We present evidence that in full QCD with two dynamical quarks confinement is produced by dual superconductivity of the vacuum as in the quenched theory. Preliminary information is obtained on the nature of the deconfining transition.

1. Introduction

A schematic phase diagram of full QCD with 2 dynamical flavours \((m_u = m_d = m_q)\) is shown in Fig. 1. The upper part of the diagram is the deconfined phase, the lower part is confined. The line is determined by the maxima of a number of susceptibilities [1,2], including the susceptibility \(\chi_L\) of the Polyakov line

\[
\chi_L = \int d^3x \langle L(\vec{x}, 0) L^\dagger(\vec{0}, 0) \rangle
\]  

and the susceptibility \(\chi_{ch}\) of the chiral order parameter

\[
\chi_{ch} = \int d^3x \langle \bar{\psi}\psi(\vec{x}, 0) \bar{\psi}\psi(\vec{0}, 0) \rangle .
\]

All of them have a maximum at the same value of \(T\), for a given \(m_q\), which defines the line in Fig. 1. For \(m_q > 3\) GeV, the maxima of \(\chi_L\) diverge proportionally to the volume \(V\), indicating a first order transition. At \(m_q = 0\) there are theoretical reasons and numerical indications that the transition is second order [3,1,2]. At intermediate values of \(m_q\) (tiny part of the line in Fig. 1) the susceptibilities do not diverge with \(V\), and this is interpreted as absence of a phase transition: the line would correspond to a crossover.

Across the transition the density of the free energy \(F\) is a function of the order parameters. The singularities of derivatives of \(F\) are related to susceptibilities of the order parameters. In our case \(L\) is the order parameter only at \(m_q = \infty\), \(\bar{\psi}\psi\) only at \(m_q = 0\). A good order parameter in the whole range of \(m_q\)'s could lead to a different assignment for the order of the transition.

Such a parameter could be the disorder parameter \(\langle \mu \rangle\) which describes condensation of magnetic charges [4]. That parameter has been constructed and tested in quenched theory [5–7] and its definition can be extended to full QCD [8]. In the spirit of \(N_c \to \infty\) arguments one expects that the mechanism of confinement is the same in quenched and full QCD, quark loops being non leading in \(1/N_c\) expansion. The operator \(\mu\) creates a magnetic charge, as defined by some given abelian projection. \(\langle \mu \rangle \neq 0\) signals dual superconductivity. In the quenched theory the specific choice of the abelian projection proves to be immaterial [7,9]. For the details about the definition of \(\mu\) we refer to [5–8]. For the quenched theory
it is found that \( \langle \mu \rangle \neq 0 \) for \( T < T_c \) and that \( \langle \mu \rangle = 0 \) for \( T > T_c \). \( \langle \mu \rangle \approx \tau^\delta \) as \( T \to T^{-} \), with \( \tau = 1 - T/T_c \). A finite size scaling analysis of the infinite volume limit yields \( \delta = 0.20(3) \) for \( SU(2) \); \( \delta = 0.50(2) \) for \( SU(3) \). The critical index \( \nu \) has the values \( \nu = 0.62(1) \) for \( SU(2) \), in agreement with Ref. [10], \( \nu = 0.33(1) \) for \( SU(3) \), corresponding to a first order transition [11]. We repeat the same analysis for 2 staggered flavours, using the Wilson action for the pure gauge sector, and \( 12^3 \times 4, 16^3 \times 4, 32^3 \times 4 \) lattices. The machines used are APEmille crates. Part of the results are already published in Ref. [8].

Figure 1. Phase diagram for two degenerate flavours, \( m_u = m_d = m_q \).

\[ T \text{ (MeV)} \]

\[ m_s = 0 \quad m_s = 3 \text{ GeV} \quad m_s = \infty \]

Figure 2. \( \rho \) as a function of \( N_s \) for \( T < T_c \). As \( N_s \to \infty \), \( \rho \to -\infty \) if the magnetic charge is non-zero and stays constant otherwise.

2. Numerical Results

As usual instead of \( \langle \mu \rangle \) itself we determine the quantity \( \rho = \frac{\partial}{\partial \beta} \log \langle \mu \rangle \) in terms of which \( \langle \mu \rangle = \exp \left( \int_0^\beta \rho(\beta') d\beta' \right) \). For \( T < T_c \) we find that \( \rho \) is practically size independent, i.e. that \( \langle \mu \rangle \neq 0 \) as \( N_s \to \infty \) (see Fig. 2). For \( T > T_c \) we find (Fig. 3)

\[ \rho = -kN_s + \text{const} \]

i.e. that \( \langle \mu \rangle \) is strictly zero as \( N_s \to \infty \). More extended checks of superselection of magnetic charge in the deconfined phase have been done, showing that for different magnetic charges created by \( \mu \), \( \rho \to -\infty \) if the net magnetic charge is non-zero, \( \rho \to \text{const} \) if it is zero. Some examples are given in Fig. 3.

Around \( T_c \) a finite size scaling analysis goes as follows. By dimensional arguments one can parametrize \( \langle \mu \rangle \) as

\[ \langle \mu \rangle = \tau^\delta \phi \left( \frac{a}{\xi}, \frac{N_s}{\xi}, m_q N_s^\tau \right) \]

where \( a \) is the lattice spacing and \( \tau = 1 - T/T_c \). If the correlation length goes large, \( \xi \sim \tau^{-\nu} \), \( a/\xi \ll 1 \) and the dependence on \( a \) can be neglected. The variable \( N_s/\xi \) can be traded for \( \tau N_s^{1/\nu} \). In the quenched theory \( m_q \) is absent and

Figure 3. \( \rho \) as a function of \( N_s \) for \( T > T_c \).
\[ \langle \mu \rangle = \tau^5 \phi(0, \tau N_s^{1/\nu}) \]

which gives for \( \rho \) the scaling behaviour \( \rho/N_s^{1/\nu} = f(\tau N_s^{1/\nu}) \), that can be tested and allows the determination of \( T_c, \nu \) and \( \delta \). In particular it implies that the height of the peak scales as \( N_s^{1/\nu} \).

In presence of dynamical quarks, \( m_q \) sets another scale, and we face a two scale problem. Simulations done at fixed \( (m_\pi/m_\rho) \) show that the height of the peak \( \rho \) roughly scales as \( N_s^{3} = V \) (see Fig. 4). A more refined analysis can be done by choosing \( m_q \) and \( N_s \) such that \( m_q N_s^{\gamma} \) in Eq. (4) is kept constant, and the problem is again reduced to a one scale problem, allowing to determine \( \nu \), which gives information on the order of the transition. The index \( \gamma \) is known to be \( \gamma \simeq 2.49 \) [1,2], so that the three sets \((m_q = 0.075, N_s = 16), (m_q = 0.043, N_s = 20)\) and \((m_q = 0.01335, N_s = 32)\) keep \( m_q N_s^{\gamma} \) constant. Preliminary results indicate that \( \nu = 1/3 \), or that the transition is compatible with first order. This is clearly visible from Fig. 5, where the height of the peak is plotted versus \( N_s^{3} \). More detailed numerical analyses involving more values of \( N_s \), \( m_q \) and the use of an improved action, to improve the quality of scaling, are needed to draw a definite conclusion.

In any case it is demonstrated that also in full QCD confinement is produced by monopole condensation.

REFERENCES