Are Post-Newtonian templates faithful and effectual in detecting gravitational signals from neutron star binaries?

E. Berti\textsuperscript{1}, J.A. Pons\textsuperscript{2}, G. Miniutti\textsuperscript{2}, L. Gualtieri\textsuperscript{2} and V. Ferrari\textsuperscript{2}

\textsuperscript{1} Department of Physics, Aristotle University of Thessaloniki, Thessaloniki 54006, Greece
\textsuperscript{2} Dipartimento di Fisica “G.Marconi”, Università di Roma “La Sapienza” and Sezione INFN ROMA1, piazzale Aldo Moro 2, I-00185 Roma, Italy

Abstract

We compute the overlap function between Post-Newtonian (PN) templates and gravitational signals emitted by binary systems composed of one neutron star and one point mass, obtained by a perturbative approach. The calculations are performed for different stellar models and for different detectors, to estimate how effectual and faithful the PN templates are, and to establish whether effects related to the internal structure of neutron stars may possibly be extracted by the matched filtering technique.
I. INTRODUCTION

The detection of gravitational waves emitted during the late inspiral and merger phases of coalescing compact binaries is one of the main targets of the ground based interferometric detectors that are currently in the final stage of construction or in the commissioning phase (LIGO, VIRGO, GEO600, TAMA). To detect these signals, a detailed knowledge of the emitted waveforms is fundamental; indeed, the performances of the matched filter which will be used to extract the signal from the detectors noise, largely depend on the capability of the theoretical templates to reproduce the true waveforms.

Although the population of neutron star-neutron star (NS-NS) binaries is expected to double that of black hole-black hole (BH-BH) binaries, and to be several times larger than that of (NS-BH) [1], BH-BH binaries having a total mass of \( \sim (20 - 40) \, M_\odot \) will likely be detected first by the initial ground-based interferometers, because, due to their larger mass, the signal is more intense in the frequency region where the detectors are more sensitive [2]. In the future, however, as the detectors sensitivity in the high frequency region improves, NS-NS coalescence should become detectable as well. According to recent investigations [3–6], the signal emitted during the latest phases preceding coalescence differs from that emitted by two black holes essentially in one respect: the modes of oscillation of the stars could be “marginally excited”. A mode is resonantly excited if the system moves on an orbit such that the Keplerian orbital frequency, \( \omega_k \), is in a definite ratio with the mode frequency \( \omega_i \), i.e. if \( \ell \omega_k = \omega_i \), where \( \ell \) is the harmonic parameter. In general the frequency of the fundamental mode is too high to be excited directly, because the stars merge before reaching the corresponding orbit [4]; however, the width of the resonance of the fundamental mode (especially for \( \ell = 2 \)) is large enough to allow the mode to be marginally excited before the resonant frequency is reached. As a consequence, more energy is emitted with respect to that due to the orbital motion, the process of inspiralling is accelerated, and this changes the phase of the emitted signals during the last orbits before merging. This effect is stronger for stiffer equations of state (EOSs), or for low mass NSs, for which the frequency of the fundamental mode is lower, and the width of the resonance is larger [6]. Thus, an accurate detection of the signal emitted in a NS-NS binary coalescence besides probing the theory of gravity, as BH-BH signals would do, would also give an insight into the equation of state of matter at high density regimes unreachable in a terrestrial laboratory.

The aim of this paper is to investigate whether the templates that are being constructed to extract the signal emitted by inspiralling binaries from the detectors noise are well suited to detect a NS-NS coalescence.

II. PERTURBATIVE APPROACH VERSUS POST-NEWTONIAN EXPANSIONS.

The basic functions needed to construct the gravitational wave (GW) templates are the orbital energy of the system, \( E(v) \), and the gravitational luminosity, \( \dot{E}_{GW}(v) \), where \( v \) is the orbital velocity. We evaluate \( \dot{E}_{GW}(v) \) by using a perturbative approach, assuming that one of the two bodies is a neutron star (hereafter we shall refer only to non rotating stars and black holes), whose equilibrium structure is described by a solution of the relativistic equations of hydrostatic equilibrium; the second body is a test-particle in circular orbit, which induces a
perturbation on the gravitational field and on the thermodynamical structure of the extended companion. By this approach, we can account for the relativistic tidal effects of the close interaction on one of the two stars, and for the effects that the internal structure has on the gravitational emission. Since the particle moves on a geodesic, its orbital energy normalized to the mass of the star, \( E(v) \), is known

\[
E(v) = \eta \left(1 - 2v^2\right) \left(1 - 3v^2\right)^{-1/2};
\]

(2.1)
in this formula \( \eta = m_1/m_2, \) \( m_2 \) is the mass of the central body, \( m_1 \ll m_2 \) is the particle mass, \( v = (\pi m_2 \nu_{GW})^{1/3} \) is the orbital velocity, and \( \nu_{GW} \) is the frequency of the emitted radiation. It is useful to define the normalized GW-luminosity that will be used in the following

\[
P(v) \equiv \dot{E}_{GW}(v)/\dot{E}_N(v),
\]

(2.2)
where \( \dot{E}_N(v) = 32\eta^2v^{10}/5 \) is the Newtonian quadrupole luminosity. In [5,6] we solved the relativistic equations of stellar perturbations and computed \( \dot{E}_{GW}(v) \) during the final stages of inspiralling of this idealized binary system. In [6] we considered 5 models of polytropic stars, labelled from A to E, with parameters chosen to cover most of the range of structural properties obtained with realistic EOSs. These parameters are given in Table I. In the following, we shall refer to the GW signals computed for the different models as the true signals.

In the standard PN approach, \( E(v) \) and \( \dot{E}_{GW}(v) \) are found by assuming that both compact objects are pointlike, and by expanding the general relativistic equations of motion and the wave generation formulas in powers of \( v/c \). In the test-particle limit \( E(v) \) is known exactly (Eq. 2.1); \( \dot{E}_{GW}(v) \) has been derived by Taylor expanding up to \( (v/c)^{11} \) the solution of the Bardeen-Press-Teukolsky (BPT) equation [7,8], describing the perturbations of a Schwarzschild black hole driven by an orbiting test-particle [9–11].

In the case of comparable masses, the equations of binary motion have been computed at the 3PN order, and the energy flux has been evaluated without ambiguities up to 2.5PN order with respect to the quadrupole formalism (PN calculations to order \( (v/c)^7 \), developed by currently used techniques, leave undetermined a parameter entering at order \( (v/c)^6 \) [12]). Important progresses have recently been made by the introduction of re-summation techniques (see [13] and references therein), which improve the convergence of the PN series (the Taylor expansion is indeed rather poorly convergent), and new filters in the frequency domain have been proposed, which combine the performance of Padé-approximants with the simplicity of the stationary phase approximation. Summarizing, the PN approach allows to compute the phase evolution of the GW signal emitted by a binary system to order \( (v/c)^{11} \) in the test particle limit, and to order \( (v/c)^5 \) in the case of comparable masses.

Since tidal effects do not affect the evolution of a BH-BH binary system, even when the black holes have comparable masses [14], PN expansions are particularly well suited to evaluate the waveform in the case of BH-BH inspiralling. However, when at least one of the coalescing bodies is a neutron star, the effects that its internal structure may have on the gravitational emission, such as the modes excitation, have not been investigated in great detail until recently. Thus, the following question arises. Suppose that, by using the PN approach described above we construct a GW template and suppose that the gravitational event involves at least one neutron star: what would we miss in using the PN template in
the data analysis? Would the signal to noise ratio deteriorate because we are using an inappropriate template, or would it be good enough to detect the signal anyway? What would be the error in estimating the binary parameters? Or, using the terminology introduced in [13], how effectual and faithful is to use BH templates when the signal comes from NS binaries?

It should be stressed that, to date, there is no fully non linear, dynamical simulation of the inspiralling and coalescence of stars or black holes with comparable masses, which provides an exact waveform. For this reason the calibration and convergence tests of PN expansions for comparable masses have been done as follows [13]: the functions $P(v)$ is computed 1) by integrating the BPT equation for a perturbed black hole excited by an orbiting test particle and 2) by computing the PN expansion of the function to the desired order, and ignoring the dependence of the expansion coefficients on the symmetric mass ratio $\eta = (m_1 \cdot m_2) / (m_1 + m_2)^2$ (note that if $m_1 \ll m_2$, $\eta \rightarrow m_1 / m_2$). In this way, the $\eta$-dependence is kept only at the leading order. The two signals are then compared. The effectualness of the PN templates is computed by a naive approximation, by using the signal obtained by integrating the BPT equation as a true signal, and the PN approximants as templates, allowing $\eta$ to assume a finite value (see for instance Table V in [13]).

In a similar way, to answer the questions above we shall use the PN templates obtained as in 2) and replace 1) by the integration of the equations of stellar perturbations excited by the orbiting test particle. We hope that the lessons we learn from the results of this comparison will be useful when more accurate models for the signals emitted by real binary systems will be available. In addition, since no generalization of the perturbative calculations to the equal mass case is presently available, establishing whether or not the structural effects are expected to be relevant from the point of view of detection would provide a motivation for looking for a suitable generalization of the perturbative approach.

### III. EFFECTS OF NEUTRON STAR STRUCTURE.

As an example of the effects that the internal structure of the neutron star has on the emitted signal, in Fig. 1a) we show the normalized GW-luminosity (2.2), $P(v)$, as a function of the orbital velocity, computed for a BH and for a neutron star perturbed by a test-particle in circular orbit. The curve for the star refers to a NS of mass $1.4 M_\odot$ and radius $R = 15$ km, labeled as model B (see Table I). The gravitational luminosity $\dot{E}_{GW}(v)$ is obtained by solving the equations of stellar perturbations as described in [6], and the BPT equation for a Schwarzschild black hole excited by the same source. For comparison, in the same figure we plot the function $P(v)$ as computed by the PN formalism at order 2.5 (both Padé and Taylor), and at order 5.5 (only Taylor). For a definition of Taylor and Padé approximants we refer to [13].

When the central object is a neutron star, the resonant excitation of the quasi-normal modes (the peaks in Fig. 1a) clearly changes the shape of the function $P(v)$ with respect to the BH case, and the effect starts to be seen at orbital velocities larger than 0.18 [6]. We see that P-approximants rapidly converge to the BH solution in the test-particle limit. As discussed in [13], T-approximants of order 2.5 are especially bad-behaved, but P-approximants seem to reproduce very well the function $P(v)$ for a BH, even at the 2.5 PN order. Templates built in this way are fast and convenient to be used in the laborious matched-filtering
process. However, for sufficiently high order, both T- and P-approximants converge to the BH solution by construction, since \( P(v) \) is computed as a series expansion about the exact numerical solution of the BPT equation.

In order to appreciate the differences between the emission of different stellar models, in Fig. 1b) we show the function \( P(v) \) computed for the models given in Table I, restricted to the region \( v < 0.28 \), which corresponds to a distance between the two stars larger than 2.5-3 stellar radii depending on the model. The normalized energy fluxes emitted by different stellar models have a different slope, and are always larger than the flux emitted by the black hole, which is also shown for comparison. The curve for model E is practically indistinguishable from the black hole curve and that for model D is also very close to the black hole result. The steepest raise of the curves of models A,B,C is a marginal effect of the excitation of the quasi normal modes.

It is clear from Fig. 1b) that the differences are small, and whether or not they could play a role in the detection depends on the characteristics of the detector. This issue is discussed in a more quantitative way in the next sections.

IV. FAITHFULNESS AND EFFECTUALNESS

We shall now evaluate how effectual and faithful the PN templates are in detecting a signal emitted by an inspiralling neutron star binary system. We shall assume that the functions \( \dot{E}_{GW}(v) \) and \( E(v) \) are known both for the PN template and for the true signal obtained by the perturbative approach. The two waveforms \( h^A(t) \) (the PN template) and \( h^X(t) \) (the true NS signal) will be computed in the so-called restricted PN approximation: the amplitude of the waveform is taken at the lowest PN order, while the orbital phase evolution of the binary during the quasi-stationary inspiralling is evaluated by numerically integrating the equations

\[
\frac{dv}{dt} = -\frac{1}{M_{tot}} \frac{\dot{E}_{GW}(v)}{dE(v)/dv}, \tag{4.1}
\]

\[
\frac{d\phi}{dt} = \frac{2}{M_{tot}} v^3, \tag{4.2}
\]

where \( M_{tot} = m_1 + m_2 \). Explicitly, the waveform from a source at a distance \( r \) from Earth is:

\[
rh(t) = 4C\eta M_{tot} v^2(t) \cos \phi(t), \tag{4.3}
\]

where \( C \) is a constant which depends on the relative orientation of the source and the detector [15]. Let us define the Wiener inner product between the waveform \( h^A \) and \( h^X \), shifted by a time lag \( \tau \), as

\[
\langle h^A, h^X \rangle = \int_{-\infty}^{\infty} \frac{dv e^{2\pi i v \tau}}{S_N(v)} \tilde{h}^A(v) \tilde{h}^X^*(v), \tag{4.4}
\]

where \( S_N(v) \) is the one-sided power spectral density of the detector’s noise, and \( \tilde{h}^A, \tilde{h}^X \) are the Fourier transforms of \( h^A(t), h^X(t) \). We compute \( \tilde{h} \) using standard FFT routines because
we prefer to avoid further uncertainties introduced when $\tilde{h}^{A,X}$ are obtained in the stationary phase approximation. The noise curves of LIGO I and VIRGO I which we are going to use are taken from [16], and those of EURO and EURO-Xylophone, the third-generation interferometric detector recently proposed, from [17]. The explicit expressions of $S_N(\nu)$ are given in Table II.

From the Wiener scalar product and the associated norm we can build the so-called ambiguity function

$$A = \max_{\tau, \Phi_A, \Phi_X} \frac{\langle h^A(t, \Phi_A, m_1, m_2), \hat{h}^X(0, \Phi_X, m_1, m_2) \rangle}{||h^A|| | |h^X||} = \max_{\tau, \Phi_A, \Phi_X} \langle \hat{h}^A, \hat{h}^X \rangle.$$  \hspace{1cm} (4.5)

where we denote by a hat the normalized waveforms: $\hat{h}^A = h^A / |h^A|$, and $\hat{h}^X = h^X / |h^X|$.

For simplicity, we shall assume that the approximate and exact waveforms $h^A$ and $h^X$ depend only on their initial phases and on the masses of the binary members (a more realistic parameterization would require the inclusion of spins, angular dependences, etc.):

$$h^A = h^A(t, \Phi^A_c, m_1, m_2), \quad h^X = h^X(t, \Phi^X_c, m_1, m_2).$$

The *faithfulness* is defined as

$$F_{AX} = \max_{\tau, \Phi_A^c, \Phi_X^c} \langle \hat{h}^A(\tau, \Phi_A^c, m_1, m_2), \hat{h}^X(0, \Phi_X^c, m_1, m_2) \rangle,$$  \hspace{1cm} (4.6)

i.e., as the maximum of the ambiguity function over the initial phases and the time lag, when the source and template parameters (in our case the masses) are matched. To maximize over the difference in times of arrival $\tau$ and over the phases we follow the approach described in the Appendix A of Ref. [13].

In practice, in a detection the source parameters are unknown. The maximum of the ambiguity function with respect to phases and times of arrival will be smaller than one, and will occur, in general, when the parameters of the source and template are not equal. In this case it is useful to compute the *effectualness*, i.e. to maximize the ambiguity function over all the parameters of the template

$$E_{AX} = \max_{\tau, \Phi_A^c, \Phi_X^c, m_1^A, m_2^A} \langle \hat{h}^A(\tau, \Phi_A^c, m_1^A, m_2^A), \hat{h}^X(0, \Phi_X^c, m_1^X, m_2^X) \rangle.$$  \hspace{1cm} (4.7)

Thus, the computation of the *effectualness* differs from that of the *faithfulness* essentially in one respect: we maximize not only over the difference in times of arrival $\tau$ and over the phases of the waves, but also over the masses of the template waveform $m_1^A, m_2^A$, or, equivalently, over the symmetric mass ratio $\eta^A = m_1^A m_2^A / (M_{tot}^A)^2$ and over the total “chirp mass” $M^A = (\eta^A)^{3/5} M_{tot}^A$.

A reasonable criterion for a template to be effectual is that the ambiguity function thus computed should be larger than 0.965, which ensures that no more than 10% of the events are lost (Number of lost events$= 1 - E_{AX}^3$).

**V. RESULTS AND CONCLUDING REMARKS**

In Fig. 2 we show the contour levels of constant ambiguity function, as a function of $M$ and $\eta$. We compute the *minimax* overlap, which is physically more relevant for detection than
the best overlap, as explained in [13]. The true waveform is that emitted by the stellar model D excited by an orbiting point particle of $1.4M_\odot$, while the PN template is Padé-6. The noise curves are, respectively, those of VIRGO I, EURO and EURO-Xylophone, indicated as EURO-X. We do not plot the curves for LIGO I, because they are very similar to those of VIRGO I. The true parameters of the binary system are $\eta = 0.25$ and $M = 1.2188M_\odot$. Model D is one with a soft EOS, and it is very compact. The frequency of the fundamental mode is too high to be excited at a significant level, and indeed in Fig. 1b) we see that its GW-luminosity is very close to that of a black hole. Thus, we expect that in this case both the chirp mass and $\eta$ will be accurately determined by the PN templates, that are constructed for black hole coalescence. This is confirmed in Fig. 2 for all the considered detectors. From the three panels of the figure we see that the most important parameter in determining the overlap function is the chirp mass, which can be inferred with an error smaller than one part in a thousand (notice the scale on the $x$-axis). This fact was already noted in refs. [18,2]. The dependence on the symmetric mass ratio is found to be somewhat weaker, and the relative error in its estimation is about 2-3 %.

It is interesting to plot a similar figure for the stellar model B. This model has a stiff EOS, and the marginal excitation of the fundamental mode before merging is visible in Fig. 1b). From Fig. 3 we see that if the detector is VIRGO, the chirp mass and the mass ratio where $A$ has a maximum are $M = 1.2185M_\odot$ and $\eta = 0.234$; if the detector is EURO, $M = 1.2178M_\odot$ and $\eta = 0.225$, and if the detector is EURO-Xylophone $M = 1.2161M_\odot$ and $\eta = 0.213$. Thus, whereas the chirp mass would still be determined to a very good accuracy the determination of $\eta$ would be less accurate if the detectors are EURO-type, i.e. very sensitive at high frequency, and if the templates remain tuned to the black hole signal. This can be understood also by looking at Table III, where we give the values of the effectualness for LIGO I, VIRGO I, EURO and EURO-X. The true signal is that emitted by the five models of NSs given in Table I; as a template we use the Padé-6 approximant (column 1 for each detector), and the signal obtained by integrating the BPT equation for a Schwarzschild black hole perturbed by an orbiting particle (column 2), because the Padé approximant is nothing but an approximation of this signal; thus we wanted to check what is the change if we use as a template the exact signal emitted by a black hole. We see that the performances of $P$-approximants, as well as those of BH approximants, degrade if the detector is very sensitive in the high frequency region. In this case, the use of templates which account for effects of stellar structure would be needed. In Table IV we show the analogous results for the faithfulness.

The required effectualness threshold of 0.965 is always achieved in LIGO I for all of the stellar models; for VIRGO I it is a little lower because the detector is more sensitive at high frequency. Thus, if the coalescing binary system is composed of two neutron stars, both LIGO and VIRGO would be able to detect it by using the standard PN templates, and to determine the masses with a sufficient accuracy, provided the event occurs close enough to be visible by these instruments.

If the noise curve is that of EURO or EURO-Xylophone, the effectualness is lower and a relevant fraction of events would be missed using the standard PN templates; for instance EURO would miss $\sim 36\%$ of the events if the stars have low mass as in the stellar model A, and $\sim 18\%$ if the EOS is that of model B. For EURO-Xylophone it would be worse: $\sim 78\%$ events missed for model A, $\sim 57\%$ for model B and $\sim 33\%$ for model C. We would like to
emphasize that this difference between neutron star models is what makes the situation more interesting. By constructing filters which include resonant effects, we would both increase the chances to detect NS-NS events and be able to estimate the oscillation frequencies of NSs. Knowing the mass of the stars, these could be used to infer their radius, as suggested by recent investigations [19,20], and set constraints on the EOS of nuclear matter in the supranuclear density regime [21]. In addition, it should be stressed that the imprint that the internal structure of the stars leaves on the GW signal may be enhanced by rotation, the effect of which is to lower some of the mode frequencies; this would shift the effect of the mode excitation toward lower frequencies and amplify the signal in the region where EURO-type detectors would be more sensitive.

ACKNOWLEDGMENTS

We would like to thank Dr. B.S. Sathyaprakash for useful discussions on the use of PN templates and for providing us the correct noise curve of EURO.

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TABLE I. Parameters of the polytropic stars we consider in our analysis: the polytropic index $n$, the central density, the ratio $\alpha_0 = \epsilon_0/p_0$ of central energy density to central pressure, the mass, the radius and the ratio $M/R$ ($\alpha_0$ and $M/R$ are in geometric units). The central energy density is chosen in such a way that the stellar mass is equal to $1.4M_\odot$, except for model A, the mass of which is about one solar mass.

<table>
<thead>
<tr>
<th>Model number</th>
<th>$n$</th>
<th>$\rho_c$ (g/cm$^3$)</th>
<th>$\alpha_0$</th>
<th>$M$ ($M_\odot$)</th>
<th>$R$ (km)</th>
<th>$M/R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.5</td>
<td>$1.00 \times 10^{15}$</td>
<td>13.552</td>
<td>0.945</td>
<td>14.07</td>
<td>0.099</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>$6.584 \times 10^{14}$</td>
<td>9.669</td>
<td>1.4</td>
<td>15.00</td>
<td>0.138</td>
</tr>
<tr>
<td>C</td>
<td>1.5</td>
<td>$1.260 \times 10^{15}$</td>
<td>8.205</td>
<td>1.4</td>
<td>15.00</td>
<td>0.138</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>$2.455 \times 10^{15}$</td>
<td>4.490</td>
<td>1.4</td>
<td>9.80</td>
<td>0.211</td>
</tr>
<tr>
<td>E</td>
<td>1.5</td>
<td>$8.156 \times 10^{15}$</td>
<td>2.146</td>
<td>1.4</td>
<td>9.00</td>
<td>0.230</td>
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</table>

TABLE II. One-sided power spectral densities, $S_N$, for the interferometers considered in this paper. For each detector $S_N$ is given as a function of the dimensionless frequency $x = \nu/\nu_0$, and is considered to be infinite below the seismic cutoff frequency $\nu_s$.

<table>
<thead>
<tr>
<th>Detector</th>
<th>$\nu_s$/Hz</th>
<th>$\nu_0$/Hz</th>
<th>$10^{46} \times S_N(x)/\text{Hz}^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIRGO I</td>
<td>20</td>
<td>500</td>
<td>$3.24 \left[(6.23x)^{-5} + 2x^{-1} + 1 + x^2\right]$</td>
</tr>
<tr>
<td>LIGO I</td>
<td>40</td>
<td>150</td>
<td>$9 \left[(4.49x)^{-56} + 0.16x^{-4.52} + 0.52 + 0.32x^2\right]$</td>
</tr>
<tr>
<td>EURO</td>
<td>10</td>
<td>1000</td>
<td>$10^{-4} \left[0.0036x^{-4} + 0.13x^{-2} + 1.3(1 + x^2)\right]$</td>
</tr>
<tr>
<td>EURO-X</td>
<td>10</td>
<td>1000</td>
<td>$10^{-4} \left[0.0036x^{-4} + 0.13x^{-2}\right]$</td>
</tr>
</tbody>
</table>

TABLE III. Ambiguity function maximized over all parameters (Effectualness) using as templates the Padé-6 approximant ($P_6$) and the black hole signal (BH), assuming that the two inspiralling masses are equal. The true signal is that emitted by the five models of neutron stars given in Table I [6]. Values quoted are the minimax overlaps.

<table>
<thead>
<tr>
<th></th>
<th>LIGO I</th>
<th>VIRGO I</th>
<th>EURO</th>
<th>EURO-X</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\langle P_6, X \rangle$</td>
<td>$\langle BH, X \rangle$</td>
<td>$\langle P_6, X \rangle$</td>
<td>$\langle BH, X \rangle$</td>
</tr>
<tr>
<td>A</td>
<td>0.971</td>
<td>0.972</td>
<td>0.911</td>
<td>0.913</td>
</tr>
<tr>
<td>B</td>
<td>0.984</td>
<td>0.984</td>
<td>0.965</td>
<td>0.968</td>
</tr>
<tr>
<td>C</td>
<td>0.992</td>
<td>0.993</td>
<td>0.982</td>
<td>0.984</td>
</tr>
<tr>
<td>D</td>
<td>0.999</td>
<td>0.999</td>
<td>0.997</td>
<td>0.998</td>
</tr>
<tr>
<td>E</td>
<td>0.999</td>
<td>1.000</td>
<td>0.995</td>
<td>0.999</td>
</tr>
</tbody>
</table>
TABLE IV. Ambiguity function maximized over the phases and the time lag (*Faithfulness*). Templates and true signals are chosen as in Table II. Values quoted are the *minimax* overlaps.

<table>
<thead>
<tr>
<th></th>
<th>LIGO I</th>
<th>VIRGO I</th>
<th>EURO</th>
<th>EURO-Xylo</th>
</tr>
</thead>
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<tr>
<td></td>
<td>(P₀, X)</td>
<td>(BH, X)</td>
<td>(P₀, X)</td>
<td>(BH, X)</td>
</tr>
<tr>
<td>A</td>
<td>0.955</td>
<td>0.957</td>
<td>0.867</td>
<td>0.869</td>
</tr>
<tr>
<td>B</td>
<td>0.977</td>
<td>0.980</td>
<td>0.923</td>
<td>0.924</td>
</tr>
<tr>
<td>C</td>
<td>0.989</td>
<td>0.991</td>
<td>0.945</td>
<td>0.953</td>
</tr>
<tr>
<td>D</td>
<td>0.998</td>
<td>0.997</td>
<td>0.994</td>
<td>0.986</td>
</tr>
<tr>
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<td>1.000</td>
<td>0.992</td>
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</tr>
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</table>

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FIG. 1. The normalized GW-luminosity, $P(v)$, is plotted versus the orbital velocity. On the left, we show the curve for the stellar model B and for a black hole. For comparison, we also show the 2.5 PN Padé and Taylor approximants, and the 5.5 PN Taylor approximant. The peaks correspond to the excitation of the $f$–mode of the neutron star for different $l$s, and the wider resonance corresponds to $l = 2$. On the right, we plot $P(v)$ for all stellar models given in Table I and for a black hole, for a smaller orbital velocity range.
FIG. 2. Constant (minimax) overlap levels are plotted as a function of the chirp mass $\mathcal{M}$ and of the symmetric mass ratio $\eta$. The true signal is emitted by the stellar model D with an orbiting test particle of $1.4M_\odot$ (the true values of $\eta$ and $\mathcal{M}$ are $\eta = 0.25$, and $\mathcal{M} = 1.2188M_\odot$), the PN template is Padé-6 and the noise curves are those of VIRGO, EURO and EURO-Xylophone (EURO-X). The diamond indicates the maximum of the ambiguity function.
FIG. 3. Constant (minimax) overlap levels are plotted as in Fig. 2, with the true signal emitted by the stellar model B + orbiting test particle (true values: $\eta = 0.25$, and $M = 1.2188 M_\odot$), and the noise curve of VIRGO, EURO and EURO-Xylophone.
REFERENCES