Light quark masses from UKQCD’s dynamical simulations with O(a)-improved Wilson fermions

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I present preliminary results on the light quark masses from a partially quenched analysis of UKQCD’s dynamical datasets.

1. Introduction

The determination of the masses of the light quarks represents an important challenge in particle physics today. They are among the least well determined parameters of the Standard Model, having uncertainties of the same order as their values as given in the particle data book.

Lattice QCD is in a position to improve this situation because it allows one to study the quark mass dependence of physical observables such as hadron masses. The physical values of these observables can then be used to fix the quark masses. Unfortunately current simulations cannot probe the up/down quark mass range directly and so some form of extrapolation is necessary. Chiral perturbation theory is at present the best procedure for providing a link to the low mass regime. The simulations studied in this paper however are still too heavy for $\chi^PT$ and a simpler extrapolation ansatz is used instead.

In this paper I present results for the average up/down quark mass, $m_l$, and the strange quark mass, $m_s$, from a partially quenched analysis of pseudoscalar and vector meson masses for each of UKQCD’s dynamical fermion datasets. The suitably averaged values of the hadron masses used to fix the quark masses are as follows: $M_\pi = 137.3$ MeV ; $M_K = 495.7$ MeV ; $M_{K^*} = 893.9$ MeV and $M_\phi = 1020$ MeV.

2. Simulation Details

All of the lattice simulations studied here have been carried out on a $16^3 \times 32$ lattice with a non-perturbatively O(a)-improved Wilson fermion action and standard Wilson glue. Details of these simulations have been discussed previously in [1,2]. Additionally the set of $\kappa_{\text{val}}$ values for the datasets at $\beta = 5.2, \kappa_{\text{sea}} = 0.13565$ and $\beta = 5.2, \kappa_{\text{sea}} = 0.1358$ have been increased to \{0.1342, 0.1350, 0.1355, 0.13565, 0.13580\}. Pseudoscalar and vector meson correlation functions have been generated with all non-degenerate combinations of these $\kappa_{\text{val}}$ values. For the $\kappa_{\text{sea}} = 0.1358$ dataset these correlators have also been calculated at $\kappa_{\text{val}} = 0.13595$.

3. Hadron masses

The procedures used here to extract hadron masses from correlation functions have been described in detail in [1,2]. In brief the masses of the pseudoscalar and vector mesons were determined from double cosh fits to their corresponding correlation functions. Local and fuzzed correlators were fitted simultaneously to constrain fit parameters and a sliding window analysis used to find an appropriate time window for each fit.

4. Chiral Extrapolations

The pseudoscalar meson masses for each dataset were extrapolated in $1/\kappa_{\text{val}}$ using the simple ansatz
\[ M_{PS}^2(\kappa_1, \kappa_2) = A_{PS}(m_1 + m_2) \]
\[ = A_{PS}\left(\frac{1}{2}\kappa_1 + \frac{1}{\kappa_2}\right) \]

The value of \( \kappa_{crit} \) was determined as a free parameter in this fit.

For the vector meson the ansatz
\[ M_V(m_1, m_2) = A_V(m_1 + m_2) + B_V \]
was found to fit the data well.

An example plot of these chiral extrapolations is given in Figure 1 for the \( \kappa_{sea} = 0 \).

\[ \kappa_{val} = 0.13595 \]
are included in the plot but were not included in the extrapolation. The lightest vector point clearly deviates from the linear behaviour suggesting large finite size effects.

For more discussion of finite size effects in these datasets I refer the reader to [3].

**5. Fixing the bare light quark masses**

Determination of the light quark masses from the results of the chiral extrapolations requires setting the scale by some physical quantity. In this work the Sommer scale parameter, \( r_0 = 0.49 \) fm, and \( M_\rho = 769.3 \) MeV were used.

The average up/down quark mass, \( m_l \), was determined by requiring that the pseudoscalar mass equal the physical pion mass at that point i.e.
\[ M_{PS}^2(m_l, m_l) = aM_\pi \]

The strange quark mass, \( m_s \), was then determined from each of \( M_K, M_{K^*} \) and \( M_\phi \) by solving
\[ M_{PS}^2(m_l, m_s) = aM_K \]
\[ M_V(m_l, m_s) = aM_{K^*} \]
\[ M_V(m_s, m_s) = aM_\phi \]

When the lattice spacing was fixed by \( r_0 \) however only the \( M_K \) method gave a sensible value for \( m_s \) and only these results are given.

**6. Renormalisation and matching**

In this work the values for the renormalised quark masses are given in the \( \overline{MS} \) scheme at a reference scale of 2 GeV. The relationship between the renormalised and bare quark masses is given by the relation
\[ m_R^{\overline{MS}}(\mu) = Z_m^{\overline{MS}}(\mu)(1 + b_m a m_q)m_q \]

where the matching has been performed at a scale, \( \mu \). Running the masses from the scale, \( \mu \), to a scale of 2 GeV was achieved using the Mathematica package, RunDec [4], which implements 4-loop running from perturbation theory.

One-loop tadpole-improved perturbative results [5,6] were used for \( Z_m \) and \( b_m \) reorganised in terms of \( \alpha^{\overline{MS}}(\mu) \)
\[ Z_m^{\overline{MS}}(\mu) = 1 - \frac{\alpha^{\overline{MS}}(\mu)}{4\pi}(8\ln(a\mu) - 15.085)|u_0^{-1} \]
\[ b_m = [-1/2 - 0.7850 \alpha^{\overline{MS}}(\mu)|u_0^{-1} \]
The tadpole factor, \( u_0 \), was taken to be \( 1/8\kappa_c \) here.

An estimation of the best value for the matching scale, \( \mu = q^* \) [7], has not been determined for
this work. Instead the cases $\mu = 1/a$ and $\mu = \pi/a$
have been investigated and the variation treated
as a systematic effect.

The values of $\alpha_M^{\overline{MS}}(\mu)$ were also determined
with the RunDec package using values for $\Lambda^{\overline{MS}}$
given in [8].

7. Results

Results for $m_l$ and $m_s$ have been plotted in Fig-
ures 2 and 3 respectively. Datasets matched in $r_0$
have been distinguished from the others and a
matched quenched dataset included for compar-
ison. Only results for $\mu = 1/a$ are plotted as choice
of matching scale was found to have around a 1%
effect. The results for $m_s(M_\rho)$ are omitted as
they were almost identical to those for $m_s(M_{K^*})$.

A large dependence on the choice of scale setting parameter can be seen for both $m_l$ and $m_s$. It
should be noted however that a smaller physical
value for $r_0$ would bring the results into better
agreement. There is also a clear difference be-
 tween $m_s(M_K)$ and $m_s(M_{K^*})$ though this would
be expected to vanish in the continuum limit.

There is some indication of a trend in the
matched datasets as the sea quark mass is de-
creased, as expected. When compared to the
matched quenched point however there is no clear
signal of unquenching from either quark mass
though this may improve with statistics.

The ratio, $m_s/m_l$, was calculated for each
scheme and found to be between 25.0 and 25.5
for $m_s(M_K)$ and around 31.0 for $m_s(M_{K^*})$. This
can be compared with the result from $\chi_{PT}$ [9] of
24.4(1.5).

8. Acknowledgments

I would like to thank the Carnegie Trust
for the Universities of Scotland, the European
Community’s Human Potential programme un-
der HPRN-CT-2000-00145 Hadrons/LatticeQCD
and PPARC under grant PP/G/S/1998/00777
for their financial support of this work. I also
thank Craig McNeile for his invaluable help.

Figure 2. Average up/down quark masses with
$\mu = 1/a$. Only statistical errors are shown.

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Figure 3. Strange quark masses with $\mu = 1/a$. Only statistical errors are shown.