Quantum computation using the Aharonov-Childs set up

where each element of the Aharonov-Childs set could be used as the basic building block for quantum computation. The Aharonov-Childs set is defined as follows:

\[ [\Lambda - \lambda] \cdot (\mathbf{U} - \mathbf{A}) \mathbf{V}^{\dagger} + i \lambda P \mathbf{V}^{\dagger} \mathbf{V} + \frac{i}{\lambda} \mathbf{V}^{\dagger} \mathbf{V} = \mathbf{0} \]
assumption that charge-charge and dipole-dipole interactions can be neglected between all pairs of AC-qubits. Except for the trivial case where $\gamma$ is an integer multiple of $\pi$, $\beta(\gamma)$ may entangle the qubits it acts on.

Universal quantum computation can be achieved by combining $B(\gamma)$ with the one-qubit logic gates

$$U(\gamma) = \exp \left[ -i \frac{\gamma}{2} \sigma^j_j \right], \text{ (phase shift gate)},$$

$$U_{\text{SWAP}}(\theta_{j}) = \exp \left[ -i \frac{\theta_{j}}{2} \sigma^j_j \right], \text{ (partial swap gate)},$$

where $\sigma^j_j = |0\rangle_j \langle 0|_j - |1\rangle_j \langle 1|_j$ and $\sigma^j_j = -\frac{i}{2} |0\rangle_j \langle 1|_j + \frac{i}{2} |1\rangle_j \langle 0|_j$. That is, any $N$-qubit logic operation can be simulated to any precision with an appropriate set of $U(\gamma), U_{\text{SWAP}}(\theta_{j})$, and $B(\gamma)$ gates. The one-qubit phase shift gate $U(\gamma)$ is achieved by encircling the two sites within a single AC-qubit around each other depending upon whether the state is $|0\rangle_j$ or $|1\rangle_j$. This could for example be achieved by addressing the site $a$, say, in such a way that the particle there is only taken around site $b$ if it is charged. This would result in $U(\gamma)$ up to an unimportant overall phase factor $e^{i\gamma/2}$. The $U_{\text{SWAP}}(\theta_{j})$ gates could be realised in principle by exposing beam-splitters to each of the AC set ups. The parameter $\theta_{j}$ determines the transmission probability $T$ of such a beam-splitter according to $T = \cos(\theta_{j}/2)$.

While the $U(\gamma)$ gate is topological and thereby fault tolerant to path deformations, the $U_{\text{SWAP}}(\theta_{j})$ gate is essentially dynamical and nontopological as it relies upon the detailed interaction between the AC-qubit and the beam-splitter. This situation is expected since the present treatment of the AC effect is basically Abelian and there are therefore non-Abelian operations necessary to achieve universality that could not be obtained by the AC effect alone (see Ref. [5] for a similar case).

Quantum computation based upon the AC set up can be realised as follows. First, translate the quantum algorithm into a set of elementary one- and two-qubit gates. Prepare an initial state by spreading out a set of AC composites along a line in the plane of motion. Perform approximate phase shifts and partial swaps on each AC set up and perform appropriate two-qubit controlled phase shifts by braiding sites from pairs of AC-qubits. The final answer of the computation is obtained by measuring the spatial location of the particles in the output.

Universal quantum computation using the AC set up only involves electromagnetic interactions between elementary systems and works even for non-interacting qubits. This should be compared with the suggestions for topological quantum computation in Refs. [3, 4, 5, 6, 7], that all involves collective effects such as anyons or spin systems with long-range correlations.

In principle, quantum computation based upon the AC set up could be realisable in combined atom-ion systems confined to a plane. A similar implementation could be achieved in three dimensions by replacing the point charge with a line of charge. However, to put this charged line in a coherent superposition would be difficult in practice and it is therefore unclear whether AC based quantum computation could have any relevance in the three-dimensional context. Moreover, it is important to keep in mind that the AC shift is essentially a relativistic effect and thus usually quite small also for such systems (see, e.g., [19, 20]), which means that the gates have to be repeated many times to achieve phase shifts of useful size. This may spoil the fault tolerance of the phase shift gates as the error probability is expected to increase with the winding number. Another challenge, associated with the implementation of the controlled phase shift gate, is the control of the nontopological charge-charge and dipole-dipole interactions that act between the various AC-qubits. These interactions could be made small under certain circumstances, but may add up when repeating the gate.
In conclusion, we have proposed to use the two-dimensional Aharonov-Casher (AC) set up as the basic building block for quantum computation. We have argued that the AC set up could be useful in the implementation of one- and two-qubit phase shift gates that are fault tolerant to path deformations when two sites are encircled around each other. Universality is achieved by adding nontopological one-qubit partial swap gates. Although it seems hard to implement quantum computation based upon the AC set up with present day technology, we believe it has a conceptual value as it demonstrates topological quantum computation using electromagnetic interactions between elementary systems.

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