A phase transition due to thick vortices in SU(2) lattice gauge theory

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SU(2) lattice gauge theory is studied after eliminating thin monopoles and the smallest thick monopoles. Kinematically this constraint allows thick vortex loops which produce long range $Z(2)$ fluctuations. The thick vortex loops are identified in a three dimensional simulation. A condensate of thick vortices persists even after the thin vortices have all disappeared. They decouple at a slightly lower temperature (higher $β$) than the thin vortices and drive a $Z(2)$ like phase transition.

1. Introduction

The role of thick vortices \cite{1,2} as a mechanism for confinement in SU(2) LGT has been extensively discussed in the last few years \cite{3}. Thick vortices are analogous to the domain walls in ferromagnets with a continuous symmetry— they are like thick Peierls contours. Thick vortices should be distinguished from thin vortices. Thin vortices have infinite action in the continuum limit. For this reason they are unlikely to play a major role in the continuum limit. On the other hand thick vortices can reduce their free energy by arbitrarily increasing their cross section \cite{4}. In addition to thin and thick vortices SU(2) gauge theory also has thin and thick monopoles. A thin vortex can end in a thin $Z(2)$ monopole which (in three space-time dimensions) is defined on an elementary 3 dimensional cube. The thin $Z(2)$ monopole density on a cube $c$ is given by

$$\rho_1(c) = (1/2)(1 - \text{sign}(\prod_{p \in \partial c} \text{tr} U(p))) .$$

(1)

Analogously, a thick vortex can end in a thick $Z(2)$ monopole. Its density (in three space-time dimensions) is given by

$$\rho_d(c_d) = (1/2)(1 - \text{sign}(\prod_{d \in \partial c_d} \text{tr} U(d))) ,$$

(2)

where the product is taken over all $dXd$ loops bordering a cube $c_d$ of side $d$. The subscript $d$ indicates that the density can be defined on any 3 dimensional cube of side $d$. In three space-time dimensions the monopoles are point like objects or they have a finite size. Both monopoles are $SO(3)$ invariant as they only depend on the SU(2)/$Z(2)$ coset of the link variables. A whole hierarchy of such monopoles can be defined with different thicknesses. Unlike the thin monopoles, which are suppressed by the action, the thick monopoles cost lesser energy because their energy is spread over a finite region. It was proposed in \cite{5} that at lower and lower temperatures vortices and monopoles of thicker and thicker cross-section are important. These ideas were encapsulated in the effective $Z(2)$ theory of confinement \cite{5} where it was proposed that the long distance properties of SU(2) LGT can be described by a $Z(2)$ theory with an effective coupling $β(d)$ ($β(d) \to \infty$ as $d \to \infty$). Thick vortices are the vortices in the effective $Z(2)$ theory.

The presence of thick monopoles prevents the thick vortices from forming closed loops and makes it difficult to study their effects separately. If the thick monopoles are removed it might be possible to see if thick vortices are present at all and then study their effects. With this aim in mind we study the SU(2) theory after eliminating the thin and thick monopoles. This model can be regarded as a generalisation of the Mack-Petkova model which eliminates only the thin monopoles.
Just as in the Mack-Petkova model the elimination of the thin and thick monopoles changes only the short distance properties of the gauge theory.

$$S = \frac{\beta}{2} \sum_p tr U(p) - \lambda_1 \sum_{c_1} \rho_1(c_1) - \lambda_2 \sum_{c_2} \rho_2(c_2) \quad (3)$$

$\lambda_1, \lambda_2$ are chosen to be large large so that the monopole densities are very small (in practice $\lambda_1 = \lambda_2 \approx 20$ resulted in identically zero densities for almost all thermalised configurations on $12^3$ lattices).

The simulation of this model presented its own share of difficulties. The environment of a single link is complicated because each link touches four thin monopoles and eight thick monopoles. Different strategies were attempted to simulate this model. Metropolis updating and a combination of heat bath and metropolis were both tried but metropolis updating was found to be more efficient provided the table of $SU(2)$ elements is tuned regularly to get a reasonable acceptance. One observes long metastabilities while thermalising the lattice and the simulation has to be kept running for unusually long times unless a good starting configuration is chosen. It was found that in the phase where thick vortices are in abundance an ordered start (which has zero density of monopoles and vortices) took a very long time to reach the equilibrium distribution. Similarly, in the phase where thick vortices are absent a random start (which has a large number of thick vortices) took a long time to reach the equilibrium distribution.

$2. \text{ The model}$

Thin and thick monopoles can be removed by introducing two (large) chemical potential terms for the thin and thick monopole densities in the Wilson action. The action considered is

$$S = \frac{\beta}{2} \sum_p tr U(p) - \lambda_1 \sum_{c_1} \rho_1(c_1) - \lambda_2 \sum_{c_2} \rho_2(c_2) \quad (3)$$

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$3. \text{ Numerical results}$

It is well known that 3 dimensional $SU(2)$ LGT does not exhibit the crossover found in the four-dimensional theory but instead has a very smooth behaviour for the plaquette and other quantities. In Fig 1 we show this smooth behaviour in the

Fig1 The 2X2 Wilson loop in the $SU(2)$ theory.

Fig2. The plaquette in the monopole suppressed model. These metastabilities do not arise in the pure $SU(2)$ theory or the Mack-Petkova model and they appear to be linked to the thick vortices present in this model. Several approaches were tried to deal with this metastability like starting from configurations "closer" to the equilibrium configuration but the problem always remained. Nevertheless, with increased computational effort consistent results can be and were obtained. The simulation was performed in three dimensions mainly for reasons of computational speed. On the lattices used ($12^3$) it was found that simulating this model is more time consuming than simulating the four dimensional pure $SU(2)$ model.
2X2 Wilson loop. Eliminating thin monopoles leads to a slight jump in the plaquette due to the presence of thin vortices. Eliminating the thick monopoles leads to another jump at a lower temperature. Our study has focused on the properties of the system around this point. Fig 2 and Fig 3 show the sharp jump in the plaquette and the 2X2 Wilson loop respectively. In the vicinity of this jump very long relaxation times are observed before a starting configuration reaches equilibrium. The sign of the 2X2 Wilson loop changes abruptly across this transition. It should be emphasized that the thin vortices have all disappeared at this point (the sign of the plaquette is always a positive quantity here) and the fluctuations present are over longer distances.

Since $Z(2)$ fluctuations at longer length scales are observed it is natural to ask if these fluctuations are caused by thick vortices. Indeed they are! Thick vortex loops can be identified as closed loops which are composed of a co-closed set of 2X2 plaquettes with a negative sign. A measurement of the thick vortices on either side of the transition shows that they form very long loops in one phase and smaller loops in the other phase. Hence, the thick vortices are producing the $Z(2)$ fluctuations at longer distances, in line with the ideas of the effective $Z(2)$ theory of confinement. The location of the phase transition was determined to be around $\beta \approx 2.5$.

4. Conclusions

Thick vortices can be directly observed once the thick monopoles are eliminated. The condensate of thick vortices persists at higher values of $\beta$ after the thin vortices have all disappeared. They are responsible for the long range $Z(2)$ fluctuations which are a crucial ingredient in the effective $Z(2)$ theory of confinement. As $\beta$ increases the lattice spacing becomes smaller and vortices with a non-zero physical thickness can still be present. In the lattice model there will be a hierarchy of effective $Z(2)$ theories operating at larger and larger $\beta$ [5]; the model studied here is just the first member of the hierarchy. Eliminating monopoles of greater thickness will in principle unravel the entire hierarchy of $Z(2)$ like theories operating at larger lattice spacings. It is a challenge to show that in the continuum limit we are left with $Z(2)$ fluctuations at physical distances of the order of a fermi as predicted in [4].

REFERENCES

Fig 1
Fig2
Fig 3