Using the constrained-fitting method based on Bayesian priors, we extract the masses of the two lowest states of octet and decuplet baryons with both parities. The calculation is done on quenched $16^3 \times 28$ lattices of $a = 0.2$ fm using an improved gauge action and overlap fermions, with the pion mass as low as 180 MeV. The Roper state $N(1440)\frac{1}{2}^+$ is clearly observed for the first time as the 1st-excited state of the nucleon from the standard interpolating field. Together with other baryons, our preliminary results indicate that the level-ordering of the low-lying baryon states on the lattice is largely consistent with experiment. The realization is helped by cross-overs between the excited $\frac{1}{2}^+$ and $\frac{3}{2}^-$ states in the region of $m_\pi \sim 300$ to 400 MeV.

The rich structure of the excited baryon spectrum, as documented by the particle data group [1], provides a fertile ground for exploring the nature of quark-quark interactions. One outstanding example is the ordering of the lowest-lying states which has the order of positive and negative-parity excitations inverted between $N$, $\Delta$ and $\Lambda$ channels. Conventional quark models have difficulty explaining the ordering in a consistent manner. There are two contrasting views. One is from the constituent quark model [2,3] which has the interaction dominated by one-gluon-change type, i.e., color-spin $\lambda_1^1 \cdot \lambda_2^2 \vec{\sigma}_1 \cdot \vec{\sigma}_2$. The other is based on Goldstone-boson-exchange [4] which has flavor-color $\lambda_1^1 \cdot \lambda_2^2 \vec{\sigma}_1 \cdot \vec{\sigma}_2$ as the dominant part. Even though evidence from valence QCD [5] supports the flavor-color picture, the challenge of reproducing the ordering still faces lattice QCD.

There exist a number of lattice studies of the excited baryon spectrum using a variety of actions [6–10]. The nucleon channel is the most-studied, focusing on two independent local fields:

$$\chi_1 = \epsilon^{abc} (u^{Ta} C \gamma_5 d^b) u^c, \quad (1)$$
$$\chi_2 = \epsilon^{abc} (u^{Ta} C d^b) \gamma_5 u^c. \quad (2)$$

$\chi_1$ is the standard nucleon operator, while $\chi_2$, which has a vanishing non-relativistic limit, is sometimes referred to as the ‘bad’ nucleon operator. Note that baryon interpolating fields couple to both positive and negative-parity states, which can be separated by well-established parity-projection techniques. The consensus so far appears to be that, first, the negative-parity splitting of $N(\frac{1}{2}^-)$ is largely established and consistent with experiment. Secondly, the Roper state $N'(\frac{1}{2}^+)$ as the 1st-excited state of the nucleon is still elusive. Since $\chi_2$ couples little to the nucleon ground state, there was initial speculation that it couples to the Roper state. But that identification has been mostly abandoned since the mass extracted from $\chi_2$ is consistently too high. What $\chi_2$ couples to remains an open question.
Figure 1. Solid symbols denote $N(\frac{1}{2}^+)$ states: ground (●) and 1st-excited (★). Empty symbols denote $N(\frac{1}{2}^-)$ states: lowest (△) and 2nd lowest □. The experimental points (★) are taken from PDG [1].

Figure 2. Similar to Fig. 1, but for $\Lambda(\frac{1}{2}^\pm)$ states.

Figure 3. Similar to Fig. 1, but for $\Sigma(\frac{1}{2}^\pm)$ states.

Figure 4. Similar to Fig. 1, but for $\Xi(\frac{1}{2}^\pm)$ states.
Figure 5. Similar to Fig. 1, but for $\Delta^{(3\pm)}$ states.

The lattice size we use is $16^3 \times 28$ with the scale of $a = 0.202(1)$ fm set from $f_\pi$, which is our preferred choice for scale [11]. We consider a wide range of quark masses: 26 masses with the lowest $m_\pi = 180$ MeV (or $m_\pi/m_\rho = 0.248$), very close to the physical limit, and with 18 masses below the strange quark mass. We analyzed 80 configurations. Details of the simulation can be found in [12].

We adapted the constrained curve-fitting method advocated in [13,14], adhering to the following guidelines: a) fit as many time slices in the correlation function $G_{\text{data}}(t)$ and as many terms in $G_{\text{theory}}(t)$ as possible; b) use prior knowledge, such as $A > 0$ and $E_n - E_{n-1} > 0$; c) seek guide for priors from a subset of data (empirical Bayes method); d) un-constrain the term of interest in $G_{\text{theory}}(t)$ to have conservative error bars. The details of the implementation are discussed in [15].

Fig. 1 to Fig. 5 show the results in various channels. To emphasize the small mass region, only the masses starting from the strange quark mass are shown. The most significant feature is that the ordering among $N(938)\frac{1}{2}^+$, $N'(1440)\frac{1}{2}^+$, $N(1535)\frac{3}{2}^-$, and $\Lambda(1405)\frac{1}{2}^-$ is consistent with experiment, which is the first time this has been seen on the lattice. It comes about with crossovers between the excited $\frac{1}{2}^+$ and $\frac{1}{2}^-$ states in the region of $m_\pi \sim 300$ to 400 MeV.

Further study is under way to check the stability of the fitting algorithm, especially in the small mass region where the signal worsens, and to automate the fitting process. To make sure that the results are not due to finite-volume effects [16] (our lowest $Lm_\pi \approx 3$), we are repeating the entire calculation on a smaller lattice of $12^3 \times 28$ with all other parameters fixed. The lowest $m_\pi$ on this lattice is about 250 MeV, small enough to probe the crossover region.

REFERENCES

8. W. Melnitchouk et al., hep-lat/0202022.

16. We thank C. Morningstar for bringing the issue to our attention.