A simple explanation of the non-appearance of physical gluons and quarks

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Abstract

We show that the non-appearance of gluons and quarks as physical particles is a rigorous and automatic result of the full, i.e. non-perturbative, nonabelian nature of the color interaction in quantum chromodynamics. This makes it in general impossible to describe the color field as a collection of elementary quanta (gluons). Neither can a quark be an elementary quantum of the quark field, as the color field of which it is the source is itself a source, making isolated noninteracting quarks, crucial for a physical particle interpretation, impossible. In geometrical language, the impossibility of quarks and gluons as physical elementary particles arises due to the fact that the color Yang-Mills space does not have a constant trivial curvature.

In QCD, the particles “gluons” and “quarks” are merely artifacts of an approximation method (the perturbative expansion) and are simply absent in the exact theory. This also coincides with the empirical, experimental evidence.
One major problem in contemporary particle physics is to explain why quarks and gluons are never seen as isolated particles. A lot of effort has gone into trying to resolve this puzzle over a period of years. Different approaches include lattice QCD, dual Meissner effect (in the QCD-vacuum), instantons, etc, but the problem is not yet fully solved. For a review, see [1].

We will take a different route than normally used, to eliminate the problem before it arises.

Usually, most particle physicists use “fields” and “particles” interchangeably, i.e. as denoting the same things. That is because the almost universal usage of Feynman diagrams gives the false impression that particles (quanta) are always exchanged, even when they cannot exist. This is an example of mistaking the approximation (perturbation theory) for the exact theory. However, this misconception seems to be so common that many physicists do not even note, or care about, the distinction. The usage of the Feynman diagram technique (propagators, Green’s functions, etc) can be justified as an approximation for mildly nonlinear theories (weak coupling limit) but breaks down for strongly coupled nonabelian theories. (And also for strongly coupled abelian theories with sources.)

In quantum chromodynamics (QCD) it is at first sight a puzzle why the color force should be short-range, and especially why gluons are not seen as free particles, as the nonbroken SU(3) color symmetry seems to demand massless quanta, which naively would have infinite reach. However, as we shall see, there generally are no quanta.

In quantum field theory an elementary particle [2], i.e. a quantum = a harmonic excitation of a fundamental field [3], is defined through the creation and annihilation operators, \( a_\dagger \) and \( a \), of the “second-quantized” theory. For instance, in quantum electrodynamics (QED), the entire electromagnetic field can be seen as a collection of superposed quanta, each with an energy \( \omega_k \). The hamiltonian of the electromagnetic field (omitting the zero-point energy) can be written

\[
H = \sum_k N_k \omega_k, \quad (1)
\]

where

\[
N_k = a_\dagger_k a_k, \quad (2)
\]
is the “number operator”, i.e. giving the number of quanta with a specific four-momentum \( k \) when operating on a free state,

\[
N_k |\ldots n_k\ldots\rangle = n_k |\ldots n_k\ldots\rangle.
\]

(3)

As all oscillators are independent,

\[
|\ldots n_k\ldots\rangle = \prod_k |n_k\rangle,
\]

(4)

where \( n_k \) is a positive integer, the number of quanta with that particular momentum. The energy in the electromagnetic field is thus the eigenvalue of the Hamiltonian (1). The reasoning for fermion fields is the same, but then the number of quanta in any given state can be only 0 or 1 (“Fermi statistics”).

Now, assuming that QCD is the correct theory of quark interactions, a problem arises, as it is generally impossible to write the color fields in terms of superposed harmonic oscillators. It is not possible to represent the solution as a Fourier expansion and then interpret the Fourier coefficients as creation/annihilation operators through “second quantization”, as the color vector potentials \( A_b^\mu (b \in 1, \ldots, 8) \) are governed by nonlinear evolution equations,

\[
D^\mu F_{\mu \nu} = j_\nu,
\]

(5)

and Fourier methods are inapplicable to nonlinear equations (see, e.g., [4]).

Without a quark current, \( j_\nu \equiv g_s \bar{\psi} \gamma_\nu \psi = 0 \), we get, in component form

\[
(\delta_{ab} \partial^\mu + g_s f_{abc} A_c^\mu)(\partial_\mu A^b_\nu - \partial_\nu A^b_\mu + g_s f_{bde} A^d_\mu A^e_\nu) = 0,
\]

(6)

where \( g_s \) is the color coupling constant (summation over repeated indices implied).

When we have an abelian dynamical group, as in QED, all the structure constants \( f_{abc} \) are zero. Eq.(6) then reduces to a linear differential equation, and a general solution can be obtained by making the Fourier expansion:

\[
A^{QED}_\mu = \int d^3k \sum_{\lambda=0}^{3} \left[ a_k(\lambda) \epsilon_\mu(k; \lambda)e^{-ik \cdot x} + a^+_k(\lambda) \epsilon_\mu^*(k; \lambda)e^{ik \cdot x} \right],
\]

(7)

where \( \epsilon_\mu \) is the polarization vector. We have also omitted an, for our purposes, inessential normalization factor.
However, for a theory based on a nonabelian group, like QCD, this can no longer be done [4], due to the nonlinear nature of Eq.(6) when $f_{abc}, f_{bde} \neq 0$,

$$A_{\mu}^{b Q C D} \neq \int d^3 k \sum_{\lambda=0}^{3} [a_{k}^{b}(\lambda) \epsilon_{\mu}(k, \lambda)e^{-ik \cdot x} + a_{k}^{b \dagger}(\lambda) \epsilon^{\ast}_{\mu}(k, \lambda)e^{ik \cdot x}].$$

Thus, the color fields can be represented by harmonic oscillators (gluons) only in the trivial, and physically empty, limit when the strong interaction coupling constant tends to zero, $g_s \to 0$ (or, within perturbation theory, equivalently when $Q^2 \to \infty$ because of asymptotic freedom). Hence, no elementary quanta of the color interaction, in the usual sense, can exist. This means that no gluon particles are possible, and that Eq.(1) does not hold for color fields. The fields are there, but their quanta, gluons (and quarks), are relevant only when probed at sufficiently (infinitely) short distances. Generally, quarks do not exchange gluons, but the fermion fields $\psi$ react to the color fields given by $A_{\mu}$. Fields are primary to particles.

So far we have strictly only banished gluons. To also banish quarks as physical particles we note that a quark field is the source of a color field, but this color field is itself a source of a color field. Hence, a quark field is never removed from other sources, is always interacting, and can never be considered to be freely propagating. This results in that the quark fields can never be represented by harmonic oscillator modes, unless $Q^2 \to \infty$, whereas physical particles, observable in nature, should exist as $Q^2 \to 0$. This means that no quark field quanta (quarks) can ever exist. In QCD, the particles “gluons” and “quarks” are merely artifacts of the approximation method used, i.e. the perturbative expansion in the interaction on a “background” of assumed free gluons and quarks. They are simply absent in the exact theory.

In QED things are very different. An electric charge gives rise to an abelian field, which is not the source of another field. Hence an electrically charged field can be removed from other sources and exist as a physical

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1Early criticism of this paper focused on that physical particles surely not are as simple as these “bare” particles, and that by necessity “dressed” particles must be used for physical states. This misses the point that the distinction “bare” vs. “dressed” is defined solely within perturbation theory. Even the “dressed” particle states rely on exactly the same kind of approximation methodology as the “bare” particles. Also, the classic explanation of the photoelectric effect uses just these “naive” quanta, in terms of photons.
particle. Thus, the observability of, e.g. an electron is ultimately due to the fact that electromagnetic quanta (photons) can exist as real particles.

A more mathematical treatment of the physical picture regarding quarks given above is provided by geometry. A case analogous to the one we are studying appears in quantum field theory on a curved spacetime [5], where it is well known, and generally accepted, that fields are more fundamental than particles. Indeed, there it can be shown that the very concept of a particle is, in general, useless [6]. Actually, nonabelian gauge fields and quantum field theory on a curved background have a lot in common. The quark fields can, in an approximation similar to quantum field theory on a curved (spacetime) background, be treated as “living” on the curved (gauge) space defined by the color fields. The total curvature, and also the dynamical coupling to “matter fields” through the covariant derivative, is given by one part coming from the Yang-Mills connection (i.e. gauge potential) and one part coming from the Riemannian (Levi-Civita) connection [7]. Only when both the gauge field curvature and the spacetime curvature are zero, or at most constant, can a particle be unambiguously defined. The former is constant for abelian quantum field theory, the latter is zero on a Minkowski background with inertial observers, and constant for some special, and static, spacetimes. The curvature in gauge space is given by the field strength tensor,

\[ F_{\mu\nu}^b = \partial_\mu A_\nu^b - \partial_\nu A_\mu^b + g_s f_{bcd} A_\mu^c A_\nu^d, \]

(9)

this being the analog in gauge space to the Riemann curvature tensor, \( R_{\mu\nu\sigma\rho} \), for spacetime. The properties of \( F_{\mu\nu} \) under a gauge transformation, \( U \), is

\[ F_{\mu\nu} \rightarrow F'_{\mu\nu} = UF_{\mu\nu}U^{-1}, \]

(10)

i.e. the gauge curvature generally transforms as a tensor in gauge space.

However, we see directly that for an abelian gauge theory, like QED,

\[ F_{\mu\nu} \rightarrow F'_{\mu\nu} = F_{\mu\nu}, \]

(11)

as \( U \) now commutes with \( F_{\mu\nu} \). This means that the curvature is constant (invariant) in gauge space for an abelian field, i.e. that \( F_{\mu\nu} \) transforms as a scalar in gauge space. It also means that the field is a gauge singlet, which only reflects that it has no “charge” and that the fields have no self-interactions. Abelian gauge fields \( \neq \) sources of fields.
For nonabelian fields, like QCD, the gauge curvature, $F_{\mu\nu}$, transforms as a tensor, i.e. is covariant, not invariant, and is thus generally different at different points in gauge space. The color-electric and color-magnetic fields, $E^b_i$ and $B^b_i$, which are the components of $F^b_{\mu\nu}$ defined by $F^b_0 = E^b_i$ and $F^b_{ij} = \epsilon_{ijk}B^b_k$ ($i,j,k \in 1,2,3$), are thus not gauge independent and cannot be observable physical fields, which is another, complementary and perhaps more physically direct way of seeing that physical gluons cannot exist, regardless of coupling strength. (In contrast to usual electric and magnetic fields which are both gauge singlets and observed.) The fact that $F^b_{\mu\nu} \neq$ color singlet, just implies that color gauge fields are sources of color gauge fields.

We conclude that the unbroken nonabelian gauge theories of gravity and QCD are strictly incompatible with the concept of elementary quanta in a traditional sense. In practice, however, this only rules out gluons and quarks as physical particles, as spacetime curvature (or, equivalently, observer accelerations) is normally completely negligible in experimental settings in particle physics. The difference can be traced to the fact that the dynamical curvature is directly related to the nonlinear coupling strength, which is enormously much larger for QCD than for gravity. Leptons can exist as physical particles as QED has abelian gauge dynamics and weak (nonabelian) SU(2) is broken, i.e. absent from the point of view of particle detectors.

It also follows, as a direct corollary to the argument above, that hadrons must be color singlets, i.e. color neutral, as they otherwise could not exist as physical particles.

It would be interesting to continue the analogy with gravity and speculate that the hadrons are “grey holes”, as the color stays inside. The curvature induced by the color fields would then give the hadron, or confinement, radius. This would require nonperturbative solutions to the coupled $\psi-A$ system, with fully dynamical quark fields, which is a very hard and unsolved problem. Strictly, also gravity should be included, perhaps in a Kaluza-Klein fashion, the lagrangian then containing both $F_{\mu\nu}F^{\mu\nu}$, now with covariant spacetime derivatives, and $R = R^\mu_\mu$, the Ricci-scalar. Although this is a nice picture, which may/may not be true, it is not necessary for the purpose of excluding gluons and quarks as physical quanta, or particles, for which the

\footnote{Thus, gluon and quark “confinement” can be considered as just a special case of the more general requirement that observables be gauge invariant, i.e. independent of the local choice of gauge “coordinates”.
}
argument given in this article is sufficient.

In conclusion, what we have done is to provide a “Gordian knot”-like theoretical explanation of the empirical “non-appearance” of gluons/quarks in the physical world.

We assume only that:

1) QCD is the correct theory of quark-field interactions

2) particles (quanta) are represented by $a$ and $a^\dagger$

which unambiguously leads to the result that QCD can have no elementary color quanta (gluons). If a specific fundamental quantum does not exist within a certain, supposedly correct, theory it neither can be detected in experiments. As the quark fields always generate color fields, which in turn act as sources of other color fields, the quark fields can never be considered to be noninteracting. Hence an expansion in harmonic oscillator modes is impossible, which means that no quark field quanta (quarks) can exist. Only if I) QCD is wrong, or II) elementary quanta are not necessarily described by harmonic oscillator modes (which would mean that the fundamental relation $E = \hbar \omega$ does not hold, and that the notion of what a quantum really is must be broadened from the traditional definition), or both, can gluons and quarks exist as physical particles. In geometrical terms the curvature of Yang-Mills (color) space makes quarks, as particles, impossible.

This proves that the theory of QCD automatically forbids particles with color charge, hence formally implying gluon/quark “confinement”. However, in a very real sense, there actually is nothing to confine in terms of particles.

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References


