We investigate light-cone structure on the world-volume of an unstable D-brane with a tachyon decaying inhomogeneously by using a field theoretical description. It is shown that (i) light-cones governing open strings are narrower than those governing closed strings and will eventually collapse inward in all directions except at kinks, where the tachyon remains at the top of its potential; and that (ii) light-cones governing open strings at a kink will be narrowed only in the direction perpendicular to the kink surface. It is also shown that (iii) future-directed light-cones governing open strings near a kink are tilted towards the kink, compared with those governing closed strings. The result (i) implies that open strings except at kinks are redshifted, compared with closed strings, and will eventually cease to be dynamical. On the other hand, the result (ii) shows that open strings on a kink surface can move freely along the kink surface and are dynamical but do not feel the existence of the spatial dimension perpendicular to the kink surface. The result (iii) indicates that open strings near a kink have tendency to move towards the kink. Hence, the light-cone structure vividly illustrates how open strings behave during the dynamical formation of a kink. We also discuss about a possibility that the early universe has a network of various dimensional D-branes, black-branes and tachyon matter. A problem associated with the network and a possible solution to the problem are discussed.

I. INTRODUCTION

Time-dependent backgrounds are recently getting more important than ever in the study of string theory. Actually, spacelike branes [1] or a rolling tachyon [2] have been attracting a great deal of interest. Some of non-BPS objects such as a D-brane-anti-D-brane pair and a non-BPS D-brane are by their nature unstable and should become time-dependent under perturbations. Since non-BPS objects are expected to play important roles in exploration of string dualities beyond the BPS level, one would like to study their properties including their dynamics. Hence, it is important to study the dynamics of tachyons living on those objects, as their unstable nature is characterized by the existence of tachyons.

It is not only in exploration of the string duality web but also in exploration of the early universe where tachyons can play important roles. The so called tachyon cosmology was recently initiated and investigated by many authors [3]. In this context a tachyon can dominate our universe at an early epoch and, thus, the dynamics of tachyons is essential.

The dynamics of a tachyon can actually be described in many different ways. For example, it was shown by boundary states of an unstable D-brane that the pressure of tachyon fluid approaches zero as the tachyon rolls down [4], it is a field theoretical description [5–7,4] that has been extensively used in the tachyon cosmology, and it was by using the boundary string field theory (BSFT) that some exact properties of the tachyon condensation were derived in refs. [8,9].

One of well-known properties of the tachyon effective action for an unstable D-brane is that the pressure at late time falls off exponentially as the tachyon field evolves from any spatially homogeneous initial configuration towards the minimum of the potential. This was shown by the boundary states [4], in the field theoretical description with a specific form of the tachyon potential [10], and in the BSFT [11,12] although there are some differences in details.

Another well-known property is the absence of plane-wave solutions around the minimum of the tachyon effective potential. This is easily inferred from evidences showing that the minimum describes a configuration without D-branes [13]. The absence of plane-wave solutions was shown in the field theoretical description [10] and the BSFT [14].

Knowing the absence of plane-wave solutions around the minimum, it is interesting to ask how this is achieved from the open string points of view. Before the tachyon decay, open strings are dynamical on the world-volume of the unstable non-BPS brane. On the other hand, after the tachyon decay, there are no open string states. Hence, it seems natural to ask “what really happens to open strings in the process of the tachyon decay?” In this respect, a non-perturbative confinement mechanism on the brane [15] and the classical equivalence between a gauge system and a string fluid [16] are very suggestive.

In ref. [17] Sen conjectured that a tachyonic kink on the world-volume of an unstable D(p + 1)-brane is a BPS Dp-brane. In this context, the tachyon field for the kink configuration is usually supposed to be static. On the other
II. TACHYON ACTION AND TWO METRICS

Let us consider the following action of an unstable D\((p + 1)\)-brane.

\[
I = - \int d^{p+2}x V(T) \sqrt{\det [g_{MN} + \partial_M T \partial_N T + 2\pi \ell_s^2 (F_{MN} + \partial_M \phi \partial_N \phi)]},
\]

where \(g_{MN}\) is the induced metric on the world-volume of the brane, \(T\) is the tachyon representing the unstable nature, \(F_{MN}\) and \(\phi\) represent a gauge field and scalar fields confined on the brane, respectively, and \(V(T)\) is the tachyon potential with vanishing minima at \(T = \pm \infty\).

Note that closed string degrees freedom (such as gravitons and RR fields) in the bulk couple to \(g_{MN}\) directly but that open string degrees of freedom on the brane \((F_{MN} \text{ and } \phi)\) always couple to \(g_{MN}\) through the combination

\[
G_{MN} = g_{MN} + \partial_M T \partial_N T.
\]

Hence, we have two different metrics on the brane: \(g_{MN}\) governing closed string degrees of freedom and \(G_{MN}\) governing open string degrees of freedom [18,19]. In particular, as we shall see explicitly, light-cone structures of the two metrics can be very different. In the remaining of this paper, we shall set \(F_{MN}\) and \(\phi\) to be zero. However, we shall keep in mind the fact that these open string fields on the brane couple to \(G_{MN}\).

In the rest of this paper, we set \(F_{MN} = 0\) and \(\phi = 0\) in the action (1):

\[
I = - \int d^{p+2}x V(T) \sqrt{\det G_{MN}}.
\]

This is the action considered by many authors in the context of tachyon cosmology. The equation of motion is

\[
\nabla^2 T - \frac{1}{2} \frac{\partial^M T \partial_M (\partial^N T \partial_N T)}{1 + \partial^M T \partial_M T} - \frac{V_T}{V} = 0,
\]

and the stress energy tensor is

\[
T_{MN} = -g_{MN} V(T) \sqrt{1 + \partial^L T \partial_L T} + \frac{V(T) \partial_M T \partial_N T}{\sqrt{1 + \partial^L T \partial_L T}}.
\]

The stress energy tensor characterizes how the tachyon couples to gravity. For example, if we simply add the Einstein-Hilbert term to the world-volume action or if we consider a spacetime filling brane then \(T_{MN}\) appears in the right
hand side of Einstein equation. On the other hand, if we introduce the Einstein-Hilbert action to a higher-dimensional bulk then gravity is described by the brane-world scenario. In any cases, $T_{MN}$ acts as a source of gravity.

In this paper, we investigate general feature of light-cone structure on the world-volume of the brane during inhomogeneous tachyon decay without specifying how $T_{MN}$ couples to gravity.

III. INHOMOGENEOUS TACHYON DECAY

We would like to investigate light-cone structure on the world-volume of an unstable D($p+1$)-brane with a tachyon decaying inhomogeneously. For simplicity, we consider inhomogeneity in only one spatial dimension. Namely, we assume $p$-dimensional plane symmetry. A general form of the tachyon field and the induced metric within this symmetry is given by

$$T = T(t, y),$$

$$g_{MN} dx^M dx^N = -N^2(t, y) dt^2 + A^2(t, y) \sum_{i=1}^{p} (dx^i)^2 + dy^2. \tag{6}$$

For this ansatz, the metric $G_{MN}$ governing open strings is

$$G_{MN} dx^M dN = -\tilde{N}^2 dt^2 + A^2 \sum_{i=1}^{p} (dx^i)^2 + B^2 (dy - \omega dt)^2, \tag{7}$$

where

$$\tilde{N}^2 = \frac{1 - \left(\dot{T}/N\right)^2 + (T')^2}{1 + (T')^2} N^2,$$

$$B^2 = 1 + (T')^2,$$

$$\omega = -\frac{\dot{T}T'}{1 + (T')^2}. \tag{8}$$

Here a dot and a prime denote derivative with respect to $t$ and $y$, respectively. Note that $\tilde{N}^2 \leq N^2$ (the equality holds if and only if $\dot{T} = 0$).

The future directed null vectors in the $\pm y$ direction are

$$u^M_{\pm} = \left( \frac{\partial}{\partial t} \right)^M \pm N \left( \frac{\partial}{\partial y} \right)^M \tag{9}$$

for $g_{MN}$, and

$$U^M_{\pm} = \left( \frac{\partial}{\partial t} \right)^M + \left( \omega \pm \frac{\tilde{N}}{B} \right) \left( \frac{\partial}{\partial y} \right)^M \tag{10}$$

for $G_{MN}$. On the other hand, the future directed null vectors in the $\pm x^i$ direction are

$$v^M_{(i)\pm} = \left( \frac{\partial}{\partial t} \right)^M \pm \frac{N}{A} \left( \frac{\partial}{\partial x^i} \right)^M \tag{11}$$

for $g_{MN}$, and

$$V^M_{(i)\pm} = \left( \frac{\partial}{\partial t} \right)^M \pm \frac{\tilde{N}}{A} \left( \frac{\partial}{\partial x^i} \right)^M \tag{12}$$

for $G_{MN}$.

From the above expressions of null vectors, we can understand many things.

(i) First, if $\dot{T} \neq 0$ then the light cones of $G_{MN}$ governing open strings are narrower in all directions than those of $g_{MN}$ governing closed strings since $\tilde{N}^2 < N^2$. In general we expect $\dot{T} \neq 0$ except at kinks if we consider the
inhomogeneous tachyon decay. Therefore, open strings except at kinks are redshifted, compared with closed strings. Moreover, if $\tilde{N} \to 0$ (or $N \to 0$) as $t \to \infty$ with $y$ fixed, then the light cones of $G_{MN}$ governing open strings (or those of $g_{MN}$ governing closed strings, respectively) eventually collapse inward in all directions. When $N \neq 0$, $\tilde{N}$ vanishes if and only if $1 - (\dot{T}/N)^2 + (T')^2 = 0$. Hence, if

$$1 - (\dot{T}/N)^2 + (T')^2 \to 0 \quad (t \to \infty, y \text{ fixed}),$$

then light-cones governing open strings eventually collapse inward in all directions while those governing closed strings remain well-defined. The asymptotic behavior (13) is actually satisfied for a homogeneous rolling tachyon ($|\dot{T}| \to 1, T' = 0$ and $N = 1$). Even for an inhomogeneous tachyon decay, once the tachyon starts rolling down, the asymptotic behavior (13) is generally expected and was conjectured to be true in ref. [20] based on numerical study of the inhomogeneous tachyon decay. On the other hand, it is evident that this condition cannot be satisfied at a kink, where $\dot{T} = 0$. Hence, we can expect the asymptotic behavior (13) to be true except in the vicinity of kinks. Therefore, open strings except in the vicinity of kinks are redshifted, compared with closed strings, and will eventually cease to be dynamical.

(ii) Secondly, at a kink, $v_{(i)\pm}^M = V_{(i)\pm}^M$ since $\dot{T} = 0$. Hence, light-cones of $G_{MN}$ governing open strings at a kink will be narrowed only in the direction perpendicular to the kink surface. This, combined with the above consideration (i), implies that open strings on a kink surface can move freely along the kink surface and are dynamical but do not feel the existence of the spatial dimension perpendicular to the kink surface.

(iii) Thirdly, near a kink but not on the kink surface we expect that $\dot{TT} > 0$, provided that the top of the tachyon potential is at $T = 0$. To see this, let us consider a simple example of inhomogeneous tachyon decay with a sufficiently smooth initial condition at $t = t_1$ satisfying

$$T(t_1, y) = -T(t_1, -y),$$

$$\dot{T}(t_1, y) = 0,$$

and

$$N(t_1, y) = N(t_1, -y), \quad \dot{N}(t_1, y) = \dot{N}(t_1, -y),$$

$$A(t_1, y) = A(t_1, -y), \quad \dot{A}(t_1, y) = \dot{A}(t_1, -y).$$

Of course, appropriate constraint equations must be imposed on the initial condition. For the initial condition, we expect the tachyon to start rolling down the potential hill except at the kink where $T$ and $\dot{T}$ remain to vanish (see Fig. 1). For this simple example, it is evident that $\ddot{TT} > 0$ near the kink but not on the kink surface. We expect that this statement is true for a wide class of inhomogeneous tachyon decay if applied to a sufficiently close vicinity of a kink. Hence, $(y - y_0)\omega < 0$ near a kink but not on the kink surface, where $y_0$ is the position of the kink and $\omega$ is defined by (8). Note that $\omega$ appears in (10) and, thus, the null vectors $U_{\pm}^M$ for $G_{MN}$ near a kink are tilted towards the kink, compared with the null vectors $v_{\pm}^M$ for $g_{MN}$. In other words, future-directed light-cones governing open strings near a kink are tilted towards the kink, compared with those governing closed strings. This indicates that open strings near a kink have tendency to move towards the kink.
In the above three considerations, the light-cone structure vividly illustrates how open strings behave during the dynamical formation of a kink.

In ref. [17] Sen conjectured that a tachyonic kink on the world-volume of an unstable D\((p+1)\)-brane is a BPS D\(p\)-brane. This conjecture seems consistent with the time-dependent kink, or the inhomogeneous tachyon decay, considered above rather than a static kink. Actually, if we consider a static configuration \(\dot{T} = 0\), \(\tilde{N} = N\) and \(\omega = 0\). Hence, it is easily seen that for a static configuration, light-cones of \(G_{MN}\) governing open strings are well-defined as far as those of \(g_{MN}\) governing closed strings are well-defined and that light-cones of \(G_{MN}\) near a kink is not tilted towards the kink at all. Hence, open strings on the static kink background can be as dynamical as closed strings: there is no sign of redshift or tendency to move towards the kink. On the other hand, for the time-dependent kink considered above, all three considerations (i) - (iii) seem to indicate that a lower-dimensional D-brane is dynamically formed at the position of the kink.

IV. D-BRANE NETWORK IN THE EARLY UNIVERSE

In the context of the tachyon cosmology, if the universe starts from the top of the tachyon potential then we expect dynamical formation of the time-dependent tachyonic kink. In fact, quantum fluctuations kick the background tachyon field to different directions at different points and, as a result, the background tachyon field falls into different vacua in different regions. In this way, the formation of a network of kinks is expected. This is exactly like the formation of a network of topological defects in the early universe.

Sen’s conjecture implies that the kink network is actually a D-brane network. Hence, if the tachyon cosmology is responsible for an early stage of the history of our (4-dimensional or higher dimensional) universe, then our universe experienced an era in which there is a network of D-branes. On one side of a D-brane and another side, the tachyon is rolling towards its (different) vacua and, hence, there is what is called tachyon matter in each region.

This is not the end of the story. After the D-brane network formation, a loop of D-brane can collapse in principle. If this happens then it would imply a formation of a network of lower-dimensional D-branes since this process is locally equivalent to a D-brane anti-D-brane annihilation. Actually, a vortex on the world volume of a coincident D\(p\)-brane and anti D\(p\)-brane pair is believed to be a D\((p-2)\)-brane [21,22]. When the lower dimensional D-branes are produced, tachyon matter is again expected to be produced among the newly created D-brane network.

Therefore, we expect that at its early stage, the (4-dimensional or higher dimensional) tachyon cosmology has a very rich structure composed of networks of various dimensional D-branes and tachyon matter. Moreover, if the string coupling is strong enough, then these D-branes can be black-branes. In this case, the D-brane network should be replaced by a network of black-branes. If the string coupling, or the dilation, is spacetime dependent then the network may be composed of mixture of not only various dimensional D-branes (without horizons) and tachyon matter but also various dimensional black-branes.
In ref. [23] it was argued that the formation of tachyonic kinks is hazardous in 4-dimensional cosmology. As mentioned in the above, the tachyonic kinks are D-branes (or black-branes). Hence, we can quantitatively investigate how hazardous the formation of the network of tachyonic kinks in the early universe is.

In the 4-dimensional standard cosmology, there is a very significant constraints on the mass per unit area, or the tension, in a network of domain walls [24]. In general a domain wall can annihilate with an anti-domain wall if the spacetime is flat and we can wait for a sufficiently long time. However, the spacetime representing the early universe is not flat but curved. The universe is actually expanding and, because of the expansion, the annihilation process in the universe cannot be completed but has to leave on the order of one domain wall stretching across each Hubble radius. Hence, the mass of the part of the domain wall network within the present Hubble volume is $M_{DW} \sim \sigma H_0^{-2}$, where $\sigma$ is the tension of the wall and $H_0$ is the present value of the Hubble parameter. This mass must be much smaller than the total mass within the present Hubble volume, $M_{tot} \sim m_{pl}^2 H_0^{-1}$, since the strongly anisotropic mass distribution of a domain wall can contribute to the cosmic microwave background (CMB) anisotropy. Actually, the ratio $M_{DW}/M_{tot}$ must be smaller than the CMB anisotropy:

$$\frac{M_{DW}}{M_{tot}} \leq 10^{-5}. \quad (16)$$

Hence, we obtain the constraint

$$\frac{\sigma}{m_{pl}^2 H_0} \leq 10^{-5}. \quad (17)$$

Now let us apply the constraint (17) to the D-brane network. For simplicity, we consider an unstable D9-brane in 10-dimension and, hence, the network of BPS D8- (and anti-D8-) branes. We compactify six extra dimensions to obtain 4-dimensional universe with a network of 2-branes. In this case, $\sigma \sim m_s^9 V_6 / g_s$ and $m_{pl}^2 \sim m_s^5 V_6 / g_s^2$, where $m_s$ is the string scale, $g_s$ is the string coupling constant and $V_6$ is the volume of the six extra dimensions. Hence, the constraint (17) is reduced to

$$\frac{g_s^2 m_{pl}}{H_0 v_6^{3/2}} \leq 10^{-5}, \quad (18)$$

where $v_6 \equiv m_s^6 V_6$ is the volume of the six extra dimensions in the string unit. We have another constraint from the fact that we do not observe the tower of Kaluza-Klein modes: $m_{KK} / m_{pl} \geq 10^{-16}$, where $m_{KK} \sim V_6^{-1/6}$ is the mass of the lightest Kaluza-Klein mode. This constraint is written as

$$\frac{g_s}{v_6^{2/3}} \geq 10^{-16}. \quad (19)$$

Combining this with the previous constraint, we obtain

$$10^{-32} v_6^{4/3} \leq g_s^2 \leq 10^{-5} \cdot \frac{H_0}{m_{pl}} v_6^{1/2}. \quad (20)$$

There are values of the string coupling constant $g_s$ allowed by this inequality if and only if $v_6^{5/6} \leq 10^{27} H_0 / m_{pl} \sim 10^{-34}$. This would require that the extra dimensions should be much smaller than the string length. Hence, the constraints cannot be satisfied by parameter values for which the low energy description is valid.

So far we have not yet taken into account effects of RR fields. However, since an RR field couples to the tachyon through the interaction of the form

$$\int f(T) dT \wedge C_{p+1}, \quad (21)$$

where $f(T)$ is a function of the tachyon field $T$ and $C_{p+1}$ is the $(p + 1)$-form potential, the RR field can be excited by the tachyon decay. Hence, the effect of the RR field is one of important ingredients of the tachyon cosmology. Progress in this direction and its cosmological implications will be reported elsewhere [25].

Now we would like to propose a possible solution to the D-brane network problem addressed in the above. A basic idea is that a RR field can play a role of a positive cosmological constant.

First, for simplicity, let us consider a spacetime filling brane $(p + 2 = D)$. In this case, after the tachyon decay, the $(p + 2)$-form RR flux is constant and plays a role of a positive cosmological constant $\Lambda_{RR}$. Since the RR flux
can be excited by the tachyon decay, we can expect a positive $\Lambda_{RR}$. This may solve the D-brane network problem in the tachyon cosmology. However, a problem now is how to reduce $\Lambda_{RR}$ to almost vanishing value after the D-brane network problem was solved. We have to rely on other mechanisms like Brown-Teitelboim mechanism [26] to reduce the value of $\Lambda_{RR}$.

Second, if we consider an unstable brane with less dimensions ($p + 2 < D$), then the ($p + 2$)-form after the tachyon decay is no longer a constant. However, it may still play a role of a positive cosmological constant for a while if the decay of the RR field is not too rapid. A question now is whether the time scale of the decay of the RR field is long enough to solve the D-brane network problem.

In both cases, further investigation of the D-brane network problem and its solution is necessary [25].

V. SUMMARY

We have investigated light-cone structure on the world-volume of an unstable D-brane with a tachyon decaying inhomogeneously by using a field theoretical description. It has been shown that (i) light-cones governing open strings are narrower than those governing closed strings and will eventually collapse inward in all directions except at kinks, where the tachyon remains at the top of its potential; that (ii) light-cones governing open strings at a kink will be narrowed only in the direction perpendicular to the kink surface; and that (iii) future-directed light-cones governing open strings near a kink are tilted towards the kink, compared with those governing closed strings. The result (i) implies that open strings except at kinks are redshifted, compared with closed strings, and will eventually cease to be dynamical. On the other hand, the result (ii) shows that open strings on a kink surface can move freely along the kink surface and are dynamical but do not feel the existence of the spatial dimension perpendicular to the kink surface. The result (iii) indicates that open strings near a kink have tendency to move towards the kink. In this way, the light-cone structure vividly illustrates how open strings behave during the dynamical formation of a kink.

Finally, we have also discussed about a possibility that the early universe has a network of various dimensional D-branes, black-branes and tachyon matter. An associated problem was pointed out and a possible solution of the problem was proposed.

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