Measuring the CP-violating phase by a long base-line neutrino experiment with Hyper-Kamiokande

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Abstract

We study the sensitivity of a long-base-line (LBL) experiment with neutrino beams from the High Intensity Proton Accelerator (HIPA), that delivers $10^{21}$ POT per year, and a proposed 1Mt water-Čerenkov detector, Hyper-Kamiokande (HK) 295km away from the HIPA, to the CP phase ($\delta_{\text{MNS}}$) of the three-flavor lepton mixing matrix. We examine a combination of the $\nu_\mu$ narrow-band beam (NBB) at two different energies, $\langle p_\pi \rangle = 2, 3\text{GeV}$, and the $\bar{\nu}_\mu$ NBB at $\langle p_\pi \rangle = 2\text{GeV}$. By allocating one year each for the two $\nu_\mu$ beams and four years for the $\bar{\nu}_\mu$ beam, we can efficiently measure the $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ transition probabilities, as well as the $\nu_\mu$ and $\bar{\nu}_\mu$ survival probabilities. CP violation in the lepton sector can be established at $4\sigma$ ($3\sigma$) level if the MSW large-mixing-angle scenario of the solar-neutrino deficit is realized, $|\delta_{\text{MNS}}|$ or $|\delta_{\text{MNS}} - 180^\circ| > 30^\circ$, and if $4|U_{e3}|^2(1 - |U_{e3}|^2) \equiv \sin^2 2\theta_{\text{CHOOZ}} > 0.03$ ($0.01$). The phase $\delta_{\text{MNS}}$ is more difficult to constrain by this experiment if there is little CP violation, $\delta_{\text{MNS}} \sim 0^\circ$ or $180^\circ$, which can be distinguished at $1\sigma$ level if $\sin^2 2\theta_{\text{CHOOZ}} \geq 0.01$.

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Neutrino oscillation experiment is one of the most attractive experiments in the first quarter of 21st century. Many experiments will measure precisely the model parameters in the neutrino oscillations. In this article, we discuss the sensitivity of a long-base-line (LBL) experiment with conventional neutrino beams to measure the CP phase in the lepton sector.

The Super-Kamiokande (SK) collaboration showed that the $\nu_\mu$ created in the atmosphere oscillates into $\nu_\tau$ with almost maximal mixing [1]. The SNO collaboration reported that the $\nu_e$’s from the sun oscillate into the other active neutrinos [2]. A consistent picture in the three active-neutrino framework is emerging.

In the three-neutrino-model, neutrino oscillations depend on two mass-squared differences, three mixing angles and one CP violating phase of the lepton-flavor mixing (Maki-Nakagawa-Sakata (MNS) [3]) matrix. These parameters are constrained by the solar and atmospheric neutrino observations. One of the mixing angles and one of the mass-squared differences are constrained by the atmospheric-neutrino observation, which we may label [4] as $\sin^2 \theta_{\text{ATM}}$ and $\delta m^2_{\text{ATM}}$, respectively. The K2K experiment, the ongoing LBL neutrino oscillation experiment from KEK to SK, constrains the same parameters [5]. Their findings are consistent with the maximal mixing, $\sin^2 2\theta_{\text{ATM}} \sim 1$ ($\sin^2 \theta_{\text{ATM}} \sim 0.5$) and $\delta m^2_{\text{ATM}} \sim (2 \sim 4) \times 10^{-3} \text{(eV}^2)$. The solar-neutrino observations constrain another mixing angle and the other mass-squared difference, $\sin^2 2\theta_{\text{SOL}}$ and $\delta m^2_{\text{SOL}}$, respectively. Four possible solutions to the solar-neutrino deficit problem [6] are found: the MSW [7, 8] large-mixing-angle (LMA) solution, the MSW small-mixing-angle (SMA) solution, the vacuum oscillation (VO) solution [9], and the MSW low-$\delta m^2$ (LOW) solution. The SK collaboration [6] and the SNO collaboration [2] suggested that the MSW LMA solution is the most favorable solution among them, for which $\sin^2 2\theta_{\text{SOL}} = 0.7 \sim 0.9$ and $\delta m^2_{\text{SOL}} = (3 \sim 15) \times 10^{-5} \text{eV}^2$. For the third mixing angle, only the upper bound is obtained from the reactor neutrino experiments. CHOOZ [10] and Palo Verde [11] found $\sin^2 2\theta_{\text{CHOOZ}} < 0.1$ for $\delta m^2_{\text{ATM}} \sim 3 \times 10^{-3} \text{eV}^2$. No constraint on the CP phase ($\delta_{\text{MNS}}$) has been reported.

Several future LBL neutrino-oscillation experiments [12]-[15] have been proposed to confirm the results of these experiments and to measure the neutrino oscillation parameters more precisely. One of those experiments proposed in Japan makes use of the beam from High Intensity Proton Accelerator (HIPA) [16] and SK as the detector [15]. The facility HIPA [16] has a 50 GeV proton accelerator to be completed by the year 2007 in the site of JAERI (Japan Atomic Energy Research Institute), as a joint project of KEK and JAERI. The proton beam of HIPA will deliver neutrino beams of sub-GeV to several GeV range, whose intensity will be two orders of magnitudes higher than that of the KEK PS beam for the K2K experiment. The HIPA-to-SK experiment with $L=295$ km base-line length and $\langle E_\nu \rangle \simeq 1 \text{ GeV}$ will measure $\delta m^2_{\text{ATM}}$ at about 3 % accuracy and $\sin^2 \theta_{\text{ATM}}$ at
about 1% accuracy from the $\nu_\mu$ survival rate, while $\nu_\mu$-to-$\nu_e$ oscillation can be discovered if $\sin^2 2\theta_{\text{CHOOZ}} = 4|U_{e3}|^2(1 - |U_{e3}|^2) \gtrsim 0.006$ [15]. As a sequel to the HIPA-to-SK LBL experiment, prospects of using the HIPA beam for a very long base-line (VLBL) experiments with the base-line length of a few thousand km have been studied [4, 17]. Use of narrow-band high-energy neutrino beams ($\langle E_\nu \rangle = 3 \sim 6\text{GeV}$) and a 100kton-level water Čerenkov detector [17] will allow us to distinguish the neutrino mass hierarchy ($m_2 - m_1$), if $\sin^2 2\theta_{\text{CHOOZ}} \gtrsim 0.03$ [4]. If the LMA solution of the solar neutrino deficit is chosen by the nature, we can further constrain the allowed region of the $\delta^{\text{MNS}}$ and $\sin^2 2\theta_{\text{CHOOZ}}$ [4]. However, because $\nu_\mu \rightarrow \nu_e$ appearance is strongly suppressed by the matter effect at such high energies, the measurement is not sensitive to the CP violating effects, $\sim \sin \delta^{\text{MNS}}$.

In this paper, we study the capability of an LBL experiment between HIPA and Hyper-Kamiokande (HK), a megaton-level water Čerenkov detector being proposed to be built at the Kamioka site [18]. Here a combination of the shorter distance ($L = 295\text{km}$) and low $\nu$-energy ($\langle E_\nu \rangle \sim 1\text{GeV}$) makes the matter effect small, and the comparison of $\nu_\mu \rightarrow \nu_e$ and $\nu_\tau \rightarrow \nu_e$ appearance experiments is expected to have sensitivity to the CP violation effects proportional to $\sin \delta^{\text{MNS}}$.

The MNS matrix of the three-neutrino model is defined as

$$\nu_\alpha = \sum_{i=1}^{3} (V^{\text{MNS}})_{\alpha i} \nu_i = \sum_{i=1}^{3} (U^{\text{MNS}})_{\alpha i} \mathcal{P}_{ii} \nu_i,$$

where $\alpha = e, \mu, \tau$ are the lepton-flavor indices and $\nu_i (i = 1, 2, 3)$ denotes the neutrino mass-eigenstates. The $3 \times 3$ MNS matrix, $V^{\text{MNS}}$, has three mixing angles and three phases in general for Majorana neutrinos. In the above parameterization, the two Majorana phases reside in the diagonal phase matrix $\mathcal{P}$, and the matrix $U$, which has three mixing angles and one phase, can be parameterized in the same way as the CKM matrix [19]. Because the present neutrino oscillation experiments constrain directly the elements, $U_{e2}, U_{e3}$, and $U_{\mu3}$, we find it most convenient to adopt the parameterization [20] where these three matrix elements in the upper-right corner of the $U$ matrix are chosen as the independent parameters. Without losing generality, we can take $U_{e2}$ and $U_{\mu3}$ to be real and non-negative while $U_{e3}$ is a complex number. All the other matrix elements of the $U$ are then determined by the unitary conditions [20].

The probability of finding the flavor-eigenstate $\beta$ at base-line length $L$ in the vacuum from the original flavor-eigenstate $\alpha$ is given by

$$P_{\nu_\alpha \rightarrow \nu_\beta} = |U_{\beta 1} U_{\alpha 1}^* + U_{\beta 2} e^{-i\Delta_{12}} U_{\alpha 2}^* + U_{\beta 3} e^{-i\Delta_{13}} U_{\alpha 3}^*|^2,$$

where

$$\Delta_{ij} \equiv \frac{m_i^2 - m_j^2}{2E_\nu} L \simeq 2.534 \frac{\delta m^2_{ij} (\text{eV}^2)}{E_\nu (\text{GeV})} L (\text{km})$$

(3)
satisfy $\Delta_{12} + \Delta_{23} + \Delta_{31} = (\delta m^2_{12} + \delta m^2_{23} + \delta m^2_{31})(L/2E_\nu) = 0$. The two independent mass-squared differences are identified with the two “measured” ones, as follows:

$$\delta m^2_{\text{sol}} = |\delta m^2_{12}| \ll |\delta m^2_{13}| = \delta m^2_{\text{ATM}}.$$

(4)

With the above identification, the MNS matrix elements are constrained by the observed survival probabilities, $P_{\nu_\mu \rightarrow \nu_\mu}$ from the atmospheric neutrinos [21], $P_{\bar{\nu}_e \rightarrow \bar{\nu}_e}$ from the reactor anti-neutrinos [10, 11], and $P_{\nu_e \rightarrow \nu_e}$ from the solar neutrinos [6]. The four independent parameters of the MNS matrix are then related to the observed oscillation amplitudes as

$$|U_{e3}|^2 = \left(1 - \sqrt{1 - \sin^2 2\theta_{\text{CHOOZ}}} \right)/2,$$

(5a)

$$\left(U_{\mu3}\right)^2 \equiv \sin^2 \theta_{\text{ATM}} = \left(1 \pm \sqrt{1 - \sin^2 2\theta_{\text{ATM}}} \right)/2,$$

(5b)

$$\left(U_{e2}\right)^2 = \left(1 - |U_{e3}|^2 - \sqrt{(1 - |U_{e3}|^2)^2 - \sin^2 2\theta_{\text{sol}}} \right)/2,$$

(5c)

$$\arg (U_{e3}) = -\delta_{\text{MNS}}.$$

(5d)

The CP phase of the MNS matrix, $\delta_{\text{MNS}}$, is not constrained. The solution eq.(5c) follows from our convention [4], $U_{e1} > U_{e2}$, which defines the mass-eigenstate $\nu_1$. In this convention, there are four mass hierarchy cases corresponding to the sign of $\delta m^2_{ij}$: I ($\delta m^2_{13} > \delta m^2_{12} > 0$), II ($\delta m^2_{13} > 0 > \delta m^2_{12}$), III ($\delta m^2_{12} > 0 > \delta m^2_{13}$), and IV ($0 > \delta m^2_{12} > \delta m^2_{13}$) [4]. If the MSW effect is relevant for the solar neutrino oscillation, then the neutrino mass hierarchy cases II and IV are not favored. When $\sin^2 2\theta_{\text{ATM}} \neq 1$, there is an additional twofold ambiguity in the determination of $U_{\mu3}$ in eq.(5b). In order to avoid the ambiguity, we adopt the $U_{\mu3}$ element itself, or equivalently $\sin^2 \theta_{\text{ATM}}$ defined in eq.(5b), as an independent parameter of the MNS matrix. Summing up, we parametrize the three-flavor neutrino oscillation parameters in terms of the 5 observed (constrained) parameters $\delta m^2_{\text{ATM}}, \delta m^2_{\text{sol}}, \sin^2 \theta_{\text{ATM}}, \sin^2 2\theta_{\text{sol}}, \sin^2 2\theta_{\text{CHOOZ}}$, and one CP-violating phase $\delta_{\text{MNS}}$, for four hierarchy cases.

Neutrino-flavor oscillation inside of the matter is governed by the Schrödinger equation

$$i \frac{\partial}{\partial t} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = H \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \frac{1}{2E_\nu} H_0 + \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix},$$

(6)

where $H_0$ is the Hamiltonian in the vacuum and $a$ is the matter effect term [7]

$$a = 2\sqrt{2}G_F n_e E_\nu = 7.56 \times 10^{-5} (\text{eV}^2) \left( \frac{\rho}{\text{g/cm}^3} \right) \left( \frac{E_\nu}{\text{GeV}} \right).$$

(7)

Here $n_e$ is the electron density of the matter, $E_\nu$ is the neutrino energy, $G_F$ is the Fermi constant, and $\rho$ is the matter density. In our analysis, we assume for brevity that the
density of the earth's crust relevant for the LBL experiment, between HIPA and HK is a constant, $\rho = 3$, with an overall uncertainty of $\Delta \rho = 0.1$;

$$\rho \text{ (g/cm}^3\text{)} = 3.0 \pm 0.1.$$ \hfill (8)

The Hamiltonian is diagonalized as

$$H = \frac{1}{2E_{\nu}} \tilde{U} \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \tilde{U}^\dagger,$$

by the MNS matrix in the matter $\tilde{U}$. The neutrino-flavor oscillation probabilities in the matter

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \left| \tilde{U}_{\beta 1} \tilde{U}_{\alpha 1}^* + \tilde{U}_{\beta 2} e^{-i \tilde{\Delta}_{12}} \tilde{U}_{\alpha 2}^* + \tilde{U}_{\beta 3} e^{-i \tilde{\Delta}_{13}} \tilde{U}_{\alpha 3}^* \right|^2,$$

(10)
takes the same form as those in the vacuum, with $\tilde{\Delta}_{ij} = (\lambda_j - \lambda_i)L/2E_{\nu}$, if the matter density can be approximated by a constant throughout the base-line. Because the effective matter potential for anti-neutrinos has the opposite sign with the same magnitude, the total Hamiltonian $\mathcal{H}$ governing the anti-neutrino oscillation in the matter is obtained from $H$ as follows [4],

$$\mathcal{H} \left( \delta m^2_{12}, \delta m^2_{13} \right) = -H^* \left( -\delta m^2_{12}, -\delta m^2_{13} \right).$$

(11)

We make the following simple treatments in estimating the signals and the backgrounds in our analysis.

- We assume a 1 Mega-ton water Čerenkov detector, which is capable of distinguishing between $e^\pm$ CC events and $\mu^\pm$ CC events, but cannot distinguish their charges.

- We do not require capability of the detector to reconstruct the neutrino energy.

Although the water Čerenkov detector has the capability of measuring the energy of the produced $\mu$ and $e$ as well as a part of hadronic activities, we do not make use of these information in this analysis. We only use the total numbers of the produced $\mu^\pm$ and $e^\pm$ events from $\nu_\mu$ or $\bar{\nu}_\mu$ narrow-band-beams (NBB). The NBBs from HIPA deliver $10^{21}$ protons on target (POT) in a typical 1 year operation, corresponding to about 100 days of operation with the design intensity [16]. Details of the NBB’s used for this study are available from the web-page [22].

In the following discussion, we examine $\nu_\mu$ NBB’s with the mean $\pi$ momentum $\langle p_\pi \rangle = 2\text{GeV}$ (NBB(2GeV)) and $\langle p_\pi \rangle = 3\text{GeV}$ (NBB(3GeV)), and $\bar{\nu}_\mu$ NBB with $\langle p_\pi \rangle = 2\text{GeV}$, (NBB(2GeV)). For our input (‘true’) value of $\delta m^2_{\text{ATM}} = 3.5 \times 10^{-3} \text{ eV}^2$, the probability $P_{\nu_\mu \rightarrow \nu_e}$ has a broad peak at $E_\nu \sim 1\text{GeV}$. NBB(2GeV) and NBB(2GeV) are chosen to
Table 1: Expected number of CC signal events from $\nu_\mu \rightarrow \nu_\mu, \nu_e$ oscillations for NBB(2GeV), NBB(3GeV) and those from $\overline{\nu}_\mu \rightarrow \overline{\nu}_\mu, \overline{\nu}_e$ oscillations for NBB(2GeV), with 1Mt-year exposure. The results are shown for the parameters of eq.(13).

maximize the transition probability, since $\langle E_{\nu} \rangle \simeq \langle p_\pi \rangle/2$. Because $P_{\nu_\mu \rightarrow \nu_e}$ does not change much in the range $E_{\nu} \simeq 0.6 \sim 1.2$ GeV, our results do not depend strongly on the true value of the $\delta m^2_{\text{ATM}}$; as long as it stays in the range $(2 \sim 5) \times 10^{-3}$ eV$^2$ [4].

The signals in this analysis are the numbers of $\nu_\mu$ and $\nu_e$ CC events from NBB(2, 3GeV) and those of the $\overline{\nu}_\mu$ and $\overline{\nu}_e$ CC events from $\overline{\text{NBB}}$(2GeV). These are calculated as

$$N_l(\nu_\mu; \langle p_\pi \rangle) = M N_A \int_{0}^{10 \text{ GeV}} dE_{\nu} \phi_{\nu_\mu}(E_{\nu}; \langle p_\pi \rangle) P_{\nu_\mu \rightarrow \nu_\ell}(E_{\nu}) \sigma_{\nu_\ell}^{CC}(E_{\nu}) ,$$

$$N_l(\overline{\nu}_\mu; \langle p_\pi \rangle) = M N_A \int_{0}^{10 \text{ GeV}} dE_{\nu} \overline{\phi}_{\nu_\mu}(E_{\nu}; \langle p_\pi \rangle) P_{\overline{\nu}_\mu \rightarrow \overline{\nu}_\ell}(E_{\nu}) \sigma_{\overline{\nu}_\ell}^{CC}(E_{\nu}) ,$$

for $l = e$ or $\mu$, where $M$ is the mass of detector (1Mega-ton), $N_A = 6.017 \times 10^{23}$ is the Avogadro number, $\phi_{\nu_\mu}(E_{\nu}; \langle p_\pi \rangle)$ and $\overline{\phi}_{\nu_\mu}(E_{\nu}; \langle p_\pi \rangle)$ are the flux of $\nu_\mu$ in NBB($\langle p_\pi \rangle$)GeV and $\overline{\nu}_\mu$ in $\overline{\text{NBB}}$(\langle p_\pi \rangle)GeV, respectively. The flux is negligibly small at $E_{\nu} > 10$GeV for the NBB’s used in our analysis. The cross sections are obtained by assuming a pure water target [23].

Typical numbers of expected CC signals are tabulated in Table 1 for the parameter sets:

$$\sin^2 \theta_{\text{ATM}} = 0.5 , \quad \delta m^2_{13} = \delta m^2_{\text{ATM}} = 3.5 \times 10^{-3} \text{ eV}^2 ,$$

$$\sin^2 2\theta_{\text{sol}} = 0.8 , \quad \delta m^2_{12} = \delta m^2_{\text{sol}} = 1.0 \times 10^{-4} \text{ eV}^2 ,$$

$$\sin^2 2\theta_{\text{CHOOZ}} = 0.06, \ 0.01 , \quad \delta_{\text{MNS}} = 0^\circ, \ 90^\circ, \ 180^\circ, \ 270^\circ ,$$

$$\rho = 3 \text{ g/cm}^3 .$$

The numbers in the Table 1 are for 1 Mt-year exposure with $10^{21}$ POT per year for 0.77 MW operation of HIPA at $L = 295$ km. From Table 1, we learn that the transition events,
$N_e$ and $\overline{N}_\pi$, are sufficiently large to have the potential of distinguishing the CP conserved cases, $\delta_{\text{MNS}} = 0^\circ$ and $180^\circ$, from the CP violating cases of $\delta_{\text{MNS}} = 90^\circ$ and $270^\circ$, even if $\sin^2 2\theta_{\text{CHOOZ}} = 0.01$. We also find that the survival events, $N_\mu$ and $\overline{N}_\pi$, barely depend on the CP phase. The ratio $\overline{N}_\pi(2\text{GeV})/N_\mu(2\text{GeV})$ is approximately $\sigma^{\text{CC}}_{\nu_\mu}/\sigma^{\text{CC}}_{\overline{\nu}_\mu} \simeq 2.9$, because both the flux and the survival rates are approximately the same for $\nu_\mu$ and $\overline{\nu}_\mu$ [4]. From the comparison of $N_\ell(2\text{GeV})$ and $N_\ell(3\text{GeV})$, we find that $N_\mu(3\text{GeV})/N_\mu(2\text{GeV}) \sim 3$ because of the rise in the cross section ($\sim 1.5$) and the increase in the survival rate ($\sim 2$). The $\nu_e$ appearance signal $N_e$ increases only slightly at higher energies because a slight decrease in the transition probability cancels partially the effect of the rising cross section. Most notably, we find that the difference between the predictions of $\delta_{\text{MNS}} = 0^\circ$ and $180^\circ$ cases is significantly larger for $N_e(\nu_\mu; \langle p_\pi \rangle = 3\text{GeV})$ than that for $N_e(\nu_\mu; \langle p_\pi \rangle = 2\text{GeV})$.

The above results can be seen clearly in Fig.1, where we show the expected number of $\overline{\nu}_e$ CC events $\overline{N}_\pi$ for $\overline{\text{NBB}}(2\text{GeV})$ with 4Mton-year plotted against those of the $\nu_e$ CC event $N_e$ for NBB(2GeV) (left) and for NBB(3GeV) (right), both with 1Mton-year. The CP-phase dependence of the predictions are shown as closed circles for the parameters of eq.(13) at $\sin^2 2\theta_{\text{CHOOZ}} = 0.06, 0.04, 0.02,$ and 0.01. Comparable numbers of $\overline{\nu}_e$ CC events ($N_e$) and $\nu_e$ CC events ($N_e$) are expected by giving 4 times more $\overline{\nu}_e$ than $\nu_e$ beams. At each $\sin^2 2\theta_{\text{CHOOZ}}$ the $\nu_\mu \rightarrow \nu_e$ events are expected to be smaller at $\delta_{\text{MNS}} = 90^\circ$ (solid-squares) than at $\delta_{\text{MNS}} = 270^\circ$ (open-squares). The trend is opposite for the $\overline{\nu}_\mu \rightarrow \overline{\nu}_e$ events, and thus anti-correlation allows us to distinguish the two cases clearly. On the other hand, the expected number at $\delta_{\text{MNS}} = 0^\circ$ (solid-circles) and that at $\delta_{\text{MNS}} = 180^\circ$ (open-circles) do not differ much for NBB(2GeV) and $\overline{\text{NBB}}(2\text{GeV})$. We find that NBB(3GeV) predicts significant differences between the two CP-invariant cases without losing event numbers.

In this report, we assume 1Mton-year exposure each with NBB(2GeV) and NBB(3GeV) and 4Mton-year exposure of $\overline{\text{NBB}}(2\text{GeV})$, and examine the capability of HIPA-to-Hyper-Kamiokande experiments to measure the CP phase, $\delta_{\text{MNS}}$, under the following simplified treatments of the backgrounds and systematic errors.

For the $\nu_e$ and $\nu_\mu$ CC signal from NBB($\nu_\mu; \langle p_\pi \rangle$), we consider the following backgrounds:

\begin{align}
N_e(\langle p_\pi \rangle)_{\text{BG}} &= N_e(\nu_e; \langle p_\pi \rangle) + N_e(\overline{\nu}_\mu; \langle p_\pi \rangle) + N_\pi(\overline{\nu}_e; \langle p_\pi \rangle) + N_e, \text{NC} \langle p_\pi \rangle) , \\
N_\mu(\langle p_\pi \rangle)_{\text{BG}} &= N_\mu(\nu_e; \langle p_\pi \rangle) + N_\pi(\overline{\nu}_\mu; \langle p_\pi \rangle) + N_\pi(\overline{\nu}_e; \langle p_\pi \rangle).
\end{align}

The first 3 terms in the r.h.s. are calculated as

\begin{equation}
N_{\nu_{\alpha}}(\nu_{\ell}^{(-)}; \langle p_\pi \rangle) = M N_A \int_{0}^{10 \text{ GeV}} dE_{\nu} \Phi_{\nu_{\alpha}}(E_{\nu}; \langle p_\pi \rangle) P_{\nu_{\alpha}}(E_{\nu}) \sigma^{\text{CC}}_{\nu_{\alpha}}(E_{\nu}) ,
\end{equation}

where $\Phi_{\nu_{\alpha}}$ and $\Phi_{\overline{\nu}_{\alpha}}$ stands, respectively, for the secondary $\nu_{\alpha}$ and $\overline{\nu}_{\alpha}$ flux of the primarily $\nu_\mu$ NBB. The last term in eq.(14a) for the $e$-like events gives the contribution of the NC
Figure 1: The CP phase dependence of $N_e(\nu_\mu; 2\text{GeV})$ for 4 Mt-year plotted against $N_e(\nu_\mu; 2\text{GeV})$ for 1 Mt-year in the left figure, and against $N_e(\nu_\mu; 3\text{GeV})$ for 1 Mt-year in the right figure. $\delta_{\text{MNS}} = 0^\circ$ (solid-circle), $\delta_{\text{MNS}} = 90^\circ$ (solid-square), $\delta_{\text{MNS}} = 180^\circ$ (open-circle), and $\delta_{\text{MNS}} = 270^\circ$ (open-square). The results are for the parameters at eq.(13).

Most importantly, we do not require the capability of the HK detector to distinguish charges of electrons and muons. In Table 2 the expected numbers of CC and NC events at HK in the absence of oscillations are shown for 1Mton-year each for the $\nu_\mu$ enriched NBB’s and 4Mton-year for $\bar{\nu}_\mu$ enriched NBB. The event numbers from the main (enriched)
neutrinos are shown by the bold letters. The numbers in the parenthesis are the fractions as compared to the corresponding main mode. From the comparison between NBB(2GeV) and NBB(2GeV), we find that the fraction of the secondary-beam contributions is much larger for the $\nu_\mu$-beam than that for the $\nu_\mu$-beam. This is essentially because $\nu_\tau$ CC cross section is about a factor of three smaller than the $\nu_\tau$ CC cross section at $E_\nu \sim 1$GeV.

In Fig.2, we show the expected $\nu_\mu \rightarrow \nu_e$ ($\nu_\mu \rightarrow \bar{\nu}_e$) signal and background event numbers for the parameters of eq.(13) for $\sin^2 2\theta_{\text{CHOOZ}} = 0.01 \sim 0.06$. The solid-circles show the number of expected signal events for $\delta_{\text{MNS}} = n \times 10^\circ \ (n = 1 \sim 36)$. The numbers of signal events are largest at around $\delta_{\text{MNS}} = 270^\circ$ for NBB(2,3GeV), while those for NBB(2GeV) are largest at around $90^\circ$, as is expected from the CP phase dependence of $N_e$ and $N_{\pi}$ shown in Fig.1. The open-triangle denotes $\nu_e \rightarrow \nu_e$ CC events, which give the largest background for the experiments with NBB(2,3GeV), and the second largest background for NBB(2GeV). The open-square denotes $\nu_e \rightarrow \bar{\nu}_e$ CC events that gives the largest background for NBB(2,3GeV), but is negligible for NBB(2,3GeV). The open-diamond denotes the background from the NC events, where $\pi^0$-s are miss-identified as electrons. They give the second largest background for NBB(2,3GeV). Backgrounds from $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ transition events for NBB(2,3GeV) and those from $\nu_\mu \rightarrow \nu_e$ transition events for NBB(2GeV) are shown by open-circle. These transition backgrounds depend on the CP phase and they tend to cancel the $\delta_{\text{MNS}}$ dependence of the signals, but their magnitudes are small. The background level starts dominating the signal at $\sin^2 2\theta_{\text{CHOOZ}} \lesssim 0.02$.

The background numbers for the $\mu$-like signals are found to be negligibly small ($\sim 10^{-2}$) for NBB(2,3GeV). Those for NBB(2GeV) are found to be about 21% of the signal almost independent of $\sin^2 2\theta_{\text{CHOOZ}}$. In Both cases, the major background comes from the secondary $\nu_\mu$ ($\bar{\nu}_\mu$) survival events.

Our analysis proceeds as follows. For a given set of the model parameters, we calculate

<table>
<thead>
<tr>
<th>NBB($p_\pi$)</th>
<th>$\nu_\mu$</th>
<th>$\nu_e$</th>
<th>$\bar{\nu}_\mu$</th>
<th>$\bar{\nu}_e$</th>
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</thead>
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<tr>
<td>NBB(2GeV)</td>
<td>2.8 x 10^4(1)</td>
<td>2.2 x 10^2(0.008)</td>
<td>1.9 x 10^2(0.007)</td>
<td>1.3 x 10^4(0.0005)</td>
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<tr>
<td>1Mton-year</td>
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<td>8.1 x 10^1(0.007)</td>
<td>5.3(0.0004)</td>
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<td>NBB(3GeV)</td>
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<td>3.1 x 10^2(0.007)</td>
<td>2.0 x 10^2(0.004)</td>
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<tr>
<td>1Mton-year</td>
<td>1.6 x 10^4(1)</td>
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<td>8.6 x 10^1(0.005)</td>
<td>6.3(0.0004)</td>
</tr>
<tr>
<td>NBB(2GeV)</td>
<td>3.0 x 10^3(0.09)</td>
<td>1.9 x 10^2(0.005)</td>
<td>3.5 x 10^4(1)</td>
<td>2.5 x 10^2(0.007)</td>
</tr>
<tr>
<td>4Mton-year</td>
<td>1.2 x 10^3(0.08)</td>
<td>6.9 x 10^1(0.005)</td>
<td>1.5 x 10^4(1)</td>
<td>1.0 x 10^2(0.007)</td>
</tr>
</tbody>
</table>

Table 2: Expected number of the CC and NC events at HK in the absence of oscillations. The results are for 1 Mton-year for the $\nu_\mu$ enriched NBBs and 4Mton-year for $\bar{\nu}_\mu$ enriched NBB from HIPA. The numbers in the parenthesis give the fraction of each mode against the main mode whose numbers are shown by bold letters.
the expected numbers of all the signal and background events for each NBB(⟨pπ⟩) and \( \overline{\text{NBB}}(⟨pπ⟩) \), by assuming 100% detection efficiencies for simplicity. The resulting numbers of μ-like and e-like events are then denoted by \( N_{μ}^{\text{true}}(⟨pπ⟩) \) and \( N_{e}^{\text{true}}(⟨pπ⟩) \) for NBB(⟨pπ⟩), and \( \overline{N}_{μ}^{\text{true}}(⟨pπ⟩) \) and \( \overline{N}_{e}^{\text{true}}(⟨pπ⟩) \) for \( \overline{\text{NBB}}(⟨pπ⟩) \).

We account for the following two effects as major parts of the systematic uncertainty in this analysis. One is the uncertainty in the total flux of each neutrino beam, for which we assign the uncertainty,

\[
(-)^{f_{ν_α}(⟨pπ⟩)} = 1 ± 0.03,
\]

independently for \( ν_α = ν_e, ν_μ, \bar{ν}_e, \bar{ν}_μ \) and for NBB(2GeV), NBB(3GeV), and \( \overline{\text{NBB}}(2\text{GeV}) \). Although it is likely that correlation exists among the flux uncertainties, we ignore possible effects of correlations in this analysis. By using the above flux factors, theoretical predictions for the event numbers, \( N_{ℓ}^{\text{fit}}(⟨pπ⟩) \) and \( \overline{N}_{ℓ}^{\text{fit}}(⟨pπ⟩) \), are calculated as

\[
N_{ℓ}^{\text{fit}}(⟨pπ⟩) = f_{ν_e}(⟨pπ⟩)N_{ℓ}(ν_e, ⟨pπ⟩) + f_{ν_μ}(⟨pπ⟩)N_{ℓ}(ν_μ, ⟨pπ⟩) + f_{\bar{ν}_e}(⟨pπ⟩)N_{ℓ}(\bar{ν}_e, ⟨pπ⟩) + f_{\bar{ν}_μ}(⟨pπ⟩)N_{ℓ}(\bar{ν}_μ, ⟨pπ⟩) + \delta_{α,e} P_{e/NC} \sum_{ν_α} f_{ν_α}(⟨pπ⟩)N_{NC}^{ν_α}(⟨pπ⟩),
\]

\[
\overline{N}_{ℓ}^{\text{fit}}(⟨pπ⟩) = \overline{f}_{ν_e}(⟨pπ⟩)\overline{N}_{ℓ}(ν_e, ⟨pπ⟩) + \overline{f}_{ν_μ}(⟨pπ⟩)\overline{N}_{ℓ}(ν_μ, ⟨pπ⟩) + \overline{f}_{\bar{ν}_e}(⟨pπ⟩)\overline{N}_{ℓ}(\bar{ν}_e, ⟨pπ⟩) + \overline{f}_{\bar{ν}_μ}(⟨pπ⟩)\overline{N}_{ℓ}(\bar{ν}_μ, ⟨pπ⟩) + \delta_{α,e} P_{e/NC} \sum_{ν_α} \overline{f}_{ν_α}(⟨pπ⟩)\overline{N}_{NC}^{ν_α}(⟨pπ⟩),
\]
where the last terms proportional to $\delta_{\ell,e}$ are counted only for $\ell = e$. As the second major systematic error, we allocate 3.3% overall uncertainty in the matter density along the base-line, eq.(8). The fit functions are hence calculated for an arbitrary set of the 6 model parameters, the 12 flux normalization factors, and the matter density $\rho$.

The $\chi^2$ function of the fit in this analysis can now be expressed as

$$
\chi^2 = \sum_{NBB} \left\{ \left( \frac{N^\mu_{\text{fit}}(\langle p_\pi \rangle) - N^\mu_{\text{true}}(\langle p_\pi \rangle)}{\sigma_\mu(\langle p_\pi \rangle)} \right)^2 + \left( \frac{N^e_{\text{fit}}(\langle p_\pi \rangle) - N^e_{\text{true}}(\langle p_\pi \rangle)}{\sigma_e(\langle p_\pi \rangle)} \right)^2 \\
+ \sum_{\nu_\alpha} \left( \frac{f_{\nu_\alpha}(\langle p_\pi \rangle) - 1.0}{0.03} \right)^2 \right\} \\
+ \sum_{NBB} \left\{ \left( \frac{N^\mu_{\text{fit}}(\langle p_\pi \rangle) - N^\mu_{\text{true}}(\langle p_\pi \rangle)}{\sigma_\mu(\langle p_\pi \rangle)} \right)^2 + \left( \frac{N^e_{\text{fit}}(\langle p_\pi \rangle) - N^e_{\text{true}}(\langle p_\pi \rangle)}{\sigma_e(\langle p_\pi \rangle)} \right)^2 \\
+ \sum_{\nu_\alpha} \left( \frac{f_{\nu_\alpha}(\langle p_\pi \rangle) - 1.0}{0.03} \right)^2 \right\} \\
+ \left( \frac{\rho - 3.0}{0.1} \right)^2 + \left( \frac{\delta m^2_{\text{fit SOL}} - \delta m^2_{\text{true SOL}}}{0.1 \times \delta m^2_{\text{true SOL}}} \right)^2, \tag{21} \right.
$$

where the summation is over NBB(2GeV), NBB(3GeV) and $\overline{\text{NBB}}$(2GeV). Even though we have only one $\overline{\text{NBB}}$ in our analysis, we retain the summation symbol in eq.(21) for the sake of clarity. The last term is added because KamLAND experiment [24] will measure $\delta m^2_{\text{SOL}}$ at 10% level for the LMA parameters of eq.(13). The individual error for each $N_\mu(\langle p_\pi \rangle)$ ($N_\mu(\langle p_\pi \rangle)$) is statistical only, whereas the error for each $N_\mu(\langle p_\pi \rangle)$ ($N_\mu(\langle p_\pi \rangle)$) is a sum of the statistical errors and the systematic error coming from the 10% uncertainty in the $e/\pi^0$ misidentification probability of eq.(17),

$$
\sigma_\mu(\langle p_\pi \rangle) = \sqrt{N^\mu_{\text{true}}(\langle p_\pi \rangle)}, \tag{22a} \\
\sigma_e(\langle p_\pi \rangle) = \sqrt{N^e_{\text{true}}(\langle p_\pi \rangle) + \left(0.1 N^e_{\text{true}}(NC; \langle p_\pi \rangle)\right)^2}. \tag{22b}
$$

The errors for the $\overline{\text{NBB}}$(2GeV) case are calculated similarly as above.

We show in Fig.3 regions allowed by the HIPA-to-HK experiment in the plain of $\sin^2 2\theta_{\text{CHOOZ}}$ and $\delta_{\text{MNS}}$. The mean values of the input data are calculated for the LMA parameters of eq.(13). In each figure, the input parameter point ($\sin^2 2\theta_{\text{CHOOZ}}^{\text{true}}, \delta_{\text{MNS}}^{\text{true}}$) is shown by a solid-circle for $\sin^2 2\theta_{\text{CHOOZ}}^{\text{true}} = 0.06$, and by a solid-square for $\sin^2 2\theta_{\text{CHOOZ}}^{\text{true}} = 0.01$. The regions where $\chi^2_{\text{min}} < 1$, 4, and 9 are depicted by solid, dashed, and dotted boundaries, respectively. The $\chi^2$ fit has been performed by restricting the solar-neutrino oscillation amplitude in the range

$$
0.7 \leq \sin^2 2\theta_{\text{SOL}}^{\text{fit}} \leq 0.9, \tag{23} \right.
$$
Figure 3: Regions allowed by the HIPA-to-HK experiment are shown in the plain of $\sin^2 2\theta_{\text{CHOOZ}}$ and $\delta_{\text{MNS}}$. The assumed experimental conditions are 1 Mt·year each for NBB(2GeV) and NBB(3GeV), and 4 Mt·year for NBB(2GeV) with $10^{21}$ POT/year. The input data are calculated for the LMA parameters of eq.(13). In each figure, the input parameter point ($\sin^2 2\theta_{\text{true}}^{\text{CHOOZ}}, \delta_{\text{true}}^{\text{MNS}}$) is shown by a solid-circle for $\sin^2 2\theta_{\text{true}}^{\text{CHOOZ}} = 0.06$, and by a solid-square for $\sin^2 2\theta_{\text{true}}^{\text{CHOOZ}} = 0.01$. The regions where $\chi^2_{\text{min}} < 1, 4, \text{ and } 9$ are depicted by solid, dashed, and dotted boundaries, respectively.
which represents the allowed range of the LMA solution at present [6]. All the other parameters, $\delta m^2_{\text{fit}, \text{SOL}}$, $\sin^2 \theta_{\text{fit}, \text{ATM}}$, $\delta m^2_{\text{fit}, \text{ATM}}$, $\sin^2 2\theta_{\text{fit}, \text{CHOOZ}}$, $\delta_{\text{fit}, \text{MNS}}$, $\rho_{\text{fit}}$ and the flux normalization factors are allowed vary freely in the fit.

From the top-right and bottom-right figures for $\delta_{\text{MNS}}^{\text{true}} = 90^\circ$ and $270^\circ$ respectively, we learn that $\delta_{\text{MNS}}$ can be constrained to $\pm 30^\circ (\pm 60^\circ)$ at the $1\sigma (3\sigma)$ level, even if $\sin^2 2\theta_{\text{fit}, \text{CHOOZ}} = 0.01$. This is because $N_e + \overline{N}_e$ constrain $\sin^2 2\theta_{\text{CHOOZ}}$ and $N_e/\overline{N}_e$ distinguishes between $\delta_{\text{MNS}} = 90^\circ$ and $270^\circ$ in Fig.3, whereas the remaining parameters ($\delta m^2_{\text{ATM}}$ and $\sin^2 \theta_{\text{ATM}}$) are constrained by the $\nu_\mu$ and $\overline{\nu}_\mu$ survival data, $N_\mu$ and $\overline{N}_\mu$. The accuracy of the $\delta_{\text{MNS}}$ measurement does not decrease significantly for $\sin^2 2\theta_{\text{fit}, \text{CHOOZ}} = 0.01$ despite the large background level, because the $\delta_{\text{MNS}}$ -dependence of the signal exceeds significantly the 3% uncertainty of the background level from the flux normalization factors in eq.(19). We find that the CP-violation signal can be distinguished from the CP-conserving cases ($\delta_{\text{MNS}} = 0^\circ$ or $180^\circ$) at $4\sigma (3\sigma)$ level for all $\delta_{\text{MNS}}$ values in the region $|\delta_{\text{MNS}}|$, $|\delta_{\text{MNS}} - 180^\circ| > 30^\circ$ if $\sin^2 2\theta_{\text{fit}, \text{CHOOZ}} \gtrsim 0.03$ ($0.01$), for the LMA parameters of eq.(13) and for the systematic errors assumed in this analysis.

The situation is quite different for the CP-conserving cases of $\delta_{\text{MNS}}^{\text{true}} = 0^\circ$ or $180^\circ$ shown in the left-hand side of Fig.3. $\delta_{\text{MNS}}$ can be constrained to better than $\pm 7^\circ (11^\circ)$ accuracy at $1\sigma$ level for $\sin^2 2\theta_{\text{CHOOZ}} \gtrsim 0.06$ ($0.01$), but the two cases cannot be distinguished at $2\sigma$ level. This is mainly because of the similarity of $N_e/\overline{N}_e$ between $\delta_{\text{MNS}} = 0^\circ$ and $180^\circ$ in Fig.1. The difference between the two cases is larger for NBB(3GeV). If we remove the NBB(3GeV) data from the fit, we find that the two cases cannot be distinguished even at $1\sigma$ level. This two-fold ambiguity between $\delta_{\text{MNS}}$ and $180^\circ - \delta_{\text{MNS}}$ is found in general for all $\delta_{\text{MNS}}$, because the difference in the predictions can be adjusted by a shift in the fitted $\sin^2 2\theta_{\text{CHOOZ}}$ value; see Fig.1.

It is remarkable that the $1\sigma$ error of $\delta_{\text{MNS}}$ is as large as $30^\circ$ for $\delta_{\text{MNS}}^{\text{true}} = 90^\circ$ and $270^\circ$ while it is less than $10^\circ$ for $\delta_{\text{MNS}}^{\text{true}} = 0^\circ$ and $180^\circ$. The large $1\sigma$ error of the CP-violating cases reflects the uncertainty of $\sin^2 2\theta_{\text{SOL}}$, where we give the equal probability for any values of $\sin^2 2\theta_{\text{fit}, \text{SOL}}$ in the allowed range of eq.(23). The $\delta_{\text{MNS}}$ dependence of the CP-violating cases come from the Jarlskog parameter [25], which depend on the product of $\sin^2 2\theta_{\text{SOL}}$ and $\sin \delta_{\text{MNS}}$. The 10% uncertainty in $\sin^2 2\theta_{\text{SOL}}$ gives rise to about $20^\circ$ uncertainty in $\delta_{\text{MNS}}$. This leads to the large $1\sigma$ error of $\delta_{\text{MNS}}$ around $\delta_{\text{MNS}}^{\text{true}} = 90^\circ$ and $270^\circ$. When $\delta_{\text{MNS}}^{\text{true}} = 0^\circ$ or $180^\circ$, the Jarlskog parameter vanishes and the uncertainty in $\sin^2 2\theta_{\text{SOL}}$ does not affect the error of $\delta_{\text{MNS}}$.

We close this article by pointing out that the low-energy LBL experiment like HIPA to-HK cannot distinguish between the neutrino-mass hierarchy cases (between I and III) because of the small matter effect at low energies. If we repeat the analysis by using the same input data but assuming the hierarchy III in the analysis, we obtain another excellent
fit to all the data where the fitted model parameters are slightly shifted from their true (input) values. VLBL experiments at higher energies at \( L > 1000\text{km} \) [4] are needed to determine the mass hierarchy.

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