SU(2) Lattice Gauge Theory at Nonzero Chemical Potential and Temperature.

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SU(2) lattice gauge theory with four flavors of quarks is simulated at nonzero chemical potential $\mu$ and temperature $T$ and the results are compared to the predictions of Effective Lagrangians. Simulations on $16^4$ lattices indicate that at zero $T$ the theory experiences a second order phase transition to a diquark condensate state which is well described by mean field theory. Nonzero $T$ and $\mu$ are studied on $12^3 \times 6$ lattices. For low $T$, increasing $\mu$ takes the system through a line of second order phase transitions to a diquark condensed phase. Increasing $T$ at high $\mu$, the system passes through a line of first order transitions from the diquark phase to the quark-gluon plasma phase.

1. Introduction

Since $SU(3)$ QCD cannot be simulated or studied analytically at moderate chemical potentials, theorists have turned to simpler models. One of the more interesting is the color $SU(2)$ version of QCD \[1\], \[2\], \[3\]. In this model diquarks do not carry color, so their condensation does not break color symmetry dynamically. The critical chemical potential is one-half the mass of the lightest meson, the pion, because quarks and anti-quarks reside in equivalent representations of the $SU(2)$ color group. Chiral Lagrangians can be used to study the diquark condensation transition in this model because the critical chemical potential vanishes in the chiral limit, and the model has a Goldstone realization of the spontaneously broken quark-number symmetry \[4–7\]. The problem has also been studied within a Random Matrix Model at non-zero $\mu$ and $T$ \[8\]. Lattice simulations of the model are also possible because the fermion determinant is real and non-negative for all chemical potentials.

Preliminary lattice simulations of the $SU(2)$ model with four species of quarks, simulation data and an Effective Lagrangian analysis of aspects of the $T$-$\mu$ phase diagram were recently published \[9,10\]. Very early work on this model at finite $T$ and $\mu$ was performed by \[11\]. A simulation study of the spectroscopy of the light bosonic modes will be presented elsewhere \[12\]. This work is based on \[13\].

In our exploratory study \[10\], we found a line of transitions surrounding a phase with a diquark condensate. Along this line there is a tricritical point, labeled 2 in Fig. 1, where the transition switches from being second order to first order. We will present evidence for metastability along the line at high $\mu$. The tricritical point 2, has a natural explanation in the context of chiral Lagrangians \[10\]. Following the formalism of \[4\] we argued that trilinear couplings among the low lying boson fields of the Lagrangian become more significant as $\mu$ and $T$ increase and they can cause the transition to become first order at a $\mu$ value in the vicinity of the results found in the simulation. A tricritical point is also found in Chiral Perturbation Theory \[14\].

The conjectured phase diagram of Fig. 1 also has a critical point labeled 1 which connects the line of first order transitions to the dashed line of crossover phenomena extending to the $\mu = 0$ axis where we expect a finite $T$ chiral crossover between conventional hadronic matter and the

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quark-gluon plasma. For sufficiently light quarks in the four flavor SU(2) model, the \( \mu = 0 \) transition is known to be first order both from theoretical arguments [15,16] and simulations [17]. For quark mass \( m = 0.05 \) we find that the transition is smoothed out to a crossover. If the critical point 1 would be absent, a not unlikely possibility, the crossover line would intersect the curve separating the diquark condensation phase from the normal phase.

2. Simulation Results and Analysis

Consider the simulation results for the \( N_f = 4 \) theory on \( 16^4 \) lattices. We simulated the \( SU(2) \) model at a relatively weak coupling \( \beta = 1.85 \), within the theory’s apparent scaling window, in order to make contact with the theory’s continuum limit. The quark mass was \( m = 0.05 \), as in [10], and a series of simulations were performed at diquark source strength \( \lambda = 0.0025, 0.005, \) and \( 0.01 \) so that our results could be extrapolated to vanishing diquark source, \( \lambda = 0 \).

In Fig. 2 we show the diquark condensate \( \langle \langle \chi \tau_2 \chi \rangle \rangle \), linearly extrapolated to \( \lambda = 0 \), plotted against the chemical potential \( \mu \).

We see evidence to a quark-number violating second order phase transition in this figure. The dashed line is a power law fit from \( \mu = 0.30 \) to \( 0.35 \) which predicts the critical chemical potential of \( \mu_c = 0.2860(2) \). The power law fit is good, its confidence level is 48 percent, and its critical index is \( \beta_{mag} = 0.54(3) \) which is consistent with the mean field result \( \beta_{mag} = 1/2 \), predicted by chiral perturbation theory \([4],[5]\) including one loop corrections as well as simulations of 2-color QCD in the strong coupling limit \([18]\).
3. Runs at finite temperature

Now consider $12^3 \times 6$ simulations. Past $8^3 \times 4$ simulations and Effective Lagrangian analyses predicted that there is a line of first order transitions at high $\mu$ and high $T$ [10].

Our best evidence for a first order transition comes from the time evolution of the observables at $\beta = 1.87$ and $\mu \geq 0.40$ which show signs of metastability. For example, in the figure 3 we show the time evolution of the diquark condensate and display several tunnelings between a state having a condensate near 0.15 and another with a condensate near 0.40.

![Diquark Condensate vs. Computer Time](image_url)

Figure 3. Diquark Condensate vs. Computer Time.

4. Conclusions

We are currently calculating the model’s pseudo-Goldstone boson mass spectrum and instanton content. We hope that these measurements will add more insight into this model’s curious phase transitions.

REFERENCES