in the finite-dimensional Hilbert space of their covariance matrices, i.e., a certain class of \textit{Jaworski operators}. Gaussian mixed states are then constructed by a decomposition with a constant variable change, that is, a Gaussian\-ization of the corresponding finite-dimensional one.

We consider quantum states of the form
\begin{equation}
    \rho = \sum_i p_i |\psi_i\rangle \langle \psi_i|,
\end{equation}
where \( |\psi_i\rangle \) are Gaussian states. Gaussian mixed states are then constructed by a decomposition with a constant variable change, that is, a Gaussian\-ization of the corresponding finite-dimensional one.

We address the decomposition of a multimode Gaussian state with respect to a product of Gaussian states, which is a natural extension of the finite-dimensional case. The problem is solved by a decomposition with a constant variable change, that is, a Gaussian\-ization of the corresponding finite-dimensional one.

\section{Introduction}

Mixed state Gaussian states have been studied extensively in the literature, particularly in the context of quantum information theory. However, the problem of finding a decomposition of a multimode Gaussian state into a product of Gaussian states, which is a natural extension of the finite-dimensional case, is still open.

We address the decomposition of a multimode Gaussian state with respect to a product of Gaussian states, which is a natural extension of the finite-dimensional case. The problem is solved by a decomposition with a constant variable change, that is, a Gaussian\-ization of the corresponding finite-dimensional one.

\section{Results}

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We address the decomposition of a multimode Gaussian state with respect to a product of Gaussian states, which is a natural extension of the finite-dimensional case. The problem is solved by a decomposition with a constant variable change, that is, a Gaussian\-ization of the corresponding finite-dimensional one.
II. MODEWISE DECOMPOSITION OF PURE GAUSSIAN STATES

To begin with, suppose a collection of $N$ canonical systems or "modes" is partitioned into two sets, i.e., Alice’s $A = \{A_1, \ldots, A_m\}$ and Bob’s $B = \{B_1, \ldots, B_n\}$, of sizes $m$ and $n$ respectively. If the quantum state of the modes is a pure Gaussian state $|\psi\rangle_{AB}$, the following theorem characterizes the entanglement between Alice and Bob:

**Theorem 1:** A Gaussian pure state $|\psi\rangle_{AB}$ for $m + n$ modes $A$ and $B$ may always be written as

$$|\psi\rangle_{AB} = \sum_{s \leq m} |\tilde{\psi}_s\rangle_{A,s} \otimes |\psi_s\rangle_{B,s}$$

for some $s \leq \min(m, n)$, where $\tilde{A} = \{\tilde{A}_1, \ldots, \tilde{A}_m\}$ and $\tilde{B} = \{\tilde{B}_1, \ldots, \tilde{B}_n\}$ are new sets of modes obtained from $A$ and $B$ respectively through local linear canonical transformations, the states $|\tilde{\psi}_s\rangle$ are two-mode squeezed states of the form

$$|\tilde{\psi}_s\rangle_{A,s} = \frac{1}{\sqrt{Z_s}} \sum_n e^{-\frac{1}{2} \beta_s |n\rangle_{A,s} \langle n|_{A,s}}$$

entangling the modes $\tilde{A}_k$ and $\tilde{B}_k$ for $k \leq s$, and $|\psi_s\rangle_{B,s}$ and $|0\rangle_{B,s}$ are products of oscillator ground states for the remaining modes in $\tilde{A}$ and $\tilde{B}$ respectively.

Before proving theorem 1, we first review some facts concerning the correspondence between Gaussian states and covariance matrices. Let us represent the canonical variables of a $k$-mode system by the vector

$$\eta = \eta_1 \oplus \eta_2 \oplus \ldots \oplus \eta_k,$$

where $\eta$ is the two component vector $\eta = (\eta_1, \eta_2)^T$, and assume throughout that $\langle \eta \rangle = 0$ for all states considered. A generally mixed Gaussian state $\rho$ describing a system of $k$-modes with $\langle \eta \rangle = 0$ is completely specified by its covariance matrix (CM), defined as

$$M = \text{Re} \langle \eta \eta^T \rangle.$$

A unitary transformation on $\rho$ preserving the Gaussian character of the state implements a linear transformation of the modes $\tilde{\eta} = S\eta$, known as a symplectic (or linear canonical) transformation $S \in Sp(2k, \mathbb{R})$. Such a transformation preserves the canonical structure of the commutation relations $[\eta, \eta^T] = iJ_{2k}$, where the $k$-mode symplectic matrix is given by

$$J_{2k} = \sum_{i=1}^{k} J_2,$$

and satisfies $J_{2k}^2 = -I_{2k}$. Hence, a symplectic transformation preserves the symplectic matrix under a similarity transformation, i.e.,

$$SJ_{2k}S^T = J_{2k} \Rightarrow -(J_{2k} S)(J_{2k} S^T) = I_{2k}.$$

Under such a transformation, the state $\rho$ is brought to a new state $\tilde{\rho}$ with CM $M = SVM^T$. In particular, there exist symplectic transformations bringing the CM to the so-called Williamson normal form (WNF)

$$W = \lambda_1 \mathbb{1}_2 \oplus \lambda_2 \mathbb{1}_2 \oplus \ldots \oplus \lambda_k \mathbb{1}_2,$$

where $\lambda_i$ are the non-negative eigenvalues of the matrix $iJ_k M$, also known as the symplectic eigenvalues. Expressed in the product Hilbert space corresponding to the new set of modes $\{\tilde{\eta}_i\}$ for which $W = \text{Re}(\tilde{\eta} \tilde{\eta}^T)$, the Gaussian state $\rho$ acquires the particularly simple form

$$\rho = \rho_1 \otimes \rho_2 \otimes \ldots \otimes \rho_k$$

where $\rho_i$ is an oscillator thermal state for the $i$-th mode

$$\rho_i = \frac{e^{-\beta_i N_i}}{\text{Tr}(e^{-\beta_i N_i})} = \frac{1}{Z_i} \sum_n e^{-\beta_i |n\rangle_{i} \langle n|_{i}}.$$  

Here, $N_i = \sum \tilde{a}_i a_i^\dagger$ is the number operator associated with $\tilde{\eta}_i = (\tilde{a}_i + \tilde{a}_i^\dagger) \sqrt{2}$, and $\beta_i$ is related to the symplectic eigenvalue $\lambda_i$ by $\beta_i = \ln[(\lambda_i + 1/2)/(\lambda_i - 1/2)]$. Note that as a consequence of the uncertainty principle, admissible Gaussian states satisfy the condition $\forall i, \lambda_i \geq 1/2$, with pure Gaussian states when $\lambda_i \geq 1/2$. For $\lambda_i = 1/2$, $\rho_i = \{|0\rangle_{i} \langle 0|_{i}\}$ is obtained as the limit of $\rho_i$ as $\beta_i \to \infty$.

We now proceed with the proof of theorem 1. The Schmidt decomposition automatically yields the diagonal form of the partial density matrices for $A$ and $B$:

$$\rho_A = \sum_s p_s |\phi_s\rangle_{A,s} \langle \phi_s|, \quad \rho_B = \sum_s p_s |\chi_s\rangle_{B,s} \langle \chi_s|,$$

which are seen to be of equal rank and spectrum thus showing that the $p_s$‘s are unique. The basis states $|\phi_s\rangle_A$ and $|\chi_s\rangle_B$ are also unique (up to phase factors) for non-degenerate $p_s$ and otherwise may be chosen to be elements of any orthonormal basis spanning the degenerate subspace. Now, if $|\psi\rangle_{AB}$ is Gaussian, then the reduced density matrices are also Gaussian. Thus, $\rho_A$ and $\rho_B$ can be written in the form in terms of the set of modes bringing the local covariance matrices into WNF. Suppose that there are $s$ modes in $A$ and $t$ modes in $B$ with symplectic eigenvalue $\lambda \neq 1/2$. Since the remaining modes factor out from the respective density matrices as projection operators onto their ground state, we may factor $|\psi\rangle_{AB}$ as $|\tilde{\psi}\rangle_{AB} |\psi_s\rangle_{A,s} \otimes |\psi_t\rangle_{B,s}$, where $|\psi_s\rangle_{A,s}$ and $|\psi_t\rangle_{B,s}$ are collective ground states onto the modes with $\lambda \geq 1/2$ and $|\tilde{\psi}\rangle_{AB}$ is the generally entangled state for the remaining modes, $\tilde{A}_1, \ldots, \tilde{A}_m$ and $\tilde{B}_1, \ldots, \tilde{B}_n$. Concentrate then on $|\tilde{\psi}\rangle_{AB}$, the partial density matrices of which may be written as

$$\tilde{\rho}_A = \sum_{n_A} e^{-\beta_{-\kappa} n_A} |n_A\rangle_{A} \langle n_A|, \quad \tilde{\rho}_B = \sum_{n_B} e^{-\beta_{-\kappa} n_B} |n_B\rangle_{B} \langle n_B|,$$

where $n_A = (n_{\alpha_1}, \ldots, n_{\alpha_m})^T$ and $n_B = (n_{\beta_1}, \ldots, n_{\beta_n})^T$ are $s$ and $t$-dimensional vectors representing occupation number distributions on each side and $\beta_{-\kappa} = \ldots$
\[ \{ \beta_{\lambda_1}, ..., \beta_{\lambda_s} \}^T \] and \( \tilde{\beta}_B = \{ \beta_{\tilde{\lambda}_1}, ..., \beta_{\tilde{\lambda}_t} \}^T \) represent the distribution of thermal parameters on each side. Now, by our previous discussion, both density matrices have the same rank and the same eigenvalues. This means that there must exist a one-to-one pairing between the occupation number distributions \( \bar{n}_A \) and \( \bar{n}_B \), and such that
\[ \bar{\beta}_A \cdot \bar{n}_A = \bar{\beta}_B \cdot \bar{n}_B. \]  
(11)

We now observe that the pairing \( \bar{n}_A \leftrightarrow \bar{n}_B \) is a homogeneous linear map, since \( \bar{n}_A = 0 \) and \( \bar{n}_B = 0 \) are paired (all \( \beta s \neq 0 \) and \( \bar{n}_A + \bar{n}_B = \bar{n}_B + \bar{n}_A \)) satisfies \( \bar{\beta}_A \cdot \bar{n}_A = \bar{\beta}_B \cdot \bar{n}_B \) if \( \bar{n}_A, \bar{n}_B \) satisfy \( \bar{\beta}_B \cdot \bar{n}_B \). However, if a linear map is one-to-one then the domain and range have the same dimensions. Thus we see that \( \bar{s} = t \), in other words, the number of modes in \( A \) and \( B \) with symplectic eigenvalue different from 1/2 are the same. Now, label the modes on each side in ascending order of \( \beta \), so that 0 < \( \beta_{\lambda_1} \leq \beta_{\lambda_2} \leq ... \leq \beta_{\lambda_s} \), \( \bar{n}_{\lambda_s} \leq \bar{n}_{\lambda_{s-1}} \leq ... \leq \bar{n}_{\lambda_1} \). Consider first the case \( \bar{n}_A = \{0, 0, ..., 0\}^T \), yielding the smallest non-zero value of \( \bar{\beta}_A \cdot \bar{n}_A \). By construction, this distribution must be paired with the smallest non-zero value of \( \bar{\beta}_B \cdot \bar{n}_B \), which is (or can be taken to be in the case of degenerate \( \beta_{\lambda_i} \) ) \( \bar{n}_B = \{0, 0, ..., 0\}^T \). We thus find that \( \bar{\beta}_A = \bar{\beta}_B \) (hence \( \lambda_{\lambda_1} = \lambda_{\lambda_1} \)), and by the linearity property we find for any \( \bar{n}_A \), the map \( n_{\lambda_1} \rightarrow \bar{n}_{\lambda_1} = n_{\lambda_1} \). At this point we can repeat the procedure but applied to the subspace of the remaining modes, in other words, solve for a map between \( \bar{n}_A = \{0, n_{\lambda_2}, ..., n_{\lambda_s} \} \) and \( \bar{n}_B = \{0, n_{\lambda_2}, ..., n_{\lambda_s} \} \) such that \( \bar{\beta}_A \cdot \bar{n}_A = \bar{\beta}_B \cdot \bar{n}_B \). By a similar argument we find that \( \beta_{\lambda_2} = \beta_{\lambda_2} \) and \( \lambda_{\lambda_2} = n_{\lambda_2} \). Iterating the procedure until all the components are exhausted, we find that the admissible solutions to \( \bar{\beta}_A = \bar{\beta}_B \) are \( \bar{n}_A = \bar{n}_B \) (with a freedom of re-ordering the labels of degenerate modes), provided that \( \bar{\beta}_A = \bar{\beta}_B \). Reconstructing the Schmidt decomposition of \( |\tilde{\psi}\rangle_B \) from \( \rho_A \) and \( \rho_B \) we see that
\[ |\tilde{\psi}\rangle_B = |\tilde{\psi}\rangle_B (0) \lambda_y B \rangle 0 \rangle_{B_y} \]  
(12)
Thus, \[ |\tilde{\psi}\rangle_{AB} = |\tilde{\psi}\rangle\rangle AB = |\tilde{\psi}\rangle_{AB} (0) \lambda_y A \rangle 0 \rangle_{B_y} \]  
of the form (2).

III. ISOPTRIC GAUSSIAN MIXED STATES

Although the previous result may be proved directly from general features of the Schmidt decomposition, it may also be embedded in the more general framework of mixed entangled Gaussian states. Theorem 1 can thus be seen to be a special case of a more general mode-wise decomposition theorem for a certain family of Gaussian mixed states. To define this family, we shall say that a covariance matrix is isotropic if there exists a symplectic transformation of the modes \( W = S M S^T \) with \( S \in Sp(2k, \mathbb{R}) \) that brings \( M \) to the form
\[ W = \lambda \rho_{2k}, \quad \lambda \geq \frac{1}{2}. \]  
(13)

An isotropic Gaussian state may thus be defined as a Gaussian state with an isotropic CM. An example of such a state would be the thermal state of a set of oscillator modes with degenerate frequencies. Note that all pure Gaussian states are isotropic with \( \lambda_0 = \frac{1}{k} (k \equiv 1) \).

The more general theorem is a consequence of the following Lemma concerning isotropic CMs:

**Lemma 1**: Let \( M \) be an isotropic CM for \( m + n \) modes \( \eta = \eta_A \oplus \eta_B \) with symplectic eigenvalue \( \lambda_0 \). Then there exist local symplectic transformations \( \tilde{\eta}_A = S_A \eta_A \) and \( \tilde{\eta}_B = S_B \eta_B \) such that upon appropriate pairing of the modes, the covariance matrix takes the form
\[ \bar{M} = M_{\lambda_0} = \oplus_{m} \bar{M}_{\lambda_0} \oplus \bar{M}_{\lambda_0} \oplus \lambda_0 \mathbf{1}_{2(m+n-s)} \]  
(14)
for some \( s \leq m + n \), where \( \bar{M}_{\lambda_0} \) is an isotropic correlation matrix for the two-mode sector \( \eta_{\lambda_0} \oplus \eta_{\lambda_0} \) of the form
\[ \bar{M}_{\lambda_0} = \left( \begin{array}{ccc} \lambda_0 & 0 & 0 \\ 0 & \lambda_0 & -\lambda_0 \\ 0 & -\lambda_0 & \lambda_0 \end{array} \right), \quad \lambda_0^2 = \lambda_0^2 - \lambda_0^2. \]  
(15)

The diagonal elements \( \lambda_0 \) are at the same time the symplectic eigenvalues of the local CMs \( M_A = \{ \tilde{\eta}_A \} \) and \( M_B = \{ \tilde{\eta}_B \} \) differing from \( \lambda_0 \), and the last block in \( \lambda_0 \) gives the CM for the remaining modes.

Given the correspondence between CMs and Gaussian mixed states, the following extension of Theorem 1 immediately follows:

**Theorem 2**: An isotropic Gaussian state \( \rho_{AB} \) of symplectic eigenvalue \( \lambda_0 \) for the \( m + n \) modes \( A \) and \( B \) may always be written in the form
\[ \rho_{AB} = \tilde{\rho}_{\lambda_0} \oplus \tilde{\rho}_{\lambda_0} \oplus ... \oplus \tilde{\rho}_{\lambda_0} \oplus \tilde{\rho}_{\lambda_0} \oplus \tilde{\rho}_{\lambda_0} \]  
(16)
where the new modes \( \{ \tilde{\lambda}_i \} \) and \( \{ \tilde{\lambda}_i \} \) are obtained by local symplectic transformations from \( A \) and \( B \), \( \tilde{\rho}_{\lambda_0} \) are mixed Gaussian two-mode states with CM of the form \( \lambda_0 \), and \( \tilde{\rho}_{\lambda_0} \) and \( \tilde{\rho}_{\lambda_0} \) are mixed states for the remaining modes in \( \tilde{\lambda} \) and \( \tilde{\lambda} \) respectively with diagonal isotropic CM of symplectic eigenvalue \( \lambda_0 \).

To prove Lemma 1, first perform local symplectic transformations \( \eta_A \oplus \eta_B \) bringing the local CMs \( M_A = \{ \tilde{\eta}_A \} \) and \( M_B = \{ \tilde{\eta}_B \} \) into WNF. The total CM thus obtained may be written as
\[ \bar{M} = \text{Re}(\eta \eta^T) = \left( \begin{array}{cc} W_A & K_B \\ K_A^T & W_B \end{array} \right), \]  
(17)
with \( W_A = \bigoplus_{k=1}^{D_2} \lambda_k \mathbb{1}_2 \) and \( W_B = \bigoplus_{k=1}^{D_3} \lambda_k \mathbb{1}_2 \). We next note a useful fact regarding isotropic CMs. If \( R \) is satisfied, then \( M = \lambda_0 S S^T \) for some symplectic transformation \( S \). However, as is easily verified, \( S^T = SS^T \) is a symmetric symplectic transformation. Consequently, \( R \) implies that an isotropic \( k \)-mode CM satisfies:

\[
-(J M)^2 = \lambda^2 \mathbb{1}_{2k}.
\]

Replacing \( \mathbb{1}_{2k} \) in \( R \) and using the fact that \([W, J] = 0\), the following equations are obtained:

\[
W_{j_1 j_2}^2 - (J_{j_1} \tilde{K}_{j_2})(J_{j_2} \tilde{K}_{j_1}) = \lambda_0^2 \mathbb{1}_{2m} \quad \text{(19a)}
\]
\[
W_{j_1 j_2}^2 - (J_{j_1} \tilde{K}_{j_2})(J_{j_2} \tilde{K}_{j_1}) = \lambda_0^2 \mathbb{1}_{2n} \quad \text{(19b)}
\]
\[
-W_{j_1 j_2} \tilde{K}_{j_2} + J_{j_1} \tilde{K}_{j_2} \tilde{K}_{j_1} = 0 \quad \text{(19c)}
\]

Consider then a \( 2 \times 2 \) sub-block \( \tilde{K}_{ij} \equiv \langle \eta_{\lambda_i} \eta_{\lambda_j}^T \rangle \) of \( \tilde{K} \).

Since \( W_A \) and \( W_B \) are diagonal, from equation \( \text{(19c)} \) we find that

\[
\lambda_{\lambda_i} \tilde{K}_{ij} = \lambda_{\lambda_j} J_{j_1} \tilde{K}_{j_1 j_2} = 0.
\]

It is not hard to verify that unless \( \lambda_{\lambda_i} = \lambda_{\lambda_j} \), this equation has no solution for \( \tilde{K}_{ij} \) other than \( \tilde{K}_{ij} = 0 \). Thus we find that modes in \( A \) and modes in \( B \) with the same local symplectic eigenvalue \( \lambda \), and group the modes according to their eigenvalues so that \( M \) takes the Jordan form \( M = \bigoplus_j \lambda_j \mathbb{1} \), where each block \( \lambda_j \) is the CM for the degenerate eigenmodes \( \eta_{\lambda_j} \oplus \eta_{-\lambda_j} \). Concentrating on a given \( \lambda \), assume that \( g_A \) and \( g_B \) are the degeneracies of \( \lambda \) in the symplectic spectra of \( W_A \) and \( W_B \) respectively, so that \( M_\lambda \) may be written as

\[
\tilde{M}_\lambda = \begin{pmatrix}
\lambda \mathbb{1}_{2g_A} & \tilde{K}_\lambda \\
\tilde{K}_\lambda^T & \lambda \mathbb{1}_{2g_B}
\end{pmatrix}
\]

(21)

Now note that \( \tilde{M}_\lambda \) is also an isotropic CM with symplectic eigenvalue \( \lambda_\lambda \). Substituting \( \text{(19a)} \) into \( \text{(19b)} \), therefore yields

\[
J_{g_A} \tilde{K}_\lambda J_{g_B} \tilde{K}_\lambda^T = (\lambda^2 - \lambda_\lambda^2) \mathbb{1}_{2g_A} \
J_{g_B} \tilde{K}_\lambda J_{g_A} \tilde{K}_\lambda^T = (\lambda^2 - \lambda_\lambda^2) \mathbb{1}_{2g_B}
\]

(22a)

(22b)

Taking the trace of equations (22a) and (22b) and using the cyclic property of the trace \( \text{Tr}[J_{g_A} \tilde{K}_\lambda J_{g_B} \tilde{K}_\lambda^T] = \text{Tr}[J_{g_B} \tilde{K}_\lambda J_{g_A} \tilde{K}_\lambda^T] \), we obtain that

\[
(\lambda^2 - \lambda_\lambda^2)(g_A - g_B) = 0.
\]

Thus we see that the respective degeneracies of the symplectic eigenvalue \( \lambda \) in the local covariance matrices \( M_A \) and \( M_B \) must be the same for \( \lambda \neq \lambda_\lambda \). In such a case, let \( g_A = g_B = g \) and from equations \( \text{(19b)} \) deduce that

\[
\tilde{K}_\lambda^T J_{2g} \tilde{K}_\lambda = \tilde{K}_\lambda J_{2g} \tilde{K}_\lambda^T = -\lambda^2 \mathbb{1}_{2g} \quad \text{(23a)}
\]
\[
J_{2g} \tilde{K}_\lambda J_{2g} \tilde{K}_\lambda^T = \tilde{K}_\lambda J_{2g} \tilde{K}_\lambda^T = \tilde{K}_\lambda^T J_{2g} \tilde{K}_\lambda \quad \text{(23b)}
\]

Next, define a matrix \( \beta = \bigoplus_i \mathbb{1}_3 \), where \( \mathbb{1}_3 \) is the standard Pauli matrix, and hence satisfying \( \beta^2 = 1_{2g} \), \( \beta^T = \beta \), \( \{\beta, J_{2g}\} = 0 \), and \( \beta J_{2g} \beta = -J_{2g} \). Re-expressing \( K \) in terms of some other \( 2g \times 2g \) matrix \( O_\lambda \) as

\[
\tilde{K}_\lambda = \sqrt{\lambda^2 - \lambda_\lambda^2} O_\lambda \beta
\]

(24)

and substituting into \( \text{(46)} \) we find that \( O_\lambda \) must satisfy

\[
O_{2g} J_{2g} O_{2g}^T = J_{2g} \quad O_{2g}^T O_\lambda = \mathbb{1}_{2g};
\]

(25)

in other words, \( O_\lambda \) must be an orthogonal symplectic transformation. One can then perform a one-sided symplectic transformation, say \( O_\lambda^T \), on the \( A \)-modes, leaving the local CMs invariant in \( M_A \) and bringing \( \tilde{K}_\lambda \) into the block diagonal form, i.e.,

\[
\tilde{K}_\lambda = O_\lambda^T \tilde{K}_\lambda = \sqrt{\lambda^2 - \lambda_\lambda^2} \bigoplus_i \mathbb{1}_3. \quad \text{(26)}
\]

In this way, we achieve a pair-wise decomposition of the degenerate subspace itself, where each pair has a covariance matrix of the form \((8)\). Finally, we note that for the degenerate subspace associated with \( \lambda = \lambda_\lambda \), in which the degeneracies on each side are not restricted to be the same, equations (46) imply that \( \tilde{K}_\lambda \tilde{K}_\lambda^T = 0 \Rightarrow \tilde{K}_\lambda = 0 \). Therefore, local modes with symplectic eigenvalue \( \lambda_\lambda \) decouple, as expected from the pure case \( \lambda_\lambda = \frac{1}{2} \).

To conclude with, we note that it is known that for a \( 1 \times 1 \)-mode Gaussian mixed state, the Peres-Horodecki partial transpose criterion (11) is both necessary and sufficient (12) for entanglement and hence for distillability (13). In the present case of a two mode CM of the form (19), the partial transpose criterion implies that the state is entangled iff \( \lambda_1 > \lambda_\lambda^2 + \frac{1}{2} \). Consequently, an isotropic Gaussian state is entangled and distillable iff at least one of the symplectic eigenvalues of its local CMs satisfies this condition.

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