Baryons in Partially-Quenched Chiral Perturbation Theory

M. J. Savage

Department of Physics, University of Washington, Seattle, WA 98195-1560, USA

I discuss the inclusion of baryons into partially-quenched chiral perturbation theory and describe one-loop calculations that have been performed.

1. Introduction

The motivation for including baryons into partially-quenched QCD (PQQCD), or more specifically partially-quenched chiral perturbation theory (PQ$\chi$PT), is clear. One wishes to take partially-quenched (PQ) lattice simulations of observables of interest that in the foreseeable future will be performed with quark masses that are heavier than those of nature, and make rigorous predictions for these observables in QCD. The way that such predictions will be made is to use PQ lattice simulations to “match” onto a PQ effective field theory (PQEFT), and then use the explicit quark mass dependence of the PQEFT to make predictions at the physical values of the light quark masses, i.e. the coefficients of the local operators in the PQEFT will be determined from the PQ lattice simulations.

1.1. Quenching and Partial-Quenching

Ideally, all lattice simulation would be unquenched and the quarks would have their physical masses. At some point in the future this will happen, but at this point in time and for the foreseeable future such simulations will not be possible, simply due to the lack of computing power. It is the quark loops that are disconnected from external sources (the quark determinant), participating through gluons alone, that are the most time consuming to simulate as they scale as a relatively high power of the inverse quark mass. In contrast, simulation of quark loops that are coupled to the external sources is much less time consuming. In quenched QCD (QQCD) [1–4], the quark determinant is set equal to unity, and thus the time-expensive disconnected quark loops are discarded, allowing for relatively rapid simulation at relatively light quark masses. From a formal standpoint, the quenched simulations correspond to introducing an equal mass “ghost” fermion for each light quark, so that disconnected loop contributions from the light quarks and the ghosts exactly cancel. Thus the $SU(2)_L \otimes SU(2)_R$ chiral symmetry of QCD is extended to $SU(2)_L \otimes SU(2)_R \times U(1)$. There is no formal limit in which quenched simulations can be used to make rigorous predictions about QCD. The discovery of PQQCD [6,7] was a major step forward in this regard. In addition to the ghosts that are added in QQCD, additional quarks are added to partially unquench the simulation, increasing the flavor symmetry of the theory to $SU(4)_L \otimes SU(4)_R \otimes U(1)$. These additional “sea”-quarks participate in the disconnected loop diagrams, but if they are heavier than the “valence”-quarks they require less time to simulate. In the limit that the mass of the sea-quarks become equal to those of the valence-quarks (and ghosts) the matrix elements of PQQCD between physical states become equal to those of QCD. Thus one can make QCD predictions from PQQCD.

In order to describe the low-momentum dynamics and the light-quark mass dependence of the low-lying hadrons in QCD one, of course, uses an EFT, such as $\chi$PT in the meson sector and heavy baryon $\chi$PT (HB$\chi$PT) in the single baryon sector. In extending the theory to PQQCD one

\[1\] However, it has recently been noted that QQCD in the large-$N_C$ limit is identical to QCD in the large-$N_C$ limit [5].
has to extend $\chi$PT to PQ$\chi$PT. This allow one to
determine the contributions that are non-analytic
in external momentum and $m_q$, and also to pa-
rameterize short-distance physics in the local
operators that enter order-by-order in the expansion.
As it is presently quite difficult to isolate
the non-analytic contributions from the analytic
ones in lattice simulations due to the relatively
large values of $m_q$ that can be simulated, knowl-
dge of the low-energy EFT is vital. The lattice
simulations are used to determine the values of
counterterms that appear at any given order in
the EFT. Thus, PQ simulations will be used to
make QCD predictions as the $m_q$-dependence
is explicit. Such determinations are beginning to
be performed in the meson sector [6,7].

2. Baryons in PQ$\chi$PT

Baryon properties in quenched $\chi$PT ($Q\chi$PT)
have been studied somewhat during the past six
years, starting with the seminal paper by Labrenz
and Sharpe [4]. In this initial work, quark-line
diagrams were used to “keep track” of the quarks
and ghosts in extending $\chi$PT to Q$\chi$PT. Later,
was it shown how to extend the representations of
the baryons from $SU(3)$ to $SU(3)_{\text{val}}$ for
three-flavor flavor QCD and from $SU(2)$ to $SU(2)_{\text{val}}$
for two-flavor QCD so that the tools developed for
HB$\chi$PT could be used directly in PQHB$\chi$PT. In
the quenched theory, the octet-baryon massess [4],
magnetic moment [8] and the matrix elements of
isovector twist-2 operators [9] have been studied
at the one-loop level in the chiral expansion.

Including baryons in PQ$\chi$PT is similar to in-
cluding them into Q$\chi$PT [10,11]. One constructs
the irreducible representations of the graded lie-
group, which for two-flavors is $SU(4)$, and as-
signs baryon fields, just as one does in QCD
and QQCD. The 70 dimensional representation
of baryons that contains the nucleons is described
by a tensor of the form $B_{ijk}$, where $i, j, k$ run from
1 to 6 (we have dropped the Dirac index). To
determine the particle assignment it is particularly
simple to use the interpolating fields discussed by
Labrenz and Sharpe,

$$
\left[ Q_i^{\alpha,a} Q_j^{\beta,b} Q_k^{\gamma,c} - Q_i^{\alpha,a} Q_j^{\gamma,c} Q_k^{\beta,b} \right] \epsilon_{abc} (C\gamma_5)_{\alpha\beta}
$$

for the representation containing the nucleons.
The objects $Q_i$ are quark field operators that in-
clude the valence, sea and ghost quarks. $C$ is
the charge conjugation operator, $\alpha, \beta, \gamma$ are Dirac indices. The interpo-
lating field for the 44 dimensional representation
that contains the 3/2-baryon resonances, $T_{ijk}$ (we have dropped lorentz and Dirac indices), is

$$
\left[ Q_i^{\alpha,a} Q_j^{\beta,b} Q_k^{\gamma,c} + \text{cyclic} \right] \epsilon_{abc} (C\gamma_\mu)_{\beta\gamma}.
$$

One finds that under the interchange of flavor
indices [4],

$$
B_{ijk} = (-)^{1+\eta_k} B_{ikj}
$$

for $k = 1, 2, 3, 4$ and $\eta_k = 0$ for

$$
T_{ijk} = (-)^{1+\eta_k} T_{ikj} = (-)^{1+\eta_k} T_{ikj}.
$$

with $\eta_k = +1$ for $k = 1, 2, 3, 4$ and $\eta_k = 0$ for

$k = 5, 6$.

The only baryons that appear as intermediate
states in one-loop diagrams involve either three
valence quarks, or two valence quarks and one
ghost quark, or two valence quarks and one sea
quark. Thus we only need to determine the loca-
tion of these baryons in the tensors $B$ and
$T$. It turns out to be convenient to classify the
baryons according to how they transform under
$SU(2)_{\text{val}} \otimes SU(2)_{\text{sea}} \otimes SU(2)_{\text{ghost}}$, and it is
straightforward to show that for the 70 one needs
to include fields $N_{\alpha, a}$, $t_{\alpha}$, $s_{\alpha bc}$, $t_{\alpha}$ and $s_{\alpha bc}$
that transform as $(2, 1, 1)$, $(1, 2, 1)$, $(3, 2, 1)$, $(1, 1, 2)$
and $(3, 1, 2)$ respectively. For the 44, one intro-
duces fields $\Delta_{abc}$, $\Delta_{bc}$ and $\Delta_{bc}$ that transform
as $(4, 1, 1)$, $(3, 2, 1)$ and $(3, 1, 2)$ respectively. For
the three-flavor case the octet of baryons are part
of the 240 dimensional irrep of $SU(6)$ while the
decuplet of baryon resonances are part of a 138
dimensional irrep.

It is straightforward to construct the lagrange
density describing the baryons and their in-
teractions with the pseudo-Goldstone bosons
associated with the spontaneous breaking of
$SU(4)_{L} \otimes SU(4)_{R}$ down to $SU(4)_{V}$. Thus
the loop expansion for the low-energy properties of the nucleons in PQχPT is the same as that in χPT except that the number of particles that can participate in loops is much larger, and one needs to keep track of minus signs associated with the loops involving fermionic mesons.

Calculations of several observables at the one-loop level, both in two- and three-flavor PQχPT, have recently been completed, in addition to a tree-level analysis of the nucleon-nucleon (NN) potential. A summary of what presently exists can be seen in Table 1. In the two-flavor calculations, both the valence- and sea-quarks are non-degenerate, while in the three-flavor calculations the two light-quarks are taken to be degenerate (both valence and sea) while the strange quark is non-degenerate in both, i.e. 2 + 1.

Instead of detailing each of these calculations, I wish to focus on a few aspects and observables that are somewhat out of the ordinary. For further details of the mainstream observables, such as the masses, magnetic moments and so forth I refer the reader to the appropriate papers.

2.1. The Nucleon-Nucleon Potential

In QCD it is well established that one pion exchange (OPE) gives the dominant contribution to the long-distance component of the NN (NN) potential. OPE yields a NN potential that is Yukawa potential at large-distances, falling like \( e^{-m_s r}/r \) and also provides a \( \sim 1/r^3 \) short-distance tensor contribution that mixes S-wave and D-waves.

Efforts have been made to explore the interactions between nucleons on the lattice by determining their scattering lengths. However, this is exceptionally challenging due to the fact that the scattering lengths are unnaturally large, \( \sim 6 \text{ fm} \) in the \( ^3S_1 - ^3D_1 \) coupled channels and \( \sim 24 \text{ fm} \) in the \( ^1S_0 \) channel. What’s more the simulations have been performed in QQCD [12] and currently no unquenched or PQ calculations exist (for a recent discussion see Ref. [13]).

In both QQCD and PQQCD the long-distance component of the NN potential is very different to that in QCD. The hairpin interaction that gives rise to a double pole in the iso-singlet propagator, that in QQCD is proportional to \( M_0^2 \), and in PQQCD is proportional to \( m_{vv}^2 - m_{ss}^2 \) and vanishes in the QCD limit, provides the dominant contribution to the long-distance component of the NN potential [14]. In the isospin limit, the \( SU(2) \) singlet \( (\eta, \text{ distinct from the } SU(4/2) \text{ singlet) propagator has the form} \)

\[
G_{\eta l}(q^2) = \frac{i(m_{ss}^2 - m_{vv}^2)}{(q^2 - m_{vv}^2 + i\epsilon)^2},
\]

which does not exhibit Yukawa type fall off at large distance but rather falls exponentially. Therefore, unfortunately, efforts to observe the long-distance behavior of the NN interaction in QQCD and PQQCD will need to subtract this large contribution.

2.2. Charges

When one considers processes involving electroweak gauge fields, there is an additional freedom in charge assignments that is not present in QCD. One requires that QCD is recovered in the limit where the mass of the sea-quarks become identical to the mass of the valence quarks, and thus the electromagnetic charge matrix in QCD, \( Q^{QCD} = \text{diag}(+\frac{2}{3}, -\frac{1}{3}) \), is extended to \( Q = \text{diag}(+\frac{2}{3}, -\frac{1}{3}, q_j, q_l, q_j, q_l) \) in PQQCD. The charges \( q_j, q_l \) are arbitrary; and in the QCD limit, observables become independent of them. However, away from the QCD limit quantities do depend upon them, and it may be useful to make specific choices for particular observables. For instance, choosing \( q_j = q_l = 0 \) means that disconnected loops coupling to a photon involve only valence-quarks, while choosing \( q_j = +\frac{2}{3} \) and \( q_l = -\frac{1}{3} \) means that disconnected loops coupling
to a photon involve only sea-quarks. Thus, numerically, there may be some advantage in choosing the latter set of charges over the former, but this remains to be explored. In addition, there are sets of charges that minimize the contribution from one-loop diagrams in PQ\chi PT. As higher orders are parametrically suppressed, this will lead to the smallest variation in the matrix element with respect to changes in $m_q$.

The non-uniqueness in the extension of electroweak operators from QCD to PQQCD is not confined to the electric charges. It is also present for the twist-2 operators [10], and for four-quark operators, such as those responsible for $K \to \pi \pi$ [15] and for $NN\pi$ parity-violation [16].

2.3. Proton Magnetic Moment

As an example of a calculation in PQ\chi PT, we discuss the proton magnetic moment, $\mu_p$, up to order $O(m_Q^{1/2})$ in the chiral expansion. The leading order contributions to the nucleon magnetic moment arise from the dimension-5 operators

$$L^{(5)} = \frac{e}{4M_N} F_{\mu\nu} \left[ \mu_\alpha (\overline{B}\sigma^{\mu\nu} B) + \mu_\beta (\overline{B}\sigma^{\mu\nu}QB) + \mu_\gamma (\overline{B}\sigma^{\mu\nu}B) \text{str} \right],$$

where the brackets $(..)$ denote contractions of the flavor indices as discussed in Ref. [4], and “str” denotes a supertrace. At tree-level, the coefficients $\mu_\alpha, \mu_\beta$ and $\mu_\gamma$ are related to the isoscalar, $\mu_0$, and isovector, $\mu_1$, magnetic moments of the nucleon,

$$\mu_0 = \frac{1}{6} (\mu_\alpha + \mu_\beta + 2\mu_\gamma) \quad \mu_1 = \frac{1}{6} (2\mu_\alpha - \mu_\beta),$$

Up to $O(m_Q^{1/2})$, the magnetic moment of the proton can be written as

$$\mu_p = \alpha_p + M_N \frac{1}{4\pi f^2} \left[ \beta_p + \beta_p' \right] + \ldots,$$

where

$$\alpha_p = \mu_0 + \mu_1,$$

$$\beta_p = -g_\Lambda^2 m_{vv} + (m_{sv} - m_{vv}) \left[ \frac{8}{9} g_A^2 + \frac{4}{9} g_A g_1 + \frac{1}{18} g_1^2 \right] + q_{ji} (m_{sv} - m_{vv}) \left[ \frac{2}{3} g_A^2 + \frac{1}{3} g_A g_1 + \frac{1}{6} g_1^2 \right],$$

$$\beta_p' = \frac{1}{27} g_\Lambda^2 \left[ -6F_{vv} + q_{ji} \frac{2}{3} (F_{sv} - F_{fv}) \right],$$

with $q_{ji} = q_j + q_i$. The function $F_{ij} = F(m_{ij}, \Delta, \mu)$ is given by

$$\pi F = \eta \log \left( \frac{\Delta - \eta}{\Delta + \eta} \right) - \Delta \log \left( \frac{m^2}{\mu^2} \right),$$

where $\eta = \sqrt{\Delta^2 - m^2 + i\epsilon}$, $\Delta$ is the $\Delta$-nucleon mass splitting in the chiral limit, and $\mu$ is the dimensional regularization renormalization scale. $g_A$ is the usual axial coupling constant, while $g_1$ is an additional axial coupling that contributes in PQ\chi PT, but whose contribution must vanish in the QCD limit. $g_{\Delta N}$ is the $\Delta N\pi$ coupling constant.

There are two distinct contributions from the loop diagrams away from the QCD limit. One is a contribution that is independent of the choice of charges that vanishes like $m_{sv} - m_{vv}$, while the other also vanishes like $m_{vv}$ but depends upon the choice of charges of the sea- and

<table>
<thead>
<tr>
<th>Observable</th>
<th>$N_f = 2$</th>
<th>$N_f = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Masses [10,11]</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Magnetic Moments [10,11]</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Matrix Elements of Twist-2 Operators [10,11]</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>$NN\pi$ Parity-Violating Interaction [16]</td>
<td>√</td>
<td>×</td>
</tr>
<tr>
<td>Nucleon Anapole Moment [16]</td>
<td>√</td>
<td>×</td>
</tr>
</tbody>
</table>
ghost-quarks. Therefore, one is able to find sets of charges for which the one-loop contribution to the proton magnetic moment from intermediate states in the 70 vanish, however, due to the non-trivial mass-dependences arising from intermediate states in the 44, the one-loop contributions cannot be eliminated entirely.

2.4. \( NN\pi \) Parity Violation

While flavor-changing parity-violating (PV) interactions are well understood theoretically and a great deal of precise data exists, knowledge of flavor-conserving parity-violation is rather primitive. Flavor-conserving parity-violation continues to be an area of intense investigation in the nuclear physics community. Its study is presently serving both to uncover the structure of the nucleon in electron-scattering experiments such as SAMPLE\[17\], and to determine PV flavor-conserving couplings between pions and nucleons\[18,19\]. The problems that are encountered in this sector are both experimental and theoretical.

On the experimental side, the PV signals, unlike those in flavor-changing processes, appear as small deviations in either a strong or an electromagnetic process, such as \( \gamma p \rightarrow ep \) or in the circular polarization of the \( \gamma \)-ray emitted in \( 18F^* \rightarrow 18F\gamma \) \[19\]. The current situation is somewhat confused by the fact that measurements of parity-violation in atoms and nuclei do not give rise to a consistent set of couplings between hadrons \[20\]. However, it is important to keep in mind that many of the “experimental” determinations of these couplings require theoretical inputs with varying degrees of reliability. Recently, it has been reemphasized that measurements of PV observables in the single-nucleon sector would significantly ameliorate the situation by eliminating many-body uncertainties \[21,22\]. Despite the inherent difficulty of such experiments, there are ongoing efforts to measure PV processes in systems with only one or two nucleons, such as the angular-asymmetry in \( \bar{n}p \rightarrow d\gamma \) \[23\]. Such measurements should provide a reliable determination of the leading-order (LO), momentum-independent weak \( \pi NN \) coupling constant, \( h_{\pi NN}^{(1)} \).

On the theoretical side, despite heroic efforts to model \[24,25\] hadronic matrix elements of the four-quark operators that appear in the low-energy effective theory of the standard model, there are no reliable calculations of the PV couplings between hadrons. A first principles calculation of \( h_{\pi NN}^{(1)} \) in lattice QCD would therefore be extremely welcome. This would require a lattice QCD simulation of a correlator with three hadronic sources interacting via a four-quark operator. Unfortunately, chiral symmetry does not allow one to relate the \( \pi NN \) correlator to a correlator without the pion. On the bright side, the structure of the four-quark weak Hamiltonian requires a flavor change in the nucleon and therefore there are no disconnected diagrams to be computed on the lattice.

The extraction of \( h_{\pi NN}^{(1)} \) from \( N \rightarrow N\pi \) requires an injection of energy at the PV weak vertex which can occur because the weak operator is inserted on one time-slice only. Therefore, we must include contributions from operators that are total derivatives, which usually vanish. Recently, chiral perturbation theory has been used to describe \( K \rightarrow \pi\pi \) with the kinematics appropriate for a lattice determination of the matrix elements of the relevant four-quark operators, \( n^{\text{latt}}_K = m_K^{\pi} \) and \( m_K^{\pi latt} = 2m_K^{\pi} \), including the necessary total derivative terms \[26\]. In QCD, the LO Lagrange density describing PV interactions is given by

\[
\mathcal{L}_{\text{wk}} = -h_{\pi NN}^{(1)} \frac{f}{4} \bar{\n} \{ X_L^\mu - X_R^\mu \} N \quad (11)
\]

\[\rightarrow \frac{i}{4} \bar{\pi} \Delta \Delta \cdot \mu \left[ X_L^\mu - X_R^\mu \right] c T_{abcd,\mu} \]

\[\rightarrow \frac{i}{4} \bar{\pi} \Delta \Delta \cdot \mu \left[ X_L^\mu - X_R^\mu \right] c T_{abcd,\mu} \]

\[\rightarrow \frac{i}{4} \bar{\pi} \Delta \Delta \cdot \mu \left[ X_L^\mu - X_R^\mu \right] c T_{abcd,\mu} \]

while the Lagrange density at NLO is (keeping only nucleon operators)

\[
\mathcal{L}_{\text{wk}}^{(NLO)} = \frac{h_{\pi NN}^{(1)}}{4} \left[ X_L^\mu - X_R^\mu \right] N , \quad (12)
\]

where \( \nu^\mu \) is the nucleon four-velocity. This is the leading contribution from a heavy baryon reduction of \( iD^\mu \overline{N} \gamma_\mu \left[ X_L^\mu - X_R^\mu \right] N \). Given baryon...
number conservation, the total derivative gives a non-zero contribution from the energy and momentum injected by the $X_L^3 - X_R^3$ insertion. Working in the frame where the initial state nucleon (proton) is at rest, $v^\mu = (1, 0, 0, 0)$, the amplitude at NLO resulting from eq. (12) and eq. (12) is

$$A_{\text{mp}} = \langle n\pi| i \int d^3 x \, L^{\Delta I=1}(E)| p\rangle$$

$$= -\bar{U}_n \left[ h^{(1)}_{\pi NN} + h^{(1)}_D \frac{E}{f} \right] U_p \ ,$$

(13)

where $E$ is the energy injected by the weak vertex. In order to produce an on-shell $n\pi$ final state, the injected energy must exceed $E \geq m_\pi + M_n - M_p$. Near threshold, where the final state neutron and pion are at rest and $E = m_\pi + M_n - M_p$, the contribution from the total-derivative operator, $h^{(1)}_D$, scales as $\sim m_q^{1/2}$, and is formally dominant over loop corrections and counterterms [16].

3. Conclusions

Lattice simulations will first make rigorous predictions about observables by matching onto the appropriate EFT and using its explicit quark mass dependence to extrapolate from the lattice quark masses to those of nature. Therefore the EFT’s must be known, which is the case in the meson sector and single baryon sector. However more work is required in the multi-nucleon sector to be sure that the candidate theory [27] (for a review see Ref. [28]), in fact, is consistent and converges. PQ simulations represent the future of this field until fully unquenched calculations can be performed at the physical values of the light quark masses, and in this work we have made the small step of including baryons in PQQCD. We have constructed PQ$\chi$PT and shown that there are some interesting features beyond QCD. This field is just beginning and calculations beyond one-loop level are certainly required in order to understand the convergence properties of PQ$\chi$PT and the uncertainties introduced in chiral extrapolations.

I am indebted to my colleagues, Silas Beane and Jiunn-Wei Chen, my collaborators on these works.

REFERENCES


