Static Potential and Local Color Fields in Unquenched Three-Dimensional Lattice QCD

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String breaking by dynamical quarks in (2+1)-d lattice QCD is demonstrated in this project, by measuring the static potential and the local color-electric field strength between a heavy quark and antiquark pair at large separations. Simulations are done for unquenched SU(2) color with two flavors of staggered quarks. An improved gluon action is used which allows simulations to be done on coarse lattices, providing an extremely efficient means to access the quark separations and propagation times at which string breaking occurs. The static quark potential is extracted using only Wilson loop operators and hence no valence quarks are present in the trial states. Results give unambiguous evidence for string breaking as the static quark potential completely saturates at twice the heavy-light meson mass at large separations. It is also shown that the local color-electric field strength between the quark pair tends toward vacuum values at large separations. Implications of these results for unquenched simulations of QCD in 4-d are drawn.

In unquenched simulations, we expect the linear rising static quark potential to saturate at large quark separations \(R\). This is the phenomenon of hadronic string breaking. However, despite extensive simulations of full QCD by several large scale collaborations \([1]\), a convincing demonstration of string breaking remains controversial. Traditionally, Wilson loops have been used to extract the static quark potential, though recently it has been suggested that this operator is not suitable because it has a small overlap with the two-meson ground state at large \(R\) \([2]\).

On the other hand, the Wilson loop produces a heavy quark-antiquark trial state without explicit light valence quarks. This trial state is of great physical interest because of the analogy with the process of hadronization, where light quarks are not present in the initial state, but rather are materialized from the vacuum.

It was proposed in Ref. \([3]\) that string breaking can in fact be observed in Wilson loops by working on coarse lattices using improved actions. On coarse lattices the computational effort can go towards generating much higher statistics, which allows much longer length and time scales to be accessed. This approach was applied in Ref. \([3]\) to lattice QCD in (2+1) dimensions, and later to 4-d QCD in Ref. \([4]\). It now appears that the overlap of the Wilson loop with the ground state is large enough, in the relevant range of \(R\), to allow string breaking to be resolved.

The present work is a follow-up on the earlier study of Ref. \([3]\). What is new here is a much more extensive study on larger lattices and with far higher statistics. The results give a much more convincing demonstration of an asymptote in the static quark potential at large quark separations. The demonstration of string breaking from measurements of the local color-electric field strength in unquenched QCD is entirely new to this work.

To reduce computational cost, we work in 3-d unquenched QCD with SU(2) color. QCD\(_3\) has been shown to share most of the fundamental properties of the realistic 4-d QCD \([5]\) and thus provides excellent insights to QCD. The coupling \(g_\alpha^2\) in QCD\(_3\) has dimension of mass and so explicitly sets the mass scale for the theory, i.e., \(m \propto g_\alpha^2\) for any mass quantity \(m\). In order to have some intuition for this mass scale, we express this scale in terms of the length scale in 4-d — “fermi” (\(fm\)). Motivated by the fact that both 3-d and 4-d QCD are linearly confining, we match
the quenched continuum extrapolation of the 3-d string tension, \( \sqrt{\sigma}/g^2 = 0.3353(18) \) [5], with the physical value of the string tension in four dimensions, \( \sqrt{\sigma} = 0.44 \text{GeV} \). This in effect sets the value of \( g_0 \) for our 3-d theory. A similar value for the coupling is obtained from other physical quantities, such as equating the unquenched \( \rho \) meson mass in this theory with the physical value in 4-d QCD.

The tree-level \( O(a^2) \)-accurate improved gluon action in 3-d has the usual form

\[
S_{\text{imp}} = -\beta \sum_{x,\mu>\nu} \xi \left[ \frac{5}{3} P_{\mu\nu} - \frac{1}{12} (R_{\mu\nu} + R_{\nu\mu}) \right],
\]

where \( P_{\mu\nu} \) is the plaquette and \( R_{\mu\nu} \) is the \( 2 \times 1 \) rectangle. The bare lattice anisotropy is entered through \( \xi_{ij} = \alpha_s/\alpha_t \) and \( \xi_{ij} = \alpha_t/\alpha_s \) \((i,j = 1,2)\). We use the staggered quark action in 3-d, which has the same form as the 4-d action and describes two flavors of four-component spinors:

\[
S_{\text{KS}} = \sum_{x,\mu} \zeta_4 \eta_4(x)\bar{\chi}(x) [U_\mu(x)\chi(x + \hat{\mu}) - U_\mu^\dagger(x - \hat{\mu})\chi(x - \hat{\mu})] + 2am_0 \sum_x \bar{\chi}(x)\chi(x),
\]

where \( \zeta_4 = a_s/a_t \) and \( \zeta_{1,2} = 1 \).

For comparison, most of the simulation parameters employed in the present project are taken from Ref. [3]. Simulations were done on a \( 22^2 \times 28 \) lattice with \( \beta = 3.0 \). Using the scale setting procedure discussed earlier, the lattice spacing is identified to be \( a_s \approx 0.2 \text{fm} \) in physical units. The input bare quark mass in lattice units is scaled according to \( m_0/g^2_0 = 0.10 \), with \( m_s/m_\rho \) found to be \( \approx 0.75 \). The input bare anisotropy is \( \alpha_t/\alpha_s = 1/2 \). The configurations were generated using the HMD algorithm (the \( \Phi \)-algorithm).

Overall 30,000 measurements were taken, corresponding to a total of 300,000 trajectories.

Only (fuzzy) Wilson loops were used to extract the static quark potential. The final results are presented in Fig. 1 for propagation times \( T/a_s = 2,6,12 \) (or \( T \approx 0.2,0.6,1.2 \text{fm} \) in physical units). The unquenched heavy-light meson mass was also computed and is shown as the dotted lines in Fig. 1. The results here give clear indication of string breaking as the unquenched potential is substantially flattened at large \( R \) and approaches the expected asymptotic value of the heavy-light meson mass.

To emphasize the flattening of the potential at large values of \( R \), we fitted the data points within \( R/a_s = 8.0 - 10.0 \) with a linear function \( V(R) = \sigma R + b \) and plotted the slope \( \sigma \) (string tension) against the propagation time \( T \) in Fig. 2. Results here unambiguously show that the slope tends toward zero at sufficiently large \( T \), \( T \approx 1.2 \text{fm} \).

The lattice observable needed to measure the field strength is given by the following correlator of plaquette \( P_{\mu\nu} \) with a Wilson loop \( W(R,T) \) [6]

\[
f_{\mu\nu}^{R,T}(\vec{x}) \sim \frac{< W(R,T) (P_{\mu\nu}(\vec{x}) - P_{\mu\nu}(\vec{x}_\infty)) >}{< W(R,T) >}.
\]

Here, \( \vec{x} \) is measured relative to the center of the Wilson loop and \( \vec{x}_\infty \) is usually taken as the site half the lattice away on a finite size lattice.

We measured the \( f_{13} \) component, which is corresponding to the energy density of the color-electric field in the direction parallel to \( R \) in the continuum limit, i.e., \( f_{13} = \mathcal{E}_\parallel \). The flux tube profiles for quark separations \( R/a_s = 5,7 \) are given in Fig. 3 where \( \mathcal{E}_\parallel(x) \) is plotted both along \( (x_\parallel) \) and perpendicular \( (x_\perp) \) to \( R \). The data is obtained with \( T/a_s = 7 \). Results here illustrate the formation of flux tubes between the color charges.
Notice that the thickness of the tube is $\simeq 1.6 \text{ fm}$ and is independent of the quark separation $R$.

Fig. 3 also demonstrates the screening of the sources by the dynamical quarks: the unquenched field strength is significantly lower than the quenched results inside the flux tube. One would also expect the unquenched field strength to drop substantially while the quenched results stay fairly constant near the string breaking distance. This idea is illustrated in Fig. 4 where the field strength at the center of the tube is plotted against the quark separation for the unquenched data. One can observe that at sufficiently large $T$, the field strength decreases gradually when $R$ increases, and eventually vanishes beyond the string breaking distance of $R/a_s \simeq 8.0 - 10.0$.

To summarize, we were able to observe string breaking by dynamical quarks in QCD$_3$. The use of improved actions on coarse lattices provided a crucial advantage in accessing the quark separations and propagation times at which string breaking occurs. The results confirm that string breaking appears in Wilson loop correlators only at sufficiently large $T$, of about 1 fm. These have clear implications for simulations of full QCD in 4-d as this scale is computationally accessible particularly on modestly coarse lattices.

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Figure 2. String tension $a_s^2 \sigma$ as a function of $T$.

Figure 3. Flux tube profiles for $R/a_s = 5, 7$. The open squares and the solid circles are the quenched and unquenched results respectively.

Figure 4. Unquenched $E_\parallel(0)$ for different $R$.

REFERENCES
1. See, e.g., S. Aoki et al., CP-PACS Collaboration, hep-lat/9809185.
3. H. D. Trottier, hep-lat/9812021.
6. See, e.g., H. D. Trottier and R. M. Woloshyn,