Confining string and P-vortices in the indirect $Z(2)$ projection of $SU(2)$ lattice gauge theory

V. G. Bornyakov\textsuperscript{a}KU|Department of Physics, Kanazawa University, Kanazawa 920-11, Japan,\textsuperscript{b}ITEP|ITEP, B. Cheremushkinskaya 25, Moscow 117259, Russia, A. V. Kovalenko\textsuperscript{c}ITEP, M. I. Polikarpov\textsuperscript{d}ITEP, D. A. Sigaev\textsuperscript{e}ITEP

\textsuperscript{a}[1] \textsuperscript{b}[2]

We study the distribution of P-vortices near the confining string in the indirect $Z(2)$ projection of $SU(2)$ lattice gauge theory. It occurs that the density of vortices is constant at large distances and strongly suppressed near the line connecting the test quark-antiquark pair. This means that the condensate of P-vortices is broken inside the confining string. We also find that the width of the P-vortex density distribution is proportional to the logarithm of the distance between the quark and antiquark.

1. Introduction

Two popular confinement mechanisms are the monopole confinement \cite{1,2} and the magnetic vortex confinement \cite{3}. The numerical study of these mechanisms on the lattice shows that both of them have attractive features, e. g. the abelian dominance \cite{4}, an explanation of the Casimir scaling \cite{5}, etc. Studies of the confining string in the maximal abelian projection \cite{6} clearly demonstrate that QCD vacuum behaves like the dual superconductor. Below we present results of an analogous study of the confining string in terms of P-vortices in the indirect $Z(2)$ projection of $SU(2)$ lattice gauge theory.

We use the standard definition \cite{5} for P-vortices in the indirect $Z(2)$ projection. After fixing the maximal abelian gauge we maximize the functional $R$ and define the link variables $Z_{x\mu}$:

$$R = \sum_{x,\mu} \cos^2 \theta_{x\mu}, \quad Z_{x\mu} = \text{sign}(\cos \theta_{x\mu}),$$

where $\cos \theta_{x\mu} = \frac{1}{2} \text{Tr} u_{x\mu}$, $u_{x\mu}$ is the abelian link matrix obtained from $SU(2)$ link matrix $U_{x\mu}$ by the abelian projection.

A plaquette is pierced by a P-vortex lying on the dual lattice if $Z(2)$ plaquette $P_{x,\mu\nu} = Z_{x\mu} Z_{x+\mu,\nu} Z_{x+\nu,\mu} Z_{x\nu} = -1$.

2. Vortices near the confining string

We consider the quantity that measures the dependence of the vortex density on the distances $r_\perp$ from the $q\bar{q}$ axis and $r_\parallel$ along this axis

$$Q(r_\perp, r_\parallel) = \frac{<W V(r_\perp, r_\parallel)>}{<W>},$$

where $W$ is a Wilson loop and vortex detector $V(r_\perp, r_\parallel) = (1 - P_{x,\mu\nu})/2$. The similar quantity was investigated for the distribution of monopole currents near the confining string \cite{6}. Our calculations were performed on $SU(2)$ vacuum configurations in the indirect $Z(2)$ gauge, which was fixed using the simulated annealing algorithm \cite{7}. Below we present results obtained on 50 statistically independent configurations generated on $24^4$ lattice for $\beta = 2.5$.

Obviously there is no correlation between $W$ and $V(r_\perp, r_\parallel)$ at large distances $r_\perp$, $Q(\infty, r_\parallel) = <V>$, which is the value of the string condensate \cite{8}. However, the vortex density decreases near the confining string. This effect is illustrated in Fig. 1. The minima on Fig. 1 correspond to the positions of the test quark and antiquark.
Figure 1. The dependence of $Q/a^2$ on $r_\perp$ and $r_\parallel$.

Figure 2. $Q/a^2$ as the function of $r_\perp$ for the Wilson loop $11 \times 5$. Notation ‘01’ means that the detecting plaquette lies in the plane ‘01’.

In Fig. 2 we show the dependence of the vortex density $Q(r_\perp, r_\parallel)/a^2$ on the distance $r_\perp$ for $r_\parallel$ fixed in the middle of the $qq\bar{q}$ axis and the vortex detector $V$ lying in various planes. The Wilson loop has temporal direction ‘0’ and spatial direction ‘1’. We see stronger suppression of $Q$ for the plane $(0,1)$ parallel to the Wison loop. The normalization $1/a^2$ is due to dimensional reasons [5], the density of P-vortices should scale as $a^2$, $a$ being the lattice spacing.

3. The radius of the confining string

We employ $Q(r_\perp, r_\parallel)$ to evaluate the radius of the string in terms of P-vortices.

The Gaussian form fitting function is used to fit our data:

$$Q(r_\perp) = A \exp \left[ -\left( \frac{r_\perp}{r_0} \right)^2 \right] + Q(\infty),$$

(2)

$A$ and $r_0$ being the fitting parameters. It is reasonable to treat $r_0$ as the radius of the confining string. Function (2) has been used in studies of the nonabelian [9] and abelian [10] flux tubes action density and predicted theoretically in [12]. We obtained fits with reasonable $\chi^2$ (see Fig. 3).

Fig. 4 shows the parameter $r_0$ as a function of the distance between the quark and antiquark. The radius of the string appears to be proportional to the logarithm of the distance between the quark and antiquark as predicted in [12] for the width of the nonabelian flux tube. It is worth mentioning that the logarithmic dependence has been found out in studies of the nonabelian flux tube [9], while for the abelian flux tube the width was found to be stable [10] (on the other hand, the accurate recent studies show the dependence
of the width of the abelian flux tube on the distance between the test quark-antiquark pair [11]).

4. Discussion

For the monopole confinement mechanism there exists the effective classical model, which is the dual abelian Higgs model. The classical equations of motion for this model describe unexpectedly well the profile of the confining string in the maximal abelian projection [6]. For P-vortices we have no classical effective model, and our fitting function (2) has a phenomenological origin. On the other hand, the qualitative similarity of the dependence of $Q(r_\perp, r_\parallel)$ on $r_\perp$ to the dependence of the action density of the nonabelian or abelian flux tube is not accidental. $Q(r_\perp, r_\parallel)$ can be interpreted as $Z(2)$ projection of the nonabelian magnetic/electric energy density. This aspect will be discussed elsewhere [13].

We can also explain the flux tube profile shown in Fig. 1 from another point of view. It is well known that monopoles are correlated with P-vortices [5]. On the other hand, it is also known that the condensate of monopoles is broken inside the confining string. Thus one can expect that the condensate of P-vortices should be also broken or at least substantially reduced inside the confining string. We really see this effect in Figs. 1–3.

ACKNOWLEDGEMENTS

The authors are grateful to M. N. Chernodub, F. V. Gubarev and V. I. Zakharov for useful discussions. M. I. P. is partially supported by grants RFBR 02-02-17308, RFBR 01-02-117456, RFBR 00-15-96-786, INTAS-00-00111, and CRDF award RPI-2364-MO-02. A. V. K. is partially supported by grants RFBR 02-02-17308 and CRDF MO-011-0. B. V. G. is supported by JSPS Fellowship grant.

REFERENCES

11. Y. Koma et al., to be published.