Abstract

The explicit form of the fermionic zero-modes in the fivebrane backgrounds of type IIA and IIB supergravity theories is investigated. In type IIA fivebrane background, there are four zero-modes of gravitinos and dilatons. In type IIB fivebrane backgrounds, there are four zero-modes of gravitinos are found. These zero-modes indicate the four-fermion condensates which have been suggested in a calculation of the tension of the D-brane in fivebrane backgrounds.

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1. INTRODUCTION

The study of the non-perturbative dynamics in the superstring theory is still underway. Especially, the dynamics of the fermionic sector is less known than the one in the bosonic sector. The analysis in the low-energy effective supergravity theories are mainly on the bosonic sector, and usually the fermionic sector is not considered directly. For example, although an interesting possibility of the supersymmetry breaking due to the gravitino condensation has been proposed in Refs. [1, 2, 3], it has not been clarified yet whether this is really possible or not. This kind of dynamics, which can be a non-perturbative effect of the quantum gravity, is very interesting in view of the low-energy physics because of its less model dependence.

An concrete model of the dynamical supersymmetry breaking due to the gravitino condensation is given in Ref. [2]. In the model the gravitino pair condensate is formed by the non-trivial gravitational background (Eguchi-Hanson metric), and the condensate triggers supersymmetry breaking through the Konishi anomaly relation. Since the analysis is based on the Euclidean space-time and the meaning of the non-trivial gravitational background is unclear (whether a gravitational instanton or an Euclidean configuration in the path integral), it is natural to ask whether the similar dynamics is possible in superstring theories.

The first step was the study in the low-energy supergravity theories. In Ref. [3] a background solution of the heterotic supergravity theory, which is very similar to the fivebrane background in Ref. [4], is constructed, and the zero-mode solutions of gravitino and dilatino are found. Although these zero-modes suggests the dynamical formation of the gravitino condensation, the condensation can not trigger the supersymmetry breaking through the mechanism of Ref. [2], since the ten-dimensional heterotic supergravity theory has no global chiral symmetry. The analysis of the fermionic zero-modes in fivebrane backgrounds of the heterotic supergravity theory has been done in Refs. [5, 6]. In Ref. [6] the author concluded that the gravitino condensation is possible, but it does not trigger the supersymmetry breaking.

Very few analysis which is based on the string world-sheet theory has been achieved. In Ref. [7] the author suggests some four-fermion condensations in fivebrane backgrounds of the type IIB string theory. The fivebrane backgrounds in the type IIB supergravity theory [8] can be described by the world-sheet conformal field theory as a solution of the type II superstring theory in a certain limit [9]. The existence of the four fermionic zero-modes
in fivebrane backgrounds in type IIA and type IIB supergravity theories is well known[8], and this number is consistent with the suggestion in Ref.[7]. Although such four-fermion condensations do not mean the dynamical supersymmetry breaking through the Konishi anomaly relation (there are no global chiral symmetry in type IIA and IIB supergravity theories), this kind of analysis is worth doing to understand the non-perturbative dynamics in the fermionic sector of superstring theories.

In this letter we present the analysis based on the low-energy type IIA and IIB supergravity theories. The aim of this analysis is to know the explicit form of four fermionic zero-modes in fivebrane backgrounds in type IIA and IIB supergravity theories. The fermion field equations in fivebrane backgrounds in each system are explicitly solved. In the next section we find four zero-modes of gravitinos and dilatinos in type IIA fivebrane backgrounds. In section III four dilatino zero-modes and no gravitino zero-modes are found in type IIB fivebrane backgrounds. In the last section a summary of the result is given.

II. FERMION ZERO-MODES IN TYPE IIA FIVEBRANE BACKGROUND

The Lagrangian of the type IIA supergravity theory (non-chiral $N = 2$, $D = 10$ supergravity theory) is derived from the $N = 1$, $D = 11$ supergravity theory by the dimensional reduction[10, 11, 12].

$$\mathcal{L} = e^{-2\Phi} \left\{ -\frac{1}{2} R(\epsilon, \omega(\epsilon)) + 2 \partial^\mu \Phi \partial_\mu \Phi - \frac{1}{6} H^{\mu\nu\rho} H_{\mu\nu\rho} - \frac{1}{2} \bar{\psi}_\mu \Gamma^{\mu\nu} D_\nu \psi_\nu - \frac{1}{2} \bar{\lambda} \Gamma^\rho D_\rho \lambda \\
- \bar{\psi}_\mu \Gamma^\mu \partial_\nu \Phi + \frac{\sqrt{2}}{4} \bar{\psi}_\mu \Gamma^\mu \Gamma^\nu \Gamma_1 \lambda \partial_\nu \Phi \\
+ \frac{1}{24} H_{\alpha\beta\gamma} \left( \bar{\psi}_\mu \Gamma^{\mu\alpha\beta\gamma} \Gamma_1 \psi_\nu + 6 \bar{\psi}_\mu \Gamma^{\beta\gamma} \Gamma_1 \psi_\nu - \sqrt{2} \bar{\psi}_\mu \Gamma^{\alpha\beta\gamma} \Gamma^\mu \lambda \right) \right\} \tag{1}$$

where we set all R-R fields, $C_\mu$ and $C_{\mu\nu\rho}$, zero. We use the convention of Ref.[13] in this section. Here, $\Phi$ is the dilaton scalar field, $R(\epsilon, \omega(\epsilon))$ is the scalar curvature composed by the zhenbein $e^a_\mu$ and spin connection $\omega_\mu^{ab}(\epsilon)$ with the space-time coordinate $\mu$ and local Lorentz coordinates $a$ and $b$, $H_{\mu\nu\rho}$ is the field strength of the two-form potential field $B_{\mu\nu}$, and $\psi_\mu$ and $\lambda$ are the gravitino and dilatino fields, respectively, which are Majorana spinor including both chirality components.

The fivebrane background is the solution of the field equations of this Lagrangian with
vanishing R-R fields and fermion fields. The explicit from of the background is

\[ H^{\mu \nu} = -e^{abc} \partial_\mu \Phi, \quad (2) \]

\[ e^a = e^b \delta^a_\mu, \quad (3) \]

\[ e^{2 \Phi} = e^{2 \Phi_0} + \frac{Q}{r^2}, \quad (4) \]

where the local Lorentz indices \(a, b, c\) and \(d\) run from 6 to 9 (transverse direction to the fivebrane), \(\Phi_0\) is a constant, \(Q\) is a constant which is quantized to be an integer, and \(r^2 \equiv \sum_{a=6,7,8,9} x^a x^a\) is the squared distance from the center of the fivebrane. The dilaton background satisfies the condition \(\Box e^{2 \Phi} = 0\), where \(\Box \equiv \delta_{ab} \delta_{cd}\) is the four-dimensional Laplacian. This background keeps half of the original (global) supersymmetry and breaks the other half of supersymmetry. We can check this fact by considering infinitesimal supersymmetry variations with parameter \(\eta(x)\). Since all the fermion background fields vanish, the variation of the bosonic fields is automatically zero. The variation of the fermionic fields are non-trivial.

\[ \delta_\eta \lambda_\pm = \mp \frac{\sqrt{2}}{4} \partial_\mu \Phi \Gamma^a (1 \mp \gamma_5) \eta_\pm, \quad (5) \]

\[ \delta_\eta \psi^{\pm}_\mu = \partial_\mu \eta_\pm + \frac{1}{8} \Omega^{abc} \Gamma_{ab} (1 \mp \gamma_5) \eta_\pm - \frac{1}{8} \Gamma^c \partial_\mu \Phi \Gamma^a (1 \mp \gamma_5) \eta_\pm, \quad (6) \]

where

\[ \Gamma_{11} \eta_\pm = \pm \eta_\pm \quad (7) \]

and \(\gamma_5 \equiv \Gamma^6 \Gamma^7 \Gamma^8 \Gamma^9\) is the four-dimensional chirality operator. The modified connection

\[ \Omega^{\pm \mu \nu} = \omega^{\mu \nu} \pm H^{\mu \nu} \quad (8) \]

satisfies the self-dual relation:

\[ \Omega^{\pm \mu \nu} = \Omega^{\pm \nu \mu} = \mp \frac{1}{2} e^{abcd} \Omega^{\pm \mu \nu \delta \epsilon}. \quad (9) \]

From Eqs. (5) and (6), we see that the global supersymmetry with \(\eta^{(\pm)}\) is not broken, but the one with \(\eta^{(\mp)}\) is broken, where \(\gamma_5 \eta^{(\mp)} = \pm \eta^{(\pm)}\).

It is easy to derive the fermion field equations form the Lagrangian of Eq.(1).

\[ \Gamma^\mu (D_\mu - \partial_\mu \Phi) \lambda_\pm + \frac{\sqrt{2}}{4} \Gamma^\mu \left( \pm \partial_\mu \Phi \Gamma^\nu \mp \frac{1}{6} H_{\alpha \beta \gamma} \Gamma^\alpha \Gamma^\beta \Gamma^\gamma \right) \psi^{\pm}_\mu = 0, \quad (10) \]
\[ \Gamma^\mu_{\nu\rho} (D_\rho - \partial_\rho \Phi) \psi_{\pm\nu} + \partial^\rho \Phi \Gamma^\mu \psi_{\pm\nu} - \partial^\rho \Phi \Gamma^\nu \psi_{\pm\nu} + \frac{1}{12} H_{\alpha\beta\gamma} \Gamma^\mu_{\alpha\beta\gamma} \psi_{\pm\nu} + \frac{1}{2} \frac{1}{2} H_{\alpha\beta\gamma} \Gamma^\mu \psi_{\pm\nu} + \sqrt{2} \left( \pm \partial_\rho \Phi \Gamma^\nu + \frac{1}{6} H_{\alpha\beta\gamma} \Gamma^\mu \psi_{\pm\nu} \right) \mu \lambda_{\mp} = 0. \] 

(11)

In order to find the fermion zero-modes in fivebrane backgrounds, we only consider the chirality components of \( \psi^{(\mp)}_{\pm\mu} \) and \( \lambda^{(\pm)}_{\mp} \), which can be induced by the broken supersymmetry variation with parameters \( \eta_{\pm}^{(\mp)} \) in Eqs.(5) and (6). Furthermore, only the four-dimensional space of \( \mu = 6, 7, 8, 9 \), where the metric is non-trivial, is considered, and all the fermion fields are considered just as four-dimensional spinor fields. The fermion fields are independent of the coordinates of the six-dimensional space-time \( \mu = 0, \cdots, 5 \), and the value of the fields vanishes when their indices take the values of the six-dimensional space-time. The relevant field equations are as follows.

\[ \Gamma^\rho (D_\rho - \partial_\rho \Phi) \lambda_{\mp}^{(\pm)} \pm \frac{\sqrt{2}}{2} \partial_\rho \Phi \Gamma^\rho \psi^{(\mp)}_{\pm\nu} = 0, \] 

(12)

\[ \Gamma_{\mu\nu} \left( D_\rho - \frac{3}{2} \partial_\rho \Phi \right) + \partial^\rho \Phi \Gamma^\mu - \partial^\rho \Phi \Gamma^\nu \right) \psi^{(\mp)}_{\pm\nu} = 0, \] 

(13)

where we used Eq.(2).

It is naively expected that the supersymmetry variation

\[ \lambda_{\mp}^{(\pm)} = \pm \frac{\sqrt{2}}{2} \partial_\rho \Phi \Gamma^\rho \eta_{\pm}^{(\mp)}, \] 

(14)

\[ \psi^{(\mp)}_{\pm\mu} = \partial_\mu \eta_{\pm}^{(\mp)} + \frac{3}{4} \partial_\rho \Phi \Gamma^\rho \eta_{\pm}^{(\mp)} - \frac{1}{4} \partial_\mu \Phi \eta_{\pm}^{(\mp)}, \] 

(15)

is the zero-mode solution. In fact, it is easily shown that Eq.(12) is satisfied for any functions \( \eta_{\pm}^{(\mp)} \) by using the relation

\[ \partial^\mu \partial_\rho \Phi + 2 \partial^\rho \Phi \partial_\rho \Phi = 0 \] 

(16)

which follows from the condition \( \Box e^{\Phi} = 0 \). However, Eq.(13) is not satisfied by Eqs.(14) and (15), and some modifications are required. In case of the constant \( \eta_{\pm}^{(\mp)} \), both Eqs.(12) and (13) can be satisfied by the modification of the gravitino expression.

\[ \lambda_{\mp}^{(\pm)} = \pm \frac{\sqrt{2}}{2} \partial_\rho \Phi \Gamma^\rho \eta_{\pm}^{(\mp)}, \] 

(17)

\[ \psi^{(\mp)}_{\pm\mu} = \psi^{(\mp)}_{\pm\mu} + \frac{3}{2} \left( \partial^\rho \Phi \Gamma_{\mu\rho} \eta_{\pm}^{(\mp)} - \frac{3}{2} \partial^\rho \Phi \Gamma_{\mu} \eta_{\pm}^{(\mp)} \right) = - \frac{5}{2} \partial_\mu \Phi \eta_{\pm}^{(\mp)}. \] 

(18)
This is the normalizable fermionic zero-mode solution in fivebrane background. The gravitino solution satisfies the gauge condition $D_\mu \psi_{0\pm\mu} = 0$. This solution can be rewritten as follows by rescaling the constant parameters $\eta_{\pm}^{(\mp)}$.

\[
\lambda_0^{(\pm)} = \mp \frac{\sqrt{2}}{3} \partial_\rho \Phi \Gamma^\rho \eta_{\pm}^{(\mp)} = \lambda_{\pm}^{(\pm)} \pm \frac{\sqrt{2}}{3} \partial_\rho \Phi \Gamma^\rho \eta_{\pm}^{(\mp)}, \quad (19)
\]

\[
\psi_{0\pm\mu} = - \partial_\mu \Phi \eta_{\pm}^{(\mp)} = \psi_{\pm\mu} - \frac{3}{4} \partial_\rho \Phi \Gamma_\rho \eta_{\pm}^{(\mp)}. \quad (20)
\]

This solution could be understood as a combination of supersymmetry variations and superconformal variations.

The parameter $\eta$ originally has 32 components as a Majorana spinor representation of the space-time symmetry $SO(9, 1)$. It is decomposed in two parts $16_+$ and $16_-$ by the ten-dimensional chirality. They are described as four representations of $SO(5, 1) \times SO(4) \subset SO(9, 1)$ as follows.

\[
16_+ = (4_+, 2_+) + (4_-, 2_-), \quad (21)
\]

\[
16_- = (4_+, 2_-) + (4_-, 2_+). \quad (22)
\]

The parameters $\eta_{\pm}^{(-)}$ and $\eta_{\pm}^{(+)}$ correspond to the representations of $(4_-, 2_-)$ and $(4_-, 2_+)$, respectively, and each has two independent component as a $SO(4)$ Weyl spinor. Therefore, we find the explicit form for the four fermionic zero-modes in type IIA fivebrane backgrounds.

### III. FERMION ZERO-MODES IN TYPE IIB FIVEBRANE BACKGROUND

Although there is no manifestly Lorentz-invariant action of the type IIB supergravity theory (chiral $N = 2$, $D = 10$ supergravity theory), the covariant field equations have been obtained\cite{[14]} The field equations of fermion fields are

\[
\gamma^{\mu} D_\mu \lambda = \frac{1}{240} i \gamma^{\mu_1 \cdots \mu_5} \lambda F_{\mu_1 \cdots \mu_5} + ((\text{fermion})^3 \text{ terms}), \quad (23)
\]

\[
\gamma^{\mu\nu} D_\rho \psi_\nu = - \frac{1}{2} \partial_\rho \lambda \gamma^{\nu+\mu} P_\nu - \frac{1}{48} i \gamma^{\nu+\sigma} \gamma^{\mu} \lambda G_{\nu+\sigma} + ((\text{fermion})^3 \text{ terms}), \quad (24)
\]

where the convention of Ref.\cite{[14]} is used ($\kappa = 1$). Here, the dilatino field $\lambda = \lambda_R + i \lambda_I$ is the complex Weyl spinor with positive ten-dimensional chirality, the gravitino field $\psi_\mu = \psi_{R\mu} + i \psi_{I\mu}$ is the complex Weyl spinor with negative ten-dimensional chirality, $F_{\mu_1 \cdots \mu_5}$ is the field strength of the real four-form potential field $A_{\mu_1 \cdots \mu_5}$, $P_\rho$ is the “field strength” of the
complex scalar field $B$, and $G_{\nu\rho}$ is the field strength of the complex two-form field $A_{\rho\sigma}$. The covariant derivative includes an unusual contribution:

$$D_\mu \lambda = \tilde{D}_\mu \lambda - \frac{3}{2} i Q_{\mu} \lambda, \quad (25)$$

$$D_\rho \psi_\nu = \tilde{D}_\rho \psi_\nu - \frac{1}{2} i Q_\rho \psi_\nu, \quad (26)$$

where $\tilde{D}$ denotes the usual covariant derivative in curved space-time and

$$Q_\mu = \frac{1}{1 - B^* B} \mathrm{Im} (B \partial_\mu B^*). \quad (27)$$

The supersymmetry transformation rule of these fermion fields is

$$\delta \lambda = i \gamma^\mu \epsilon \tilde{P}_\mu - \frac{1}{24} i \gamma^{\mu\nu\rho} \epsilon \tilde{G}_{\nu\rho} + \frac{3}{2} \mathrm{Im} (B \tilde{\epsilon} \lambda^\ast) \lambda, \quad (28)$$

$$\delta \psi_\mu = D_\mu \epsilon + \frac{1}{480} i \gamma^{\rho_1 \cdots \rho_5} \epsilon \tilde{P}_{\rho_1 \cdots \rho_5} + \frac{1}{96} \left( \gamma_\mu \, \epsilon \tilde{G}_{\nu\rho} - 9 \gamma^{\rho} \tilde{G}_{\rho\mu} \right) \epsilon^*$$

$$+ \left( \text{(fermion) }^2 \text{ terms} \right), \quad (29)$$

where the parameter $\epsilon = \epsilon_R + i \epsilon_I$ is the complex Weyl spinor with negative ten-dimensional chirality with

$$D_\mu \epsilon = \tilde{D}_\mu \epsilon - \frac{1}{2} i Q_\mu \epsilon, \quad (30)$$

and hat means the supercovariantization.

The fivebrane background is a classical configuration which preserves half of the original (global) supersymmetry. The background field configuration is as follows[8]. For $P_\mu$, $G_{\nu\rho}$ and $F_{\rho_1 \cdots \rho_5}$ fields,

$$P_\mu = \frac{1}{2} \partial_\mu \Phi, \quad (31)$$

$$G_{abc} = -2 \epsilon_{abcd} \partial^d \Phi, \quad (32)$$

$$F_{\rho_1 \cdots \rho_5} = 0, \quad (33)$$

where $a, b, c$ and $d$ are the local Lorentz coordinates which run from 6 to 9 and $\Phi$ is the real scalar field satisfying Eq. (4). Eq. (31) can be understood as the background $B = B_R + i B_I$ field of

$$B_R = \tanh \frac{\Phi}{2}, \quad (34)$$

$$B_I = 0, \quad (35)$$

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and this results $Q_\mu = 0$. The background metric is

$$
e^a_\mu = \begin{cases} 
e^{-\Phi/4}\delta^a_\mu, & a,\mu = 0, \ldots, 5, \\
ne^{3\Phi/4}\delta^a_\mu, & a,\mu = 6, \ldots, 9, \\
0, & \text{other } a \text{ and } \mu. 
\end{cases} \quad (36)$$

Note that we are using the Einstein metric.

The supersymmetry variation of bosonic fields vanish, since all the fermionic background fields are zero. The supersymmetry variation of fermionic fields is non-trivial.

$$\delta \lambda = \partial_\mu \Phi i\gamma^\mu \frac{1 - \Gamma_5}{2}\epsilon_R - i\partial_\mu \Phi i\gamma^\mu \frac{1 + \Gamma_5}{2}\epsilon_I , \quad (37)$$

$$\delta \psi_\mu = \left( \partial_\mu + \frac{1}{8}\partial_\mu \Phi \Gamma_5 \right) \epsilon_R + i \left( \partial_\mu - \frac{1}{8}\partial_\mu \Phi \Gamma_5 \right) \epsilon_I + \frac{3}{4}\partial_\rho \Phi \gamma_{\mu \nu} \frac{1 - \Gamma_5}{2}\epsilon_R + \frac{3}{4}\partial_\rho \Phi \gamma_{\mu \nu} \frac{1 + \Gamma_5}{2}\epsilon_I, \quad (38)$$

where $\Gamma_5 \equiv \gamma^6\gamma^7\gamma^8\gamma^9$ is the four-dimensional chirality operator. Note that all $\gamma^\mu$ are imaginary. We see that the supersymmetry with $\epsilon_R^{(\pm)}$ and $\epsilon_I^{(\pm)}$ is broken, and the supersymmetry with $\epsilon_R^{(+)} = e^{-\Phi/8}\eta_R^{(+)}$ and $\epsilon_I^{(-)} = e^{-\Phi/8}\eta_I^{(-)}$ is preserved, where $\Gamma_5\epsilon_R^{(\pm)} = \pm\epsilon_R^{(\pm)}$ and $\eta_R^{(+)}$ and $\eta_I^{(-)}$ are constant spinors.

The fermion field equations in fivebrane backgrounds are simple. From Eqs.(23) and (24),

$$\gamma^\mu D_\mu \lambda = 0, \quad (39)$$

$$\gamma^{\mu \nu} D_\rho \psi_\nu + \frac{1}{4}i\partial_\rho \Phi \gamma^{\rho \nu} \lambda^* - \Gamma_5 \lambda = 0. \quad (40)$$

In order to find the fermionic zero-modes in fivebrane backgrounds, we only consider the fermion components $\psi_R^{(-)}$, $\psi_R^{(+)}$, $\lambda_R^{(\pm)}$ and $\lambda_R^{(-)}$ which can be induced by the broken supersymmetry variation. Furthermore, we take the following ansatz: only the four-dimensional space of $\mu = 6, 7, 8, 9$ should be considered, and all the fermion fields are considered just as four-dimensional spinor fields. The fermion fields are independent of the coordinates of the six-dimensional space-time $\mu = 0, \ldots, 5$, and the value of fields vanishes when their indices take the values of the six-dimensional space-time. Then, the field equations become quite simple.

$$\gamma^\mu D_\mu \lambda = \gamma^\mu \left( \partial_\mu + \frac{9}{8}\partial_\mu \Phi \right) \lambda = 0, \quad (41)$$

$$\gamma^{\mu \nu} D_\rho \psi_\nu = \gamma^{\mu \nu} \left( \partial_\mu + \frac{3}{8}\partial_\rho \Phi \gamma_{\rho \sigma} \right) \psi_\nu = 0, \quad (42)$$

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where $\lambda$ can be $\lambda_{\mu}^{(+)}$ or $\lambda_{\mu}^{(-)}$ and $\psi_\mu$ can be $\psi_R^{(-)}$, $\psi_R^{(+)}$, or $\psi_I^{(+)}$.

The candidate of the solution of Eqs.(41) and (42) is the following broken supersymmetry variation.

$$\lambda' = \partial_{\mu} \Phi i \gamma_{\mu} \epsilon_R^{(-)} - i \partial_{\nu} \Phi i \gamma_{\nu} \epsilon_I^{(+)}.$$  

$$\psi_\mu' = \left( \partial_{\mu} - \frac{1}{8} \partial_{\nu} \Phi + \frac{3}{4} \partial_{\nu} \Phi \gamma_{\nu\mu} \right) \epsilon_R^{(-)} + i \left( \partial_{\nu} - \frac{1}{8} \partial_{\mu} \Phi + \frac{3}{4} \partial_{\nu} \Phi \gamma_{\mu\nu} \right) \epsilon_I^{(+)}.$$  

Eq.(43) is the solution of Eq.(41), if $\epsilon_R^{(-)} = e^{\frac{i}{2} \Phi} \eta_R^{(-)}$ and $\epsilon_I^{(+)} = e^{\frac{i}{2} \Phi} \eta_I^{(+)}$, where $\eta_R^{(-)}$ and $\eta_I^{(+)}$ are constant spinors. Namely, the solution is

$$\lambda_0 = \partial_{\mu} \Phi i \gamma_{\mu} \eta_R^{(-)} + i \partial_{\nu} \Phi i \gamma_{\nu} \eta_I^{(+)},$$  

and this is a normalizable solution. Eq.(44) is not the solution of Eq.(42). The real part of Eq.(44) is a linear combinations of all three possible independent terms in the lowest order of the derivative expansion. The solution should be a linear combination of these three terms. The same is true for the imaginary part of Eq.(44). However, we can explicitly show that any linear combination can not be a solution of Eq.(42). Therefore, we conclude that there is no gravitino zero-mode in the lowest order of the derivative expansion.

The parameter of the supersymmetry transformation $\epsilon$ originally has 32 components. Each of real and imaginary part of $\epsilon$ belongs to the 16$_-$ Majorana-Weyl representation of the space-time symmetry SO(9,1). Each of them is described as two representations of SO(5,1) x SO(4) C SO(9,1) as in Eq.(22). The spinors $\eta_R^{(-)}$ and $\eta_I^{(+)}$ correspond to the representations of (4$_+$, 2$_-$) and (4$_-$, 2$_+$), respectively, and each has two independent component as a SO(4) spinor. Therefore, we find the explicit form of the four dilatino zero-modes in type IIB fivebrane backgrounds.

IV. CONCLUSION

We have found the explicit form of the four fermionic zero-modes in both type IIA and IIB fivebrane backgrounds. In type IIA fivebrane backgrounds both dilatinos and gravitinos have zero-modes. On the other hand, in type IIB fivebrane backgrounds only dilatinos have zero-modes and there are no gravitino zero-modes. If we believe the naive consideration based on the path integral in the low-energy supergravity theory, this result suggests four-fermion condensations in fivebrane backgrounds.
type IIB fivebrane backgrounds four-fermion condensations have been suggested in Ref.[7] through the calculation of the D3-brane tension in type IIB fivebrane backgrounds using the world-sheet conformal field theory. To understand which four-fermion condensations forms, we need further knowledge of the higher-order terms in the low-energy supergravity theory.

The naive consideration based on the path integral also suggests that there should be no fermion pair condensates in fivebrane background, since there are four fermion zero-modes. This can be checked by calculating the fermion propagator in the string theory using the world-sheet conformal field theory. If this naive consideration is correct, there should be no massless fermion propagation. The result of this calculation will be given in future works.

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