Bayesian approach to the first excited nucleon state in lattice QCD

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We present preliminary results from the first attempt to reconstruct the spectral function in the nucleon and $\Delta$ channels from lattice QCD data using the maximum entropy method (MEM). An advantage of the MEM analysis is to enable us to access information of the excited state spectrum. Performing simulations on two lattice volumes, we confirm the large finite size effect on the first excited nucleon state in the lighter quark mass region.

In quantum field theory, the spectral functions (SPFs) of the two-point correlation function is expected to expose rich physical information which is not only for the ground state, but also for excited states. Nevertheless, the reconstruction of SPF, $A(\omega)$ from given Monte Carlo data of the Euclidian time correlator: $G(t) = \int d\omega A(\omega) \exp(-\omega t)$ is a typical ill-posed problem. The maximum entropy method (MEM) is a useful method to circumvent such ill-posed problem by making a statistical inference of the most probable image of $A(\omega)$ based on Bayesian statistics.

Recently the MEM analysis is widely employed on various problems in lattice simulations after the first success in our research area \cite{1,2}. As for the light hadron spectroscopy, the CP-PACS collaboration analyzed their own high-precision quenched lattice QCD data using the MEM. They show the reliability of the MEM through checking consistency between the standard analysis and the MEM analysis after the continuum extrapolation \cite{3}. However above applications were carried out only for mesonic hadrons. In this article, we apply the MEM analysis to lattice QCD data for both spin-1/2 and spin-3/2 baryons in order to study the excited state spectrum of baryons.

We are interested in a long standing puzzle regarding the excited state spectrum of the nucleon, namely, the level order of the positive-parity excited nucleon $N'(1440)$ and the negative-parity nucleon $N^*(1535)$. The pattern of the level order between positive and negative-parity excited states can be found universally in the $\Delta$, $\Sigma$ and flavor-octet $\Lambda$ channels. Recent quenched lattice QCD studies show that the wrong ordering between $N'$ and $N^*$ actually happens in the relatively heavy-quark mass region \cite{4}. Thus, we address a serious question whether or not the level switching between $N'$ and $N^*$ would occur in the light-quark mass region. Finding of this possibility might be directly connected to the understanding of the mysterious Roper resonance.

We remark that the simulation for the light-quark mass requires large lattice size since the “wave function” of the bound state enlarges as the quark mass decreases. Once the “wave function” is squeezed due to the small volume, the kinetic energy of internal quarks raises and thus the total energy of the bound state should be pushed up. This is an intuitive picture for the finite size effect on the mass spectrum. Such effect is expected to become serious for the (radial) excited state rather than the ground state. Indeed, ref.\cite{4} reported that the $N'$ mass in the light-quark region is significantly heavier than the mass extrapolated from the heavy quark region. Their lattice simulation was performed on relatively small volume ($L \approx 1.5$fm).

To study the finite size effect, numerical simulations are performed on two lattice sizes $16^3 \times 32$
At zero spatial momentum, the $\Delta$ correlator with spatial Lorentz indices is expressed in the form

$$G_{ij}(t) = \left( \delta_{ij} - \frac{1}{3} \gamma_i \gamma_j \right) G_{3/2}^\Delta(t) + \frac{1}{3} \gamma_i \gamma_j G_{1/2}^\Delta(t),$$

where contributions from negative-parity states are omitted for simplicity. Defining $G_{ij} = \frac{1}{2} \text{Tr}(\gamma_i \gamma_j G_{\mu}^\Delta)$, one can obtain each amplitude: $G_{3/2}^\Delta$ and $G_{1/2}^\Delta$ from appropriate combinations of $G_{ij}$. In this article, we only analyze the $J^P = 1/2^+$ part of the $N$ correlator and the $J^P = 3/2^+$ part of the $\Delta$ correlator.

We define the dimensionless SPF for baryons through $A(\omega) = \rho(\omega) \omega^5$ [2]. In the continuum perturbative QCD, the asymptotic (renormalized) value of $\rho^{\text{ren}}(\mu)$ for the large renormalization scale $\mu(\gg 1\text{GeV})$ can be evaluated within the one-loop level

$$\rho^{\text{ren}}(\mu) = \frac{c_1}{(2\pi)^4} \left( 1 + c_2 \frac{\alpha_s(\mu)}{\pi} \right).$$

Here the value of $c_1 = 5/128$ (1/10) and $c_2 = 71/12$ (52/45) for our used $N$ ($\Delta$) operator in the MS-scheme can be retrieved from the number quoted in ref.[5]. In our MEM analysis, the prior knowledge (default model; $m_0$) is deduced from $\rho^{\text{ren}}(\mu)/(Z_{3q}^{\text{latt}}(\mu,a))^2$ at $\mu = a^{-1}$. $Z_{3q}^{\text{latt}}$ denotes the renormalization factor of the specified baryon interpolating operator on the lattice. For the $N$ operator, the tadpole improved perturbative result is give by $(1 - 0.73\alpha_s)/8$ in the chiral limit [6]. We roughly choose $m_0 = 4 \times 10^{-3}$ for $N$ and $8 \times 10^{-3}$ for $\Delta$ according to the above procedure. The results are insensitive to the variation of $m_0$ around this choice.

The MEM analysis is performed by using data up to the half of the temporal extent except for the source location. Most of the results presented here use $N_\omega \sim O(600)$ and $\omega_{\text{max}} a \approx 2\pi$. The detail procedure of the MEM can be found in [2].

In Fig.1, we show SPFs in the nucleon and $\Delta$ channels on the larger volume ($L \approx 2.2\text{fm}$). The solid, dashed and dotted lines are for $\kappa=0.155$, 0.153 and 0.1515 respectively. In both channels, there are one sharp peak, one broad peak and two bumps at each $\kappa$. Two bumps at each $\kappa$ have large overlap with the ones at different $\kappa$ while
the peak positions are relatively shifted to the left as $\kappa$ increases toward the chiral limit. Those states might be the unphysical bound states of a physical quark and two doublers, which have been found in the mesonic case [3]. The additive simulations at different lattice spacing are necessary to confirm this speculation.

In order to examine the finite size effect, we compare SPFs of the nucleon on the larger volume with the ones on the smaller volume at $\kappa=0.155$ and 0.1515 in Fig.2. The crosses on each peak or bump represent the statistical significance of SPF obtained by the MEM. We see a large finite volume effect on the second peak for the lighter quark ($\kappa=0.155$) as compared with the first peak and two bumps. It indicates that the (physical) excited state is significantly affected by the finite size effect in comparison to the ground state.

We plot the $N$ and $N'$ masses, which correspond to the peak positions of first two peaks, as a function of the pion mass squared in Fig.3. The errors are estimated by the jackknife method. We mention that the $N$ masses are quite consistent with the ones determined from the single exponential fits. In addition, for the $N'$, the heaviest two points are also good agreement with the corresponding results of ref.[4]. Taking a simple linear extrapolation in Fig.3, we find $M_N=0.51(1)$ and $M_{N'}=0.90(10)$ in lattice units. Our results may be compared with the previously published results for the $N^*$ at the same lattice spacing; $M_{N^*}=0.85(5)$ [4] for lattice size $L\approx1.5\text{fm}$ with domain wall fermions and 0.89(2) [7] for lattice size $L\approx2.2\text{fm}$ with clover fermions. We find that the level spacing between $N^*$ and $N'$ reduces significantly in the chiral limit. However the level switching between them might not happen in lattice simulations with $L\lesssim2.2\text{fm}$.

We have applied the maximum entropy method to lattice QCD data for both spin-1/2 and spin-3/2 baryons to study the positive-parity excited state spectrum. We succeeded in extracting SPFs for baryons as well as mesons. Based on the systematic analysis utilizing two lattice sizes, we confirmed the large finite size effect on the first excited nucleon state in the light quark mass region originally pointed out in ref.[4].

REFERENCES