the curvature and Hubble’s constant.

In summary, we predict for the first time expected degree of constraints on dark energy models from intermediate-redshift galaxies such as LRGs, and show that $\Omega_{Q0}$ and $w_{\text{eff}}$ can be constrained surprisingly well.

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The prior input of the curvature and the Hubble’s constant are important to reduce the bounds on the dark energy density, but not so important on the EOS parameters.

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