Strong-Electroweak Unification at About 4 TeV

Paul H. Frampton (1,2)
(1) TH Division, CERN, CH1211 Geneva 23, Switzerland.
(2) University of North Carolina, Chapel Hill, NC 27599, USA.

It is shown how an SU(N)\textsuperscript{M} quiver gauge theory can accommodate the standard model with three chiral families and unify all of SU(3)\textsubscript{C}, SU(2)\textsubscript{L} and U(1)\textsubscript{Y} couplings with high accuracy at one unique scale estimated as \(M \simeq 4\) TeV.

Even before the standard model (SM) SU(2) \times U(1) electroweak theory was firmly established by experimental data, proposals were made [1,2] of models which would subsume it into a grand unified theory (GUT) including also the dynamics [3] of QCD. Although the prediction of SU(5) in its minimal form for the proton lifetime has been excluded, ad hoc variants thereof [4] remain viable. Low-energy supersymmetry improves the accuracy of unification of the three 321 couplings [5,6] and such theories encompass a "desert" between the weak scale \(\sim 100\) GeV and the much-higher GUT scale \(\sim 10^{16}\) GeV.

Recent developments in string theory are suggestive of a different strategy for unification of electroweak theory with QCD. Both the desert and low-energy supersymmetry are abandoned. Instead, the standard SU(3)\textsubscript{C} \times SU(2)\textsubscript{L} \times U(1)\textsubscript{Y} gauge group is embedded in a semi-simple gauge group such as SU(3)\textsuperscript{N} as suggested by gauge theories arising from compactification of the IIB superstring on an orbifold AdS\textsubscript{5} \times S\textsuperscript{5}/\Gamma where \(\Gamma\) is the abelian finite group \(\mathbb{Z}_N\) [7]. In such nonsupersymmetric quiver gauge theories the unification of couplings happens not by the logarithmic evolution [3] over an enormous desert covering a dozen orders of magnitude in energy scale. Instead the unification occurs abruptly at \(\mu = M\) through the diagonal embeddings of 321 in SU(3)\textsuperscript{N} [8]. The key prediction of such unification shifts from proton decay to additional particle content, not at the weak scale but at \(\simeq 4\) TeV.

Let us first consider the electroweak group which in the standard model is still un-unified as SU(2) \times U(1). In the 331-model [9,10] where this is extended to SU(3) \times U(1) there appears a Landau pole at \(M \simeq 4\) TeV because that is the scale at which \(\sin^2\theta(\mu)\) slides to the value \(\sin^2(M) = 1/4\). It is also the scale at which the custodial gauged SU(3) is broken in the framework of [11].

Such theories involve so far only an electroweak unification so in a bottom-up approach we should examine more carefully the running of all three of the SM couplings with \(\mu\) as explicated in e.g. [6]. Taking the values at the Z-pole \(\alpha_Y(M_Z) = 0.0101, \alpha_2(M_Z) = 0.0338, \alpha_3(M_Z) = 0.118 \pm 0.003\) (the errors in \(\alpha_Y(M_Z)\) and \(\alpha_2(M_Z)\) are less than 1%) they are take to run between \(M_Z\) and \(M\) according to the SM equations

\[
\alpha_Y^{-1}(M) = (0.01014)^{-1} - (41/12\pi)\ln(M/M_Z) = 98.619 - 1.0876y
\]

\[
\alpha_2^{-1}(M) = (0.0338)^{-1} + (19/12\pi)\ln(M/M_Z) = 29.586 + 0.504y
\]

\[
\alpha_3^{-1}(M) = (0.118)^{-1} + (7/2\pi)\ln(M/M_Z) = 8.474 + 1.114y
\]

where \(y = M/M_Z\).

The scale at which \(\sin^2\theta(M) = \alpha_Y(M)/(\alpha_2(M) + \alpha_Y(M))\) satisfies \(\sin^2\theta(M) = 1/4\) is found from Eqs.(1,2) to be \(M \simeq 4\) TeV as stated in the introduction above.

But let us focus on the ratio \(R(M) \equiv \alpha_3(M)/\alpha_2(M)\) using Eqs.(2,3). One finds that \(R(M_Z) \simeq 3.5\) while \(R(M_5) = 3, R(M_{5/2}) = 5/2\) and \(R(M_2) = 2\) correspond to \(M_3, M_{5/2}, M_2 \simeq 400\)GeV, 4TeV, and 140TeV respectively. The proximity of \(M_{5/2}\) and \(M\), accurate to a few percent, suggests to us strong-electroweak unification at \(\simeq 4\) TeV.

There remains the question of embedding such unification in an SU(3)\textsuperscript{N} of the type described in [7,8]. Since the required ratios of couplings at \(\simeq 4\) TeV is: \(\alpha_3 : \alpha_2 : \alpha_Y : 5 : 2 : 2\) it is natural to examine \(N = 12\) with diagonal embeddings of Color (C), Weak (W) and Hypercharge (H) in SU(3)\textsuperscript{2}\times SU(3)\textsuperscript{5}\times SU(3)\textsuperscript{5} respectively.

To accomplish this we need to specify the embedding of \(\Gamma = Z_{12}\) in the global SU(4) R-parity of the \(N = 4\) supersymmetry of the underlying theory. Defining \(\alpha = \exp(2\pi i/12)\) this specification can be made by \(4 \equiv (\alpha^{A_1}, \alpha^{A_2}, \alpha^{A_3}, \alpha^{A_4})\) with
\[ \Sigma A_\mu = 0 \pmod{12} \] and all \( A_\mu \neq 0 \) so that all four supersymmetries are broken from \( N = 4 \) to \( N = 0 \).

Having specified \( A_\mu \) we must calculate the content of complex scalars by investigating in \( SU(4) \) the \( 6 \equiv (a^{a_1}, a^{a_2}, a^{a_3}, a^{-a_3}, a^{-a_2}, a^{-a_1}) \) with \( a_1 = A_1 + A_2, a_2 = A_2 + A_3, a_3 = A_3 + A_1 \) where all quantities are defined (mod 12).

Finally one must identify the nodes (as C, W or H) on the dodecahedral quiver such that the complex scalars

\[ \Sigma_{i=1}^{i=3} \Sigma_{\alpha=1}^{\alpha=12} \left( N_\alpha, \bar{N}_{\alpha \pm a_i} \right) \quad (4) \]

are adequate to allow the required symmetry breaking to the \( SU(3)^3 \) diagonal subgroup, and the chiral fermions

\[ \Sigma_{\mu=1}^{\mu=4} \Sigma_{\alpha=1}^{\alpha=12} \left( N_\alpha, \bar{N}_{\alpha + A_\mu} \right) \quad (5) \]

can accommodate the three generations of quarks and leptons.

It is not trivial that one can accomplish all of these requirements so let us show the explicit example.

For the embedding we take \( A_\mu = (1,2,3,6) \) and for the quiver nodes take the ordering:

\[ -C - W - H - C - W^4 - H^4 - \quad (6) \]

with the two ends of (6) identified.

The scalars follow from \( a_1 = (3,4,5) \) and the scalars in Eq.(4)

\[ \Sigma_{i=1}^{i=3} \Sigma_{\alpha=1}^{\alpha=12} \left( 3_\alpha, \bar{3}_{\alpha \pm a_i} \right) \quad (7) \]

are sufficient to break to all diagonal subgroups as

\[ SU(3)_C \times SU(3)_W \times SU(3)_Y \quad (8) \]

The fermions follow from \( A_\mu \) in Eq.(5) as

\[ \Sigma_{\mu=1}^{\mu=4} \Sigma_{\alpha=1}^{\alpha=12} \left( 3_\alpha, \bar{3}_{\alpha + A_\mu} \right) \quad (9) \]

and the particular dodecahedral quiver in (6) gives rise to exactly three chiral generations which transform under (8) as

\[ 3[(3, \bar{3}, 1) + (\bar{3}, 1, 3) + (1, 3, \bar{3})] \quad (10) \]

After further symmetry breaking at scale \( M \) to \( SU(3)_C \times SU(2)_L \times U(1)_Y \) the surviving chiral fermions are the quarks and leptons of the SM. The appearance of three families depends on both the identification of modes in (6) and on the embedding of \( \Gamma \subset SU(4) \). The embedding must simultaneously give adequate scalars whose VEVs can break the symmetry spontaneously to (8). All of this is achieved successfully by the choices made. The three gauge couplings evolve according to Eqs.(1,2,3) for \( M_Z \leq \mu \leq M \). For \( \mu \geq M \) the (equal) gauge couplings of \( SU(3)^{12} \) do not run if, as conjectured in [7,8] there is a conformal fixed point at \( \mu = M \).

This is a non-gravitational theory with conformal invariance when \( \mu > M \) and where the Planck mass it taken to be infinitely large. The ubiquitous question is: What about gravity which breaks conformal symmetry in the ultraviolet (UV)? This is a question about the holographic principle for flat spacetime.

In any case, from the phenomenological viewpoint the equal couplings of \( SU(3)^{12} \) can, instead of remaining constant at energies \( \mu > M \), decrease smoothly by asymptotic freedom to a conformal fixed point as \( \mu \to \infty \). This possibility is less restrictive and may fit in better with the AdS/CFT correspondence. The "desert" can then reside in a so-far unexplored domain of some fifteen orders of magnitude in energy scale between 4 TeV and the Planck scale.

As for experimental tests of such a TeV GUT, the situation at energies below 4 TeV is predicted to be the standard model with a Higgs boson still to be discovered at a mass predicted by radiative corrections [12] to be below 267 GeV at 99% confidence level.

No superparticles of known particles are predicted in the vicinity of the Higgs mass.

Proton decay is not gauge-mediated and is not predicted at the same lifetime as in [2].

On the other hand, there are many particles predicted at \( \approx 4 \) TeV beyond those of the minimal standard model. The include as spin-0 scalars the states of Eq.(7), and as spin-1/2 fermions the states of Eq.(9). Also predicted are gauge bosons to fill out the gauge groups of (8), and in the same energy region the gauge bosons to fill out all of \( SU(3)^{12} \). All these extra particles are necessitated by the conformality constraints of [7,8] to lie close to the conformal fixed point.

As alternative to \( SU(3)^{12} \) another approach to TeV unification has as its group at \( \approx 4 \) TeV \( SU(6)^3 \) where one \( SU(6) \) breaks diagonally to color while the other two \( SU(6)'s \) each break to \( SU(3)_k \) where level \( k = 5 \) characterizes irregular embedding.
The triangular quiver $-C-W-H-$ with ends identified and $A_{\mu} = (\alpha, \alpha, \alpha, 1)$, $\alpha = \exp(2\pi i/3)$, preserves $N = 1$ supersymmetry. I have chosen to describe the $N = 0 \text{SU}(3)^2$ model in the text mainly because the symmetry breaking to the standard model is more transparent.

The TeV unification fits $\sin^2 \theta$ and $\alpha_3$, predicts three families, and resolves the GUT hierarchy. If such unification holds in Nature there is to be a very rich level of physics just one order of magnitude above presently accessible energy.

This work was supported in part by the Office of High Energy, US Department of Energy under Grant No. DE-FG02-97ER41036.