The recent experimental results on neutrino oscillation and on muonium-antimuonium conversion require extension of the minimal 3-3-1 model. We review the constraints imposed to the model by those measurements and suggest a pattern of leptonic mixing, with charged leptons in a non-diagonal basis, which accounts for the neutrino physics and circumvents the tight muonium-antimuonium bounds on the model. We also illustrate a scenario where this pattern could be realized.

PACS numbers: 12.15.Ff, 14.60.Pq, 12.60.Cn.

I. INTRODUCTION

The minimal 3-3-1 model is an alternative to the standard model (SM) of the electro-weak interactions that, among other noteworthy features, presents an interesting leptonic phenomenology due to the presence of bileptons. The charged currents involving the vector bileptons allow for a number of rare processes, among them is the muonium-antimuonium conversion.

Presently, there are two sources of experimental constraints claiming an extension of the minimal model. One of them stems from the recent experimental results that corroborate the neutrino oscillation hypothesis.

In this regards, it was shown in Ref. [6] that the model disposes of a potential that leads to a type II seesaw mechanism when we consider terms that break explicitly the lepton number. This means that the model leads naturally to small neutrino mass, as usually required by the neutrino oscillation hypothesis. However small neutrino mass is not the whole issue in neutrino physics once neutrino oscillation requires neutrino mixing. It was shown in Ref. [7] that the minimal model is unable of generating the pattern of neutrino mixing required by solar and atmospheric neutrino oscillation.

The other source of constraints on the model comes from the limits imposed on the $M - M'$ conversion. In Ref. [8], it was posed that such a constraint implies a lower bound on the doubly charged vector bilepton in the minimal 3-3-1 model, $M_{U^{++}} \geq 850$ GeV, which is in clear contradiction to the predicted upper bound required by self-consistency of the model, namely, $M_{U^{++}} \leq 600$ GeV.

As we have two different sources of constraints implying that for the minimal 3-3-1 model to be viable it has to be extended, and both involve its leptonic sector, it seems interesting to look for extensions that could lead to the desired pattern of neutrino mixing and simultaneously get rid of the $M - M'$ bounds. This is the aim of this work, and we shall carry on this proposal dealing with the leptonic mixing.

This work is organized as follows. In section II we present the leptonic sector of the minimal 3-3-1 model, highlighting the aspects that are relevant to our analysis. In section III, we show how it is possible to simultaneously be consistent with neutrino data and circumvent the bound imposed by the non-observation of $M - M'$ conversion setting an appropriate mixing in the leptonic sector. We also present the texture of the leptonic mass matrices that would lead to such a mixing and suggest a suitable way in which this scenario can be realized, addressing its phenomenological situation. We finally conclude in section IV.

II. LEPTONIC SECTOR OF THE MINIMAL 3-3-1 MODEL

Before addressing the main point of our investigation, it is worthwhile to review how the lepton masses are generated in the minimal 3-3-1 model. The Yukawa interactions that lead to these masses are given by

\[ \mathcal{L}_Y^l = \frac{G_{ab}}{2} \overline{\Psi_{aL}}(\Psi_{aL})^cS^\dagger \Psi_{bL} + \frac{F_{ab}}{2} \overline{\Psi_{iaL}}(\Psi_{iaL})^c \Psi_{jbL} \eta^c + \text{H.c.} \]

where $a, b$ label the different families and $i, j, k = 1, 2, 3$ label the elements of each multiplet, with $\Psi_{aL} = (\nu_a, \epsilon_a, \epsilon^c_a)_L$, $\eta = (\eta^0, \eta^-, \eta^+)_L$, and the sextet $S$,

\[ S = \begin{pmatrix} \sigma_1^0 & h_1^- & h_1^+ \\ h_1^+ & H_1^0 & \sigma_2^0 \\ h_2^- & \sigma_2^0 & H_2^+ \end{pmatrix}. \]
In general non-diagonal matrices $M_l$ and $M_\nu$ can be diagonalized as follows,

$$M_l^D = V_{eL}^\dagger M_l V_{eR}, \quad M_\nu^D = V_{eL}^\dagger M_\nu V_\nu,$$

(3)

where $M_{l,\nu}$ is the mass matrix in the interaction basis, and $M_{l,\nu}^D$ is the mass matrix in the mass basis. The matrices $V$ transform the lepton fields from the interaction eigenstates into mass eigenstates and, in principle, they are different for left-handed and right-handed fields. These diagonalization matrices combine themselves in the charged current of the model which, after the 3-3-1 breaking to the $SU(3)_c \otimes U(1)_{EM}$, is given by (omitting family indices):

$$L_{IC}^{CC} = -\frac{g}{\sqrt{2}} \gamma^\mu O^W \nu_L W^-_\mu - \frac{g}{\sqrt{2}} (\epsilon_R)^{\mu} O^V \gamma^\mu \nu_L V_\mu$$

$$-\frac{g}{\sqrt{2}} (\epsilon_R)^{\mu} O^U \gamma^\mu \nu_L U_\mu + H.c.,$$

(4)

where $O^W = V_{eL}^\dagger V_{eR}, O^V = V_{eL}^\dagger V_{\nu},$ and $O^U = V_{eL}^\dagger V_{\nu}$ are the mixing matrices. While the experimental data concerning neutrino oscillation give information on the possible values for the elements of $O^W$, we have no such information on the elements of $O^V$ and $O^U$. We also note that since the right handed mixing matrix $V_{eR}$ enters into the charged current interactions through bilepton-lepton coupling, the leptonic mixing results more complex than we would expect in simple extensions of the SM.

The mixing matrices appearing in this work are all of the Maki-Nakagawa-Sakata (MNS) type[1] and we will be considering the simplest case of a zero CP violating phase, since this phase is irrelevant throughout our analysis. This implies that these matrices are real, and the hermitian conjugate of the matrices involved are simply their transpose. We then adopt the following parameterization for these matrices $O^W, O^V, O^U$,

$$
\begin{pmatrix}
  c_{12} c_{13} & s_{12} c_{13} & s_{13} \\
-s_{12} c_{23} & c_{12} s_{23} & c_{23} \\
-s_{12} s_{23} & -c_{12} s_{23} & c_{13} c_{23}
\end{pmatrix},
$$

(5)

where, as usual, $s_{ij}$ and $c_{ij}$ denote sines and cosines of their arguments $\theta_{ij}$. This parameterization will be used in section IV when we determine the appropriate pattern for lepton mixing which would allow us to get rid of the $M - \overline{M}$ bound, and still be consistent with neutrino physics.

III. COULD $M - \overline{M}$ CONVERSION BE ABSENT IN 3-3-1 MODEL?

As we saw in section III the leptonic sector of the 3-3-1 model presents three mixing matrices $O^W, O^V, O^U$. It is opportune to observe that these mixing matrices are not completely independent of each other, since they involve products of the lepton diagonalization matrices $V_{eL}, V_{eL}$, and $V_{eR}$. In the case of a diagonal charged lepton basis $V_{eL}$ and $V_{eR}$ are also diagonal, leading to a diagonal mixing matrix for the interactions among $U^{\pm\pm}$ and the leptons. Hence, as we want to find a way of overcoming the $M - \overline{M}$ bound by fixing an appropriate form for the matrix $O^U$, we must consider the leptonic sector in a non-diagonal charged lepton basis. Then, our first task is to determine the three matrices $V_{eL}, V_{eL}$, and $V_{eR}$. The data from neutrino physics provide certain knowledge about the mixing matrix $O^W$. From it we can infer $V_{eL}$ and $V_{eR}$. However we do not dispose of sufficient data to determine $V_{eR}$ and this gives us some room to make a key assumption in this work, namely, imposing that $V_{eR}$ mimics $V_{eL}$. The reason behind such an assumption is that it realizes our proposal of circumventing the $M - \overline{M}$ bounds in the 3-3-1 model in a neat way, as we expose next. We then finish this section by commenting about the phenomenological status of this scenario.

Let us first determine the pattern of $O^W$. The recent analysis of atmospheric neutrino still favors $\nu_\mu - \nu_\tau$ oscillation with an almost maximal mixing 0.92 < sin$^2 2\theta_{atm}$ < 1.0 at 90 % C.L. [11]. We also have that the oscillation among $\nu_e - \nu_\mu$ is almost settled as the explanation for the solar neutrino problem. Here the recent results allow 0.25 < sin$^2 2\theta_{sun}$ < 0.40 and 0.6 < cos$^2 \theta_{sun}$ < 0.75 (90 % C.L.) [12]. Since the CHOOZ experiment failed to see the disappearance of $\bar{\nu}_e$, we also have 0 < sin$^2 2\theta_{he} < 0.1$ (90 % C.L.) [13]. For our proposal, we can fix the angles $\theta_{atm}$ ( $\theta_{23}$) and $\theta_{he}$ ( $\theta_{13}$), which is straightforwardly done by taking the best fit for $\theta_{atm}$ = 45°, while $\theta_{he}$ = 0°, in agreement with the above presented results. The angle involved in the solar neutrino oscillation ($\theta_{12}$) is the one that allows for a certain range of values, and it can be kept as a free parameter in our investigation. We are left then with the so called maximal mixing pattern for $O^W$,

$$O^W = \begin{pmatrix}
  c & s & 0 \\
-s & c & 0 \\
0 & 0 & 1
\end{pmatrix},$$

(6)

where we have used the short form sin$\theta_{sun}$ = $s$ and cos$\theta_{sun}$ = $c$.

A non-diagonal charged lepton mass basis means that both, $V_{eL}$ and $V_{eR}$, are non trivial. The only way of separating Eq. 6 in these two matrices is to have maximal mixing between $\nu_\mu$ and $\nu_\tau$ coming from the charged lepton sector and the mixing in the $\nu_e$ to $\nu_\mu$ oscillation coming from the neutrino sector. In order to disentangle the contributions of the distinct diagonalization matrices to $O^W$ given by Eq. 6, we dissociate it as:

$$O^W = \begin{pmatrix}
  c & s & 0 \\
-s & c & 0 \\
0 & 0 & 1
\end{pmatrix} \times \begin{pmatrix}
  1 & 0 & 0 \\
0 & 1 & 0 \\
-s & c & 0 \\
0 & 0 & 1
\end{pmatrix},$$

(7)
which is the only way of doing it since otherwise we would inevitably get a large $\theta_{\text{chz}}$. From this matrix equation we can easily recognize the contribution from the charged lepton sector (remembering that $O^W = V_{eL}^T V_{\nu}$),

$$V_{eL} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (8)$$

and the contribution from the neutrino sector,

$$V_{\nu} = \begin{pmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (9)$$

In this way the neutrino sector gets responsible for the mixing related to the solar neutrino oscillation, and the charged lepton sector to the maximal mixing related to the atmospheric neutrino oscillation.

Let us take $O^V = V_{eR}^T V_{\nu}$ and dissociate it as the product of three rotation matrices:

$$O^V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \times \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (10)$$

It is easy to recognize the last matrix at the right hand side of Eq. (10) as the neutrino mixing matrix given in Eq. (9). In this way let us take the usual notation $c_{12} = c$ and $s_{12} = s$. The other two matrices must form the $V_{eR}$ mixing matrix,

$$V_{eR}^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \times \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} = \begin{pmatrix} c_{13} & 0 & s_{13} \\ -s_{23}s_{13} & c_{23} & c_{23}c_{13} \\ -c_{23}s_{13} & -s_{23}c_{13} & c_{23}c_{13} \end{pmatrix}. \quad (11)$$

Since there is no information about the angles in the matrix $V_{eR}$ we can, for convenience, assume that the charged current mediated by $V^\pm$ does not contrast with the physics experimentally established by the charged current mediated by $W^\pm$. For that reason we adopt in Eq. (11) $\theta_{13} = 0$, and $\theta_{23}$ maximal. This choice accomplishes the goal of delivering the 3-3-1 model out of the $M - \overline{M}$ bound if we require that the maximal angle be negative, that is, $\theta_{23} = -45^\circ$. With these assumptions we get

$$V_{eR} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{c}{\sqrt{2}} & \frac{s}{\sqrt{2}} \\ 0 & \frac{-s}{\sqrt{2}} & \frac{c}{\sqrt{2}} \end{pmatrix}. \quad (12)$$

The matrices in Eqs. (8), (9), and (12) determine the leptonic mixing in the 3-3-1 model. $O^W$ is given by Eq. (6) and the other two are

$$O^U = V_{eR}^T V_{\nu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad (13)$$

$$O^V = V_{eL}^T V_{\nu} = \begin{pmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (13)$$

The $M - \overline{M}$ conversion is proportional to the product of $O^U_{e\mu}$ and $O^U_{\mu\tau}$. According to the pattern of leptonic mixing given above we have $O^U_{e\mu} = 0$, which completely eliminates the $M - \overline{M}$ conversion from the 3-3-1 model at tree level and, consequently, the bound on the vector bispinor mass.

Finally, once we have the pattern of leptonic mixing, it is imperative to establish the textures of the lepton mass matrices that lead to such a pattern. With $V_{\nu}$, $V_{eR}$ and $V_{eL}$ given by Eqs. (8), (9) and (12), respectively, we are able to obtain the textures of the neutrino and charged lepton mass matrices through Eq. (13).

Taking $M^D = \text{diag}(m_1, m_2, m_3)$, the mixing $V_{\nu}$ given in Eq. (9) leads to the following texture for the neutrino mass matrix:

$$V_{\nu} = M^D_V V_{\nu}^T = \begin{pmatrix} m_1 c^2 + m_2 s^2 & (m_2 - m_1) c s & 0 \\ (m_2 - m_1) c s & m_1 s^2 + m_2 c^2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \quad (14)$$

similarly, taking $M^D_l = \text{diag}(m_e, m_\mu, m_\tau)$, the mixing matrices $V_{eL}$ and $V_{eR}$ given by the respective Eqs. (8) and (12), lead to the following texture for the charged lepton mass matrix,

$$V_{eL} = V_{eL} M^D_{eR} V_{eR}^T = \begin{pmatrix} m_e & 0 & 0 \\ 0 & (m_\mu - m_\tau)/2 & -(m_\mu + m_\tau)/2 \\ 0 & (m_\mu + m_\tau)/2 & -(m_\mu - m_\tau)/2 \end{pmatrix}. \quad (15)$$

Extensions of the minimal 3-3-1 model that realize our proposal must recover the above textures.

In order to assure the feasibility of this proposal, let us implement an illustrative scenario which leads to the pattern of lepton mixing presented in Eq. (13) and (14). Let us first focus on the matrix $M^D_l$ in (15). Perceive that its unique non-diagonal element different from zero is antisymmetric. This can be obtained by the scalar triplet $\eta$ once its Yukawa interaction in (14) gives rise to antisymmetric entries. The diagonal elements can be generated by the sextet $S$. Then the Yukawa interactions in (14), with specific choice of the Yukawa couplings $G_{ab}$ and $F_{ab}$, can generate the matrix $M^D_l$ in (15).

It is the generation of $M^D_l$ given in (15) that requires extension of the minimal model. As we are considering only an illustrative scenario, one immediate possibility of
This extension in conjunction with the type II seesaw mechanism developed in Ref. [6], and appropriated choice of the Yukawa couplings, $G_{ab}$, can generate the pattern of $M_\nu$, given in (14).

We finish this section discussing features potentially restrictive for our proposal due to the form of $O^U$ in (13), since not only rare decays are possible, but also the coupling of the doubly charged vector bilepton to fermions is maximal (equal to $g/\sqrt{2}$) in some cases, leading to possibly serious constraints on its mass. For this reason we discuss now the possible phenomenological constraints on the proposed extension to the minimal model.

The phenomenology of vector bileptons is well studied in Ref. [14], and here we just update the main results relevant for this work. First, the rare decay involving the doubly charged vector bilepton, from the form of $O^U$ in Eq. (13), is the one which mixes the muon and tau, $\tau^+ \to \mu^- e^+ e^-$. This decay roughly gives (assuming $g \sim 0.1$ as for the analysis of $M - \bar{M}$) $M_{U^{++}} \geq 0.16 \text{ TeV}$, representing no threat to the model consistency. Because $O^U_{14} = 1$ we can also have contributions to the Bhabha scattering. However, in the realm of the 3-3-1 model we have to consider the contribution of the $Z'$ as well. The contributions of $U^{++}$ and the $Z'$ have negative interference terms that blurs the possible effects of the bilepton, implying that no limits on its mass can be extracted from the experimental data. Finally, another possible source of constraint on $U^{++}$ mass would be the anomalous magnetic moment of the muon, $(g - 2)_\mu$. However, due to the uncertainties related to the hadronic contribution, the deviation from the SM prediction is not enough to put any severe constraint on the vector bilepton mass. In this sense, the bound imposed by consistency of the model remains valid in our suggested scenario.

IV. CONCLUDING REMARKS

In this work we focused on the possibility of the leptonic mixing to help finding a way out to avoid the $M - \bar{M}$ bound on $M_{U^{++}}$. By assuming that right-handed and left-handed charged leptons both present maximal mixing, we arrived at a pattern of leptonic mixing which accommodates the recent experimental result in neutrino physics and eliminates the contribution of the bileptons to the $M - \bar{M}$ conversion [15]. Of course we have no knowledge of the actual values for the angles in the right-handed charged lepton mixing matrix, but the specific choice we made represents a possible conciliation of minimal 3-3-1 model with both, neutrino physics and the bound posed by $M - \bar{M}$ conversion.

For completeness we presented the texture for neutrino and charged lepton mass matrices in accordance with the pattern of leptonic mixing used in solving the $M - \bar{M}$ problem. It serves as a sort of guide to build extensions of the model capable of recovering our pattern of mixing. Finally we suggested a simple scenario that could realize our proposal. It is suitable to remark that the inconsistency raised by the $M - \bar{M}$ bound in 3-3-1 model may be pointing the need of considering charged leptons a non-diagonal basis.

Acknowledgments. The authors would like to thank Vicente Pleitez for the critical reading of the manuscript and for useful suggestions. This work was supported by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP)(AG,PSRS) and by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) (CASP).