Higgs Doublets as Pseudo Nambu-Goldstone Bosons in Supersymmetric $E_6$ Unification

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Abstract

The idea to have Higgs doublets as pseudo Nambu-Goldstone (PsNG) multiplets is examined in the framework of supersymmetric $E_6$ unified theory. We show that extra PsNG multiplets other than the expected Higgs doublets necessarily appear in the $E_6$ case. If we demand that the extra PsNG multiplets neither disturb the gauge coupling unification nor make the color gauge coupling diverge before unification occurs, only possibility for the extra PsNG is $10 + 10$ of $SU(5)$. This is realized when the symmetry breaking $E_6 \to SO(10)$ occurs in the $\phi(27) + \phi(\bar{27})$ sector while $E_6 \to SU(4)_C \times SU(2)_L \times U(1) \times U(1)$ in the $\Sigma(78)$ sector. The existence of $10 + \bar{10}$ multiplets with mass around 1 TeV is therefore a prediction of this $E_6$ PsNG scenario. Implication of their existence on the proton decay is also discussed.

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There are many attractive features of grand unified theories (GUT), such as gauge unification, miraculous anomaly cancellation within a family, charge quantization, etc. In the present form of GUT, however, there are also many unsolved problems. One of the most serious difficulties would be the so-called hierarchy problem; we need extremely light Higgs doublets which are responsible for breaking electroweak symmetry, and their masses should be kept light against radiative corrections. The most attractive way to protect against such radiative correction is to introduce supersymmetry (SUSY), which is not yet confirmed by experiments; no superpartner has been observed. Another aspect of the hierarchy problem is the so-called doublet-triplet (DT) splitting problem. It is not yet made clear how we can naturally split only $SU(2)_L$ doublets from their GUT partner color triplet states. There have been many attempts to solve this problem;\(^1\)\(^-\)\(^4\) missing partner mechanism, sliding singlet mechanism, Dimopoulos-Wilczek mechanism, etc.. Among various approaches we concentrate in this paper our attention on the simplest idea which has long been investigated, namely the idea that Higgs doublets are realized as pseudo Nambu Goldstone (PsNG) bosons.\(^5\)\(^-\)\(^10\) Actually in supersymmetric grand unified scenario, once PsNG multiplets appear, they are kept massless so far as SUSY remains unbroken because of nonrenormalization theorem. Usually light PsNG multiplets are not welcome since their additional contributions to RGE harm the gauge unification. However if we can identify them as the usual Higgs doublets, we can make active use of such property of PsNG modes in explaining the light Higgs doublets.

This letter aims to examine this idea of PsNG in supersymmetric $E_6$ unified theories. The idea of PsNG has been first proposed by Inoue, Kakuto and Takano in 1986\(^5\) adopting a global $SU(6)$ whose subgroup $SU(5)$ is gauged. Later it was made more realistic by Barbieri, Dvali and Moretti\(^8\) by taking local $SU(6)$ symmetry and utilizing two Higgs sectors possessing no cross couplings. Dvali and Pokorski\(^10\) pointed out that the anomalous $U(1)_X$ symmetry can play a role in making two Higgs sectors separated from each other in the superpotential term. An extension to $E_6$ gauge symmetry was considered in Ref. 9) with a negative result.

Consider a supersymmetric grand unified theory based on a gauge group $G$. Suppose that the theory possesses two ‘Higgs scalar fields’, $\phi$ and $\Sigma$, each of which need not be of irreducible representation of $G$ so that they each may actually stand for a set of fields. The point is that we assume that they have no direct cross couplings in the superpotential,

$$ W = W_1(\phi) + W_2(\Sigma), \quad (1) $$

so that the superpotential has an enhanced symmetry $G_\phi \times G_\Sigma$, invariance under separate rotations of $\phi$- and $\Sigma$-sectors. In principle $G_\phi$ and $G_\Sigma$ can be (accidentally) larger than the
gauge group $G$, but here we assume that both are $G$; $G_\phi = G_\Sigma = G$. Suppose that $\phi$ and $\Sigma$ develop their vacuum expectation values (VEVs) $\langle \phi \rangle$ and $\langle \Sigma \rangle$ and the symmetries are broken into

$$
G_\phi = G \rightarrow H_\phi \quad \text{by} \quad \langle \phi \rangle,
$$

$$
G_\Sigma = G \rightarrow H_\Sigma \quad \text{by} \quad \langle \Sigma \rangle.
$$

(2)

Then, the Nambu-Goldstone (NG) multiplets corresponding to the cosets $G/H_\phi$ and $G/H_\Sigma$ appear from the $\phi$ and $\Sigma$ sectors, respectively. But the actual symmetry of the full system is only $G$ and it is broken to the intersection subgroup $H_\phi \cap H_\Sigma$, so that the true NG multiplets are only those of $G/(H_\phi \cap H_\Sigma)$. The other multiplets not contained in $G/(H_\phi \cap H_\Sigma)$ are therefore all pseudo Nambu-Goldstone (PsNG) multiplets, whose number is counted as $^*)$

$$
\# \text{ of PsNG multiplets} = \dim[G/H_\phi] + \dim[G/H_\Sigma] - \dim[G/(H_\phi \cap H_\Sigma)]
$$

$$
= \dim G + \dim[H_\phi \cap H_\Sigma] - \dim H_\phi - \dim H_\Sigma.
$$

(3)

Before entering the main subjects, we here comment on the the fact that exactly the same contents of PsNG multiplets also appear under a slightly different setup which was originally considered by K. Inoue and A. Kakuto and H. Takano. The setup they considered is as follows: the gauge symmetry $G_{\text{local}}$ of the system is $H_\Sigma$, and the superpotential of the Higgs fields $\phi$ of the system possesses a global symmetry $G_{\text{global}} = G$ larger than the required local symmetry $G_{\text{local}}$ and $\phi$ develops a VEV which retains only a symmetry $H_\phi$. We call this setup ‘global $G$ setup’ while the above one our ‘local $G$ setup’. Note that we can exchange $H_\phi$ and $H_\Sigma$ in this global $G$ setup since our local $G$ setup is symmetric under the exchange $H_\phi \leftrightarrow H_\Sigma$.

The reason why the same contents of PsNG multiplets appear in both setups is as follows: Suppose that the VEV $\langle \Sigma \rangle$ is much larger than the VEV $\langle \phi \rangle$ in our local $G$ setup. Then we can consider an effective theory at the energy scale lower than $\langle \Sigma \rangle$ but higher than $\langle \phi \rangle$. There the original local symmetry $G$ is already spontaneously broken to $H_\Sigma$ and the associated NG multiplets of $G/H_\Sigma$ are all absorbed in the $G$-gauge multiplet. The rest components of $\Sigma$ become massive of order $\langle \Sigma \rangle$ and decouple. Therefore the system at this stage is just the same as that of the global $G$ setup with Higgs fields $\phi$. Indeed the superpotential of $\phi$ retains the symmetry $G$ as a global symmetry while the local gauge symmetry of the system is only $H_\Sigma$. This finishes the proof. In this proof we have assumed $\langle \Sigma \rangle \gg \langle \phi \rangle$. But the number counting of broken generators is clearly independent of such an ordering, so the proof is generally valid.

$^*)$ This counting corresponds to the so-called maximum realization case. $^{11)}$
First let us use the following notation\textsuperscript{10} for the generated NG multiplets according to the representations under the standard theory gauge symmetry $G_S = SU(3)_C \times SU(2)_L \times U(1)_Y$:

\begin{align*}
\hat{Q}_Y &= (3, 2)_Y + (3, 2)_{-Y}, \\
\hat{T}_Y &= (3, 1)_Y + (3, 1)_{-Y}, \\
\hat{D}_Y &= (1, 2)_Y + (1, 2)_{-Y} = \hat{D}_{-Y}, \\
S_Y &= (1, 1)_Y.
\end{align*}

where the two numbers in each bracket stand for the dimensions of the representations of $SU(3)_C$ and $SU(2)_L$, and the attached suffix for the value of the hypercharge $Y$. We will also use notation like $\hat{Q}$ when we do not specify the hypercharge value.

First of all let us find the representations of the true NG multiplets which appear when the group $E_6$ breaks down to the standard theory gauge group $G_S = SU(3)_C \times SU(2)_L \times U(1)_Y$. The adjoint representation 78 of $E_6$ is decomposed into irreducible representations of the subgroup $SO(10)$ as

\begin{align}
78 &= 45 + 1 + 16 + \overline{16}, \quad (8)
\end{align}

and the $SO(10)$ adjoint 45 and the spinor 16 are further decomposed into $SU(5)$ representations as

\begin{align*}
45 &= 24 + 1 + 10 + \overline{10}, \\
16 &= 10 + \overline{5} + 1. \quad (9)
\end{align*}

As is well-known, these $SU(5)$ representations 24, 10 and $\overline{5}$ are decomposed under the standard theory gauge symmetry $G_S$ as\textsuperscript{12}

\begin{align*}
24 &= (8, 1)_0 + (1, 3)_0 + (1, 1)_0 + (3, 2)_{-5/6} + (\overline{3}, 2)_{5/6}, \\
10 &= (3, 2)_{1/6} + (\overline{3}, 1)_{-2/3} + (1, 1)_{-1}, \\
\overline{5} &= (\overline{3}, 1)_{1/3} + (1, 2)_{-1/2}. \quad (10)
\end{align*}

Therefore, when $E_6$ breaks down to $SO(10)$, the NG multiplets appearing are given by

\begin{align*}
E_6 \rightarrow SO(10) : \quad &16 + \overline{16} + 1 = (10 + \overline{10}) + (\overline{5} + 5) + 3 \times 1, \\
&= (\hat{Q}_{1/6} + \hat{T}_{2/3} + S_1 + S_{-1}) + (\hat{T}_{-1/3} + \hat{D}_{1/2}) + 3S_0, \quad (11)
\end{align*}

and, when $SO(10)$ further breaks down to $SU(5)$ and then to the standard theory gauge group $G_S$, the appearing NG multiplets are

\begin{align*}
SO(10) \rightarrow SU(5) : \quad &10 + \overline{10} + 1 = (\hat{Q}_{1/6} + \hat{T}_{2/3} + S_1 + S_{-1}) + S_0, \\
SU(5) \rightarrow G_S : \quad &(3, 2)_{-5/6} + (\overline{3}, 2)_{5/6} = \hat{Q}_{-5/6}. \quad (12)
\end{align*}
The net NG multiplets appearing when $E_6$ breaks down to the standard theory gauge group $G_S$ is thus found to be

$$2(\hat{Q}_{1/6} + \hat{T}_{2/3} + S_1 + S_{-1}) + \hat{Q}_{-5/6} + (\hat{T}_{-1/3} + \hat{D}_{1/2}) + 4S_0. \quad (13)$$

Next, as another breaking pattern, we consider the breaking of $E_6$ into its maximal subgroup $SU(6) \times SU(2)$. The adjoint 78 decomposes under $SU(6) \times SU(2)$ as

$$78 = (1, 3) + (35, 1) + (20, 2), \quad (14)$$

where the $SU(6)$ 20 of broken generator $(20, 2)$ is further decomposed under the subgroup $SU(4) \times SU(2) \subset SU(6)$ into

$$20 = (4, 1) + (\bar{4}, 1) + (6, 2) \quad \leftarrow \boxed{\begin{array}{c} \begin{array}{c} 4 \end{array} \bigoplus \begin{array}{c} 1 \end{array} \bigoplus \begin{array}{c} 6 \end{array} \bigoplus \begin{array}{c} 2 \end{array} \end{array} \right) \quad (15)$$

(The undotted and dotted boxes in the Young tableau on the right-hand side stand for the indices of $SU(4)$ and $SU(2)$ of the subgroup $SU(4) \times SU(2) \subset SU(6)$, respectively.) If the first factor group $SU(6)$ contains both $SU(3)_C$ and $SU(2)_L$ of the standard theory gauge group $G_S$, in which case $SU(6)$ is denoted as $SU(6)_{C,L}$, the NG multiplets associate with the breaking $E_6 \rightarrow SU(6)_{C,L} \times SU(2)$ are given by

$$2 \times 20 = 2 \times \left\{(3, 1) + (\bar{3}, 1) + (1, 1) + ((3, 2) + (\bar{3}, 2))\right\} = 2\hat{Q} + 2\hat{T} + 4S. \quad (16)$$

Here we have not specified the hypercharge values since there are various possibilities how $U(1)_Y$ generators are embedded in the unbroken subgroup. On the other hand, if $SU(3)_C$ is contained in the first $SU(6)$ while $SU(2)_L$ in the second $SU(2)$, i.e., $E_6$ breaks down to $SU(6)_C \times SU(2)_L$, then the resultant NG multiplets are given by

$$(20, 2) = 3 \times (3, 2) + 3 \times (\bar{3}, 2) + 2 \times (1, 2) = 3\hat{Q} + \hat{D}. \quad (17)$$

Now let us consider the breaking patterns of $E_6$ into subgroups $H$ where $H$ contains the standard theory gauge group $G_S = SU(3)_C \times SU(2)_L \times U(1)_Y$. In order to exhaust all the possibilities of the breaking patterns $E_6 \rightarrow H$ in a systematic way, we first classify the cases by identifying only the part $\tilde{H}$ of the subgroup $H$ containing the $SU(3)_C$ and $SU(2)_L$ groups of $G_S$. That is, we do not identify how the hypercharge $U(1)_Y$ is contained in the full $H$ and neglect the part (factor group) of $H$ which contains neither $SU(3)_C$ nor $SU(2)_L$. For instance, the choices of $H = SU(4)_C \times SU(2)_L \times SU(2) \times U(1)$ and $H = SU(4)_C \times SU(2)_L \times [U(1)]^k \quad (k = 0, 1, 2)$ are all classified into the case $\tilde{H} = SU(4)_C \times SU(2)_L$. The suffices $C$ and $L$ attached to the group name always mean that the $SU(3)_C$ and $SU(2)_L$ groups of $G_S$ are contained in that group, as we have defined in the above. This greatly simplify the task.
We classify the possibilities of the choice of $\tilde{H}$ according to its rank. The maximal regular subgroups of $E_6$ are $SU(6) \times SU(2)$, $SO(10) \times U(1)$ and $[SU(3)]^3$. However, since we only specify the factor groups that contain $SU(3)_C$ and $SU(2)_L$, then only possibilities of $\tilde{H}$ are clearly $SU(6)_C \times SU(2)_L$ and $SU(6)_{C,L}$ for the first $SU(6) \times SU(2)$, $SO(10)_{C,L}$ for the second $SO(10) \times U(1)$, and $SU(3)_C \times SU(3)_L$ for the third $[SU(3)]^3$. Lower rank cases of $\tilde{H}$ can be found by considering further breaking of these cases. In this way we find all the possibilities for $\tilde{H} \supset G_S$ and tabulate them in Table I. There we also list the representations of the NG multiplets under $\tilde{H}$ appearing in each breaking $E_6 \to \tilde{H}$.

Table I. Possible choices for $\tilde{H} \supset G_S$ and NG fields for the breaking $E_6 \to \tilde{H}$. The columns $Q$, $T$ and $D$ denote the numbers of times those representations of NG multiplets appear in $E_6/\tilde{H}$. $SU(3)_C \times SU(2)_L$ singlets are neglected.

<table>
<thead>
<tr>
<th>rank</th>
<th>Name</th>
<th>$\tilde{H}$</th>
<th>repr. under $\tilde{H}$ of the coset $E_6/\tilde{H}$</th>
<th>$Q$</th>
<th>$T$</th>
<th>$D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>E</td>
<td>$SU(6)_C \times SU(2)_L$</td>
<td>$[20,2]$</td>
<td>3</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>A</td>
<td>$SO(10)_{C,L}$</td>
<td>$16 + \overline{16}$</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>$SU(5)_C \times SU(2)_L$</td>
<td>$(10,2) + (\overline{10},2) + (5,1) + (\overline{5},1)$</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$SU(6)_{C,L}$</td>
<td>$2 \times 20$</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>C</td>
<td>$SU(5)_{C,L}$</td>
<td>$2 \times (10 + \overline{10}) + 5 + \overline{5}$</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>$SU(4)_C \times SU(2)_L$</td>
<td>$2 \times ((6,2) + (4,1) + (\overline{4},1)) + (4,2) + (\overline{4},2)$</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$SU(3)_C \times SU(3)_L$</td>
<td>$3 \times (3,3) + 3 \times (\overline{3},\overline{3})$</td>
<td>3</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>final</td>
<td>$SU(3)_C \times SU(2)_L$</td>
<td>$3 \times (3,2 + 1) + 3 \times (\overline{3},2 + 1) + 2 \times (1,2)$</td>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Since we specify how $\tilde{H}$ contains $SU(3)_C$ and $SU(2)_L$, we can count the numbers of appearing NG multiplets of representations $\hat{Q}$, $\hat{T}$ and $\hat{D}$, which are also shown in Table I. We can not count the numbers of $SU(3)_C \times SU(2)_L$-singlet NG multiplets nor the hypercharges of the $SU(3)_C \times SU(2)_L$ non-singlet NG multiplets. They can be specified later in concrete cases after narrowing down the possibilities.

Now with Table I, we can find all the possible choices of $\tilde{H}_\phi$ and $\tilde{H}_\Sigma$. The conditions which should be satisfied are: i) an $SU(2)_L$ doublet $\hat{D}$ appear as a PsNG multiplet, and ii) other PsNG multiplets, if exist, should fall into an $SU(5)_{GG}$ multiplet so as not to disturb the gauge coupling unification.

From Table I, we see that at most only one $\hat{D}$ NG multiplet can appear for any choices of $\tilde{H}$ and one $\hat{D}$ appears as a true NG multiplet in the $E_6 \to G_S$ breakdown. In order to satisfy the condition i), therefore, we must have one $\hat{D}$ NG multiplet for each of breakings...
$E_6 \to H_\phi$ and $E_6 \to H_\Sigma$, and so the candidates for $\tilde{H}_\phi$ and $\tilde{H}_\Sigma$ are restricted to the cases A, B, C, D and E.

For any choice of a pair $(H_\phi, H_\Sigma)$ from A, B, C, D and E, we immediately see that extra PsNG multiplets appear other than the desired $\hat{D}$ in this $E_6$ case. Note that the sum of the numbers of appearing $\hat{Q}$ and $\hat{T}$ in the pair should be larger than or equal to three for both $\hat{Q}$ and $\hat{T}$ since the true NG multiplets are $3\hat{Q} + 3\hat{T} + \hat{D}$. If the sum is less than 3 for either $\hat{Q}$ or $\hat{T}$, it implies that the intersection $H_\phi \cap H_\Sigma$ is larger than $G_S$ in contradiction to the assumption. Since no extra $\hat{D}$ other than the two (a true NG and a PsNG) multiplets appears, the only possibility for the $SU(5)$ multiplet into which other PsNG multiplets could fall is $10 \oplus \overline{10} \supset \hat{Q} + \hat{T}$, which contains no $\hat{D}$ and equal numbers of $\hat{Q}$ and $\hat{T}$. Therefore the sums of the numbers of appearing $\hat{Q}$ and $\hat{T}$ should be equal in order to satisfy the condition ii).

It is immediate to see that the only possible choices of such a pair satisfying this condition are (A,D) and (C,D). The former choice (A,D) yields $4\hat{Q} + 4\hat{T} + 2\hat{D}$ so that it gives a $10 + \overline{10}$ extra PsNG multiplets, while the latter case (C,D) gives $5\hat{Q} + 5\hat{T} + 2\hat{D}$ containing two pairs of $10 + \overline{10}$ extra PsNG multiplets. However we can see that the presence of $2\hat{Q} + 2\hat{T}$ PsNG multiplets makes the $SU(3)_C$ gauge interaction asymptotically non-free and the coupling constant becomes infinity before reaching the unification scale. Indeed, we have the formula for the running coupling $\alpha = g^2 / 4\pi$ at one loop,

$$\frac{1}{\alpha(\mu)} = \frac{1}{\alpha(M)} + \frac{b}{2\pi} \ln \left( \frac{M}{\mu} \right),$$

$$b = -\frac{9}{3} T(adj) + \sum_R N_R T(R), \quad \text{tr}(T_R^a T_R^b) = T(R) \delta_{ab}$$

(18)

where $N_R$ is the number of chiral multiplets of representation $R$, and the quadratic Casimir $T(adj) \equiv C_2(G) = N$ for $G = SU(N)$ and $T(\square) = 1/2$ for the fundamental representation $\square$ and $T(\overline{\square}) = (N - 2)/2$ for the representation $\overline{\square}$. For $SU(3)_C$ gauge coupling and for three generations ($6$ $3 + \overline{3}$ chiral multiplets) plus two $10 + \overline{10}$ PsNG multiplets ($2 \times (2 + 1) = 6$ $3 + \overline{3}$ chiral multiplets)), we have $b = -9 + (6 + 6)(1/2 + 1/2) = 3 > 0$, which makes $\alpha_s(\mu)$ diverge at around $\mu = 6 \times 10^9\text{GeV}$. We thus see that the only possibility is the choice (A,D). It is interesting that the presence of $\hat{Q} + \hat{T}$ in this case just makes the $\beta$ function of $SU(3)_C$ gauge coupling vanish at one-loop; $b = -9 + (6 + 3)(1/2 + 1/2) = 0$.

We thus have seen that the breaking pattern choice (A,D) is the only possibility. However, this is only a necessary condition. It is quite non-trivial whether there is actually a concrete model of breaking pattern (A,D) which also satisfies the $U(1)_Y$ quantum number requirements, which we have not examined above.

It is sufficient to find a model that satisfies all the requirements. We consider a model
in which \( E_6 \) is spontaneously broken to \( SO(10)_{C,L} \) by fundamental and anti-fundamental repr. Higgs fields \( \phi(\mathbf{27}) \) and \( \phi(\overline{\mathbf{27}}) \), while it is broken down to \( SU(4)_C \times SU(2)_L \times U(1)_A \times U(1)_B \) by an adjoint Higgs \( \Sigma(\mathbf{78}) \):

\[
\begin{align*}
A : & \quad E_6 \rightarrow H_\phi = SO(10)_{C,L} \quad \text{by} \quad \phi(\mathbf{27}) \quad \text{and} \quad \phi(\overline{\mathbf{27}}), \\
D : & \quad E_6 \rightarrow H_\Sigma = SU(4)_C \times SU(2)_L \times U(1)_A \times U(1)_B \quad \text{by} \quad \Sigma(\mathbf{78}).
\end{align*}
\]

(It should be noted that the breaking by adjoint \( \Sigma \) cannot lower the rank of \( H_\Sigma \) than that of \( E_6 \).) We shall specify these \( SO(10)_{C,L} \), \( SU(4)_C \) and \( U(1)_A \times U(1)_B \) in more detail below by identifying which components of \( \phi(\mathbf{27}) \) and \( \Sigma(\mathbf{78}) \) acquire the VEVs. The requirements is that the intersection \( H_\phi \cap H_\Sigma \) should be the standard model group \( G_S \).

For that purpose, it is convenient to name all the twenty seven components of the fundamental representation \( \phi(\mathbf{27}) \). \( \mathbf{27} \) is decomposed as \( \mathbf{27} = \mathbf{16} + \mathbf{10} + \mathbf{1} \) under Georgi-Fritsch-Minkowski’s \( SO(10)_{\text{GFM}} \subset E_6 \). Decomposing them further under Georgi-Glashow’s \( SU(5)_{\text{GG}} \subset SO(10) \), we name the 27 components as follows:

\[
\begin{align*}
16 &= \begin{bmatrix} u^c, (u^i_i, 1), e^c \end{bmatrix}, \quad 5^* + 1, \\
10 &= \begin{bmatrix} (d^c, e, -\nu), \nu^c \\
(D^c, E^c, -N^c), \quad (D^c, E, -N) \end{bmatrix} = 1 = S.
\end{align*}
\]

The simplest scenario for the breaking A is realized by the VEV of the \( SO(10) \)-singlet component \( S \) of \( \phi(\mathbf{27}) \):

\[
\langle \phi(\mathbf{1}) = S(\phi) \rangle = v_\phi.
\]

In this case the unbroken subgroup \( H_\phi \) is Georgi-Fritsch-Minkowski’s \( SO(10)_{\text{GFM}} \) which contains Pati-Salam \( SU(4)_{\text{PS}} \simeq SO(6) \) and \( SU(2)_L \times SU(2)_R \simeq SO(4) \) as its subgroup. But the choice of \( SO(10) \) in \( E_6 \) even with a constraint \( SO(10) \supset SU(5)_{\text{GG}} \) is not unique at all but has a freedom of an \( SU(2) \) rotation. Indeed as pointed out in Ref. 12), there is a maximal subgroup \( SU(6) \times SU(2)_E \) in \( E_6 \), where \( SU(6) \supset SU(5)_{\text{GG}} \) and the \( (5 + 1) \times 2 \) components in \( \mathbf{27} \)

\[
\begin{pmatrix}
d^c, e, -\nu \\
D^c, E, -N, -\nu^c
\end{pmatrix} \leftrightarrow \begin{align*}
E_3 &= +1/2, \\
E_3 &= -1/2.
\end{align*}
\]

give an \( SU(2)_E \) doublet of \( SU(6) \) \( 6 \)-plets. That is, the two \( 5^* \)-plets and two singlets \( \mathbf{1} \) of \( SU(5)_{\text{GG}} \) in Eq. (21) are rotated into each other under the \( SU(2)_E \). Since the generators of \( SU(2)_E \) are orthogonal to those of \( SU(5)_{\text{GG}} \), the \( SU(2)_E \)-rotated \( SO(10) \) from \( SO(10)_{\text{GFM}} \) with any angle \( \theta \)

\[
SO(10)_\theta = e^{i\theta \mathbf{E}}SO(10)_{\text{GFM}}e^{-i\theta \mathbf{E}}
\]

\[\text{Eq. (24)}\]
contains $SU(5)_{GG}$ as its subgroup. Thus the VEV

$$\begin{align*}
\langle S \rangle &= v_\phi \cos(\theta/2) \\
\langle \nu^c \rangle &= v_\phi \sin(\theta/2)
\end{align*}$$

$\leftrightarrow$

$$\begin{align*}
\langle S_\theta \rangle &= S \cos(\theta/2) + \nu^c \sin(\theta/2) \\
\langle \nu^c_\theta \rangle &= \nu^c \cos(\theta/2) - S \sin(\theta/2)
\end{align*}$$

(25)

breaks $E_6$ down to a twisted $SO(10)$, $H_\phi = SO(10)_{\theta}$ (24) with $\theta = (0, \theta, 0)$. As a matter of fact, however, there is no loss of generality at this stage even if we assume that the $H_\phi$ symmetry is $SO(10)_{GFM} = SO(10)_{\theta=0}$ with $\theta$ set equal to zero. This is because we have no reference frame at this stage and we are free to define those $SU(2)_E$-rotated fields $S_\theta$ and $\nu^c_\theta$ simply to be $S$ and $\nu^c$. We can thus call $SO(10)_{\theta}$ simply $SO(10)_{GFM}$. If we have another reference frame, such as another VEV than $\langle \phi \rangle$, then, this freedom of twisting $SO(10)$ becomes to have a physical meaning and we will actually use it below.

Next consider the D breaking (20) by the adjoint Higgs $\Sigma(78)$. In order to specify the $SU(4)_C$ and $U(1)_A \times U(1)_B$ in the breaking pattern D, it is convenient to consider a maximal subgroup $SU(6)_C \times SU(2)_L$ in $E_6$, under which the fundamental 27 decomposes into

$$\begin{pmatrix}
15,1
\end{pmatrix} = \begin{pmatrix}
-\varepsilon_{ijk}D^k & -u^c_i & -d^c_i & -D^c_i \\
u^c_i & 0 & S & \nu^c \\
d^c_i & -S & 0 & e^c \\
D^c_i & -\nu^c & -e^c & 0
\end{pmatrix}, \quad \begin{pmatrix}
6,2
\end{pmatrix} = \begin{pmatrix}
u^c & E^c & -N^c \\
N & E & 0 \\
e & e & 0
\end{pmatrix}.$$  

(26)

Here the first three entries and the last three entries of the 6 of $SU(6)_C$ are the fundamental representations 3 of $SU(3)_C$ and 3 of $SU(3)_R$, respectively. The three components of 3 of $SU(3)_R$ are arranged in the order for later convenience. We define and name three $SU(2)$ subgroups of the $SU(3)_R$ as follows by identifying their doublets:

$$SU(2)_R: \begin{pmatrix}E^c & -N^c \\
N & E \end{pmatrix}, \quad SU(2)_R^c: \begin{pmatrix}E^c & -N^c \\
\nu & e \end{pmatrix}, \quad SU(2)_E: \begin{pmatrix}N & E \\
\nu & e \end{pmatrix}.$$  

(27)

The $SU(4)_C$ in the D breaking (20) should be $SU(4)_{C,E}$ orthogonal to the $SU(2)_E$, whose fundamental representation 4 is given by the first four entries in the $SU(6)$ representation (26). The reason is as follows.

The true NG multiplets for the breaking $E_6 \rightarrow G_S$ are given in Eq. (13)

$$2(\hat{Q}_{1/6} + \hat{T}_{2/3} + S_1 + S_{-1}) + \hat{Q}_{-5/6} + (\hat{T}_{-1/3} + \hat{D}_{1/2}) + 4S_0.$$  

(28)

In addition to these we expect in this (A,D) breaking scenario that there appear the following PsNG multiplets:

$$(10 + \overline{10}) + \hat{D}_{1/2} + xS_0 = (\hat{Q}_{1/6} + \hat{T}_{2/3} + S_1 + S_{-1}) + \hat{D}_{1/2} + xS_0.$$  

(29)
where the number $x$ of $G_S$-singlets $S_0$ can be arbitrary. On the other hand, the NG multiplets coming from the $\phi$-sector in which $E_6 \to SO(10)_{GFM}$ occurs are given in Eq. (11):

$$
(\hat{Q}_{1/6} + \hat{T}_{2/3} + S_1 + S_{-1}) + (\hat{T}_{-1/3} + \hat{D}_{1/2}) + 3S_0.
$$

(30)

Therefore the NG multiplets appearing from the $\Sigma$-sector should be

$$
2(\hat{Q}_{1/6} + \hat{T}_{2/3} + S_1 + S_{-1}) + \hat{Q}_{-5/6} + \hat{D}_{1/2} + (1 + x)S_0.
$$

(31)

Note that the breaking in the $\Sigma$-sector is $E_6 \to SU(4)_C \times SU(2)_L \times U(1)_A \times U(1)_B$ while the eventual breaking accompanied by the true NG multiplets is $E_6 \to G_S = SU(3)_C \times SU(2)_L \times U(1)_Y$. So the difference between (31) and (28),

$$
\hat{T}_{-1/3} + (3 - x)S_0
$$

(32)

must correspond to the NG multiplets associated with the breaking $SU(4)_C \times U(1)_A \times U(1)_B \to SU(3)_C \times U(1)_Y$. For the latter breaking we generally have $\hat{T}_Y + 2S_0$ as NG multiplets (so that $x$ is fixed to be 1). In order for this color triplet $\hat{T}_Y$ for the breaking $SU(4)_C \to SU(3)_C$ to carry the desired hypercharge $Y = -1/3$, the difference of $Y$ quantum number of the first three color triplet components from that of the fourth component of $SU(4)_C$ 4 should be $-1/3$. Noting the hypercharge quantum numbers $Y((u^i, d^i)) = 1/6$, $Y((E^c, -N^c)) = +1/2$, $Y((N, E)) = -1/2$ and $Y((\nu, e)) = -1/2$, we see that the only possibility for $SU(4)_C$ is $SU(4)_{C,LE}$ for which the 4 is given by

$$
(6, 2) = \begin{pmatrix}
    u^i & d^i \\
    E^c & -N^c
\end{pmatrix}.
$$

(33)

Indeed then the generator which converts the fourth entry $E^c$ to $u$-quark $u^i$ is $SU(3)_C$ color triplet 3 and carries hypercharge $Y(u^i) - Y(E^c) = 1/6 - 1/2 = -1/3$ as required.

Now let us identify the VEV of $\Sigma(78)$ which realizes such D breaking $E_6 \to SU(4)_{C,LE} \times SU(2)_L \times U(1)_A \times U(1)_B$. As we have seen in Eq. (14), the adjoint $\Sigma(78)$ is decomposed under $SU(6)_C \times SU(2)_L$ as $78 = (1, 3) + (35, 1) + (20, 2)$, the VEV $\langle \Sigma \rangle$ realizing such a breaking is developed in the $SU(6)_C$ adjoint component $(35, 1)$:

$$
\langle \Sigma(35, 1) \rangle = \begin{pmatrix}
    a & 0 & 0 \\
    0 & b & 0 \\
    0 & 0 & c
\end{pmatrix} (4a + b + c = 0)
$$

(34)

Here this $6 \times 6$ matrix is written on the same basis as in Eq. (26) so that the bottom right $2 \times 2$ submatrix corresponds to $SU(2)_E \times U(1)$. Note that we have used $SU(6)_C$ rotations to bring the generic VEV of hermitian $6 \times 6$ matrix $\Sigma(35, 1)$ into the above diagonal form; in
particular, an $SU(2)_E$ rotation is used to make the bottom right $2 \times 2$ submatrix diagonal. This means that the previous $\phi$-sector unbroken subgroup $H_\phi$ no longer remains to be the $SO(10)_{GFM}$ with $\theta = 0$ in this basis but becomes $SO(10)_\theta$ with $\theta \neq 0$. For $\theta \neq 0$ to have a physical meaning, the $SU(2)_E$ must be broken by $b \neq a$ as we assume here. Then two unbroken $U(1)$ charges, called $U(1)_A$ and $U(1)_B$ in the above, are given in this basis by

$$U(1)_A : \quad A \equiv \begin{pmatrix} 1_4 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \quad U(1)_B : \quad B \equiv \begin{pmatrix} 0_4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = E_3. \quad (35)$$

The latter charge $B$ is chosen to be the third component $E_3$ of $SU(2)_E$.

It should be emphasized that $\theta$ must not be zero. Otherwise, the intersection $H_\phi \cap H_\Sigma$ would contain an extraneous $U(1)$ other than the standard theory gauge symmetry $G_S$. Indeed, if $H_\phi = SO(10)_{GFM}$, its five Cartan generators are all diagonal in the particle basis which we have defined in Eq. (21), while $H_\Sigma = SU(4)_{C,LE} \times SU(2)_L \times U(1)_A \times U(1)_B$ is rank 6 and contains all the Cartan generators in $E_6$, which are also diagonal on the same basis. Therefore the $U(1)_V$ contained in $SO(10)_{GFM} \supset SU(5)_{GG} \times U(1)_V$ can be necessarily written as a linear combination of the six Cartan generators in $H_\Sigma$ and hence remains as an unbroken symmetry contained in the intersection $H_\phi \cap H_\Sigma$ in contradiction to the assumption. If $\theta \neq 0$, on the other hand, the directions of Cartan generators in $H_\phi$ and $H_\Sigma$ are twisted and no such $U(1)$ remains. [This can be seen by looking at, e.g., $e^{-i\theta E_2} E_3 e^{i\theta E_2} = E_3 \cos \theta + E_1 \sin \theta$.]

Finally let us confirm the quantum numbers including the hypercharge of the NG multiplets which actually appear in this D breaking $E_6 \to SU(4)_{C,LE} \times SU(2)_L \times U(1)_A \times U(1)_B$ realized by the $\Sigma$-VEV (34). Noting the hypercharge $Y$ is given by

$$Y = \begin{pmatrix} (1/6)1_3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & (1/2)1_2 \end{pmatrix}, \quad (36)$$

on the fundamental representation 6 of $SU(6)_C$ in the basis (26), we can find the $SU(3)_C \times U(1)_Y$ quantum numbers of the 20 = $\begin{array}{c} 1 \\ 0 \end{array}$ by inspecting Eq. (15):$^*$

$$20 = (4, 1) + (\bar{4}, 1) + (6, 2) \quad \text{under } SU(4)_{C,LE} \times SU(2)_E$$

$$= (3_{-5/6} + 1_{-1/2}, 1) + (\bar{3}_{5/6} + 1_{1/2}, 1) + (3_{1/6} + \bar{3}_{-1/6}, 2) \quad (37)$$

so that the $G_S$ quantum numbers of $(20, 2)$, which appears for the breaking $E_6 \to SU(6)_C \times SU(2)_L$, are given by

$$\boxed{(20, 2) = \hat{Q}_{-5/6} + \hat{D}_{1/2} + 2\hat{Q}_{1/6}.} \quad (38)$$

$^*$ For instance, the hypercharge $Y = -5/6$ for $(3_{-5/6}, 1)$ can be found as follows. It corresponds to $\begin{array}{c} 1 \\ 0 \end{array}$ with color index $\alpha = 1, 2, 3$ and $\cdot = 5$ or 6, hence carrying the hypercharge $(1/6) + (-1/2) + (-1/2) = -5/6$. 

11
When $SU(6)_C \times SU(2)_L$ is further broken to $SU(4)_{C,E} \times SU(2)_L \times U(1)_A \times U(1)_B$, the appearing NG multiplets are

\[ (4, \overline{2}) + (\overline{4}, 2) + 2S_0 \quad \text{under } SU(4)_{C,E} \times SU(2)_E \]

\[ = (3, \overline{2})_{2/3} + (1, \overline{2})_1 + (\overline{3}, 2)_{-2/3} + (1, 2)_{-1} + 2S_0 \]

\[ = 2 \times (T_{2/3} + S_1 + S_{-1}) + 2S_0 \]  \hspace{1cm} (39)

where $2S_0$ comes from the breaking $SU(2)_E \to U(1)_{E_3}$. We thus see that the resultant NG multiplets (38) plus (39) indeed realizes the expected one in Eq. (31).

**Proton Decay:**

The important prediction of the present idea of Higgs doublets as PSNG multiplets is that there necessarily appear additional PSNG multiplets $10_H + \overline{10}_H$ of $SU(5)_{GG}$ which we expect will get masses $M_{10}$ around $O(1)$ TeV after SUSY is broken. Aside from the direct observation of them, their effect may be seen through proton decay. Let us evaluate the order of the proton decay caused by their effect.

We expect generically the presence of the following dimension 4 and 5 operators in the low energy effective superpotential: in terms of the $SU(5)$ language,

\[ W_4 = f^{ij}_{k} \overline{5}_i 5_j 10_H \supset f^{ij}_{k} [\epsilon^{\alpha\beta\gamma} d^c_{i\alpha} d^c_{j\beta} u^c_{i\gamma} + d^c_{i\alpha}(e_j u^\alpha_H - \nu_j d^\alpha_H)] \]

\[ W_5 = \frac{f^{ij}_{k}}{M_{pl}} 10_i 5_j \overline{10}_H \supset f^{ij}_{k} [\epsilon^{\alpha\beta\gamma} u^c_{i\alpha} d^c_{j\beta} + (u^c_{i\alpha} e_j - d^\alpha_{i\gamma})] \langle H_u \rangle \overline{d}_{i\gamma}, \]  \hspace{1cm} (40)

where $10_i$ and $\overline{5}_i (i = 1, 2, 3)$ denotes three generations of matters, $10_H$ and $\overline{10}_H$ are our new light Higgs, and $5_H$ is the usual Higgs $H_u$ in which the color triplet part is in fact missing. If the colored components in $10_H$ and in $\overline{10}_H$ are connected by propagator $\langle 10_H \overline{10}_H \rangle$ and the usual Higgs doublet $5_H$ is replaced by the VEV $\langle H_u \rangle$, then we have an effective superpotential which breaks baryon number:

\[ W_6 = \frac{f^{ijkl}_{n}}{M_{pl}} \epsilon^{\alpha\beta\gamma} u^c_{i\alpha} d^c_{j\beta} \times d^c_{k\gamma} \nu_l, \quad f^{ijkl}_{\text{eff}} = f^{ij}_{k} f^{kl}_{4} \langle H_u \rangle / M_{10} \]  \hspace{1cm} (41)

Note that if the Higgs VEV $\langle H_u \rangle$ is replaced by the Higgs superfield, then this term gives an dimension 6 operator but the suppression is not by the square of Planck mass $M_{pl}$ but by a single power of $M_{pl}$. Another mass scale $M_{10}$ comes from the propagator of $10_H$ Higgs which is light and does not give any significant suppression; $\langle H_u \rangle / M_{10} \approx 1 - 10^{-1}$. So this operator is potentially dangerous so that the proton decay by this operator should be suppressed by the smallness of the coupling constant.

Similarly to the analysis of the generic dimension 5 operators as performed by Kakizaki and Yamaguchi,\(^{14}\) we can think that the coupling constants $f^{ij}_{4}$ and $f^{ij}_{5}$ obey a Froggatt-
suppression mechanism similar to the usual Higgs Yukawa coupling constant responsible for the fermion masses. Then, using letters \( q_i, l_i, u^c_i, d^c_i \) and \( h_u, h_d \) to denote the Froggatt-Nielsen \( U(1) \) charges of \( i \)-th generation quarks and leptons and the up- and down-type Higgs doublets \( H_u \) and \( H_d \),

\[
f_{ij}^{kl} = f_4 f_5 \lambda^{u^c_i + d^c_k + l_i + h_u}, \quad f_{4,5} \sim O(1)
\]

\[
= (f_4 f_5) y_d^{ij} \lambda^{(u^c_i - q_i) - h_d} y_d^{jk} \lambda^{(l_i - q_l) - h_d + h_u},
\]

where \( \lambda \sim \sin \theta_C = 0.22 \) and \( y_d^{ij} = \lambda^{q_i + u^c_j + h_d} \) is the \( ij \) matrix element of the down-type quark yukawa coupling. Then the largest operator is

\[
W_6 = \frac{f_{1123}^{1111}}{M_{pl}} u_R d_R s_R \nu \tau, \tag{43}
\]

so that the main decay mode is \( p \rightarrow K^+ \nu \tau \). The bound for the proton lifetime \( \tau_{\text{proton}} > 2 \times 10^{33} \text{yr} \) gives a constraint

\[
| f_{ij}^{kl} | \lesssim \left( 10^{-7} \sim \lambda^{11} \right) \times \left( \frac{M_{pl}}{10^{19} \text{GeV}} \right) \tag{44}
\]

We have for the main decay mode

\[
f_{1123}^{1111} \sim (f_4 f_5) \frac{\langle H_u \rangle}{M_{10}} \times \left( y_d^{11} \lambda^{(u^c_i - q_i) - h_d} y_d^{32} \lambda^{(l_i - q_l) - h_d + h_u} = y_d^{11} y_d^{32} \lambda^{p + 2h_u - 3h_d} \right) \tag{45}
\]

where use has been made of the ‘GUT-inspired’ relations \( q_i = u^c_i \) and \( l_i = d^c_i \) by Kakizaki and Yamaguchi and of the definition of \( p \):

\[
y_b / y_t = \lambda^{d^c_i + h_d - u^c_i - h_u} \equiv \lambda^p \rightarrow l_3 - q_3 = p + h_u - h_d \tag{46}
\]

If we use \( p = 2 \) corresponding \( \tan \beta \simeq 3 \), and semi-empirical relations \( y_d^{11} = \lambda^5 y_b, y_d^{32} = y_d^{33} = y_b \) and \( y_t \sim 1 \), we have

\[
f_{1123}^{1111} \sim (f_4 f_5) \frac{\langle H_u \rangle}{M_{10}} \times \lambda^{11 + 2h_u - 3h_d} \tag{47}
\]

Therefore, since it is natural to expect that the factors \( (f_4 f_5) \frac{\langle H_u \rangle}{M_{10}} \) and \( \lambda^{2h_u - 3h_d} \) are of order 1, we could see the proton decay in near future.

**Yukawa couplings:**

The particular property of our Higgs doublets as PsNG multiplets is their representations under \( SO(10) \subset E_6 \). As is seen from the discussion above, in particular Eq. (11), the down-type Higgs \( H_d \) is contained in \( (16, 5) \) in \( \phi(27) \) and up-type Higgs \( H_u \) in \( (1\overline{6}, 5) \) in \( \phi(27) \), where the two numbers in the brackets denote representations under \( SO(10) \) and \( SU(5) \). This
is in sharp contrast with the usual GUTs in which $H_u$ and $H_d$ are assigned to be $(10, 5)$ and $(\bar{10}, \bar{5})$. This property leads to some peculiarities in obtaining fermion mass terms in this model. The down-type quark mass terms come from the usual trilinear terms but they actually exist only when the down-type quarks contain ‘$SU(2)_E$ twisted components’ $\Psi(10, \bar{5})$:  

$$
\Psi_i(27)\Psi_j(27)\phi(27) \rightarrow \Psi_i(16, 10)\Psi_j(10, \bar{5})\phi(16, \bar{5}).
$$

(48)

Since $\phi(27)$ does not contain the up-type Higgs $H_u \subset 5$, these trilinear terms do not contain up-type quark masses at all. Up-type quark mass terms come from dimension 5 operators:  

$$
\Psi_i(27)\Psi_j(27)\phi(27)\phi(27) \rightarrow \Psi_i(16, 10)\Psi_j(16, 10)\phi(16, \bar{5}) \left< \phi(\bar{16}, \bar{1}) \right>. 
$$

(49)

Note that the VEV $\langle \phi(\bar{16}, \bar{1}) \rangle$ is non-vanishing only when the $SU(2)_E$ rotation (24) in the Higgs sector exists, $\theta \neq 0$. The induced top Yukawa coupling is thus not of dimension 4 coupling but comes from a higher dimensional operator. The resultant Yukawa couplings are thus accompanied with $\langle \phi(\bar{16}, \bar{1}) \rangle/M_P$. This eventually suppress the top Yukawa coupling by the power $\lambda$ or so. Note that the bottom Yukawa coupling can in principle be dimension 4. However we expect the so-called family twisting structure$^{12, 16, 13}$ and so the bottom Yukawa couplings may be accompanied by some Froggatt-Nielsen factor, so that the ratio of Yukawa couplings of the top and the bottom quarks can become smaller. Also note that in our scenario the unified gauge coupling is larger than the usual case and it may be possible to get a reasonable top quark mass as an quasi infrared fixed point; the running Yukawa coupling approaches to the order of color gauge coupling faster than in the usual case$^{*}$.

We conclude this note by adding some comments.

The PsNG Higgs approach based on the model with $G = SU(6) \times SU(2)_R$ gauge symmetry instead of $E_6$ may also be interesting,$^{17}$ in which the breaking pattern is given by  

$$
[SU(6) \times SU(2)_R]_\phi \rightarrow SU(5)_{GG},
$$

(50)

$$
[SU(6) \times SU(2)_R]_\Sigma \rightarrow SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)
$$

(51)

and there appears no extra PsNG multiplet than the desired Higgs doublets. This breaking pattern can be realized by the $\phi$ and $\Sigma$ Higgs sectors which consist of $\phi(\bar{6}, 2)$ and $\Sigma(15, 1)$ in addition to their conjugates, respectively. Note that $\Sigma(15, 1)$ contains no $SU(5)$ singlet component but has an $SU(4)$ singlet. So it can naturally breaks $SU(6)$ down to $SU(4)$ instead $SU(5)$. Moreover these Higgs fields $\phi(\bar{6}, 2)$ and $\Sigma(15, 1)$ can be combined into a

$^{*}$ The same problem already happens in the $SU(6)$ case considered by Dvali and Pomarol.$^{10}$ They introduced an additional fermion fields $20$, and top quark is represented as a mixture state of $15$ with $20$. This provides a dimension 4 top Yukawa coupling.
fundamental representation 27 of $E_6$ representation. So if $E_6$ is broken by some mechanism, for example by Hosotani mechanism, it may be possible to make a realistic scenario by using only the fundamental representation Higgs.

The notion of PsNG bosons were first investigated intensively by S. Weinberg\textsuperscript{18) in the context of dynamical symmetry breaking. The essential difference between his PsNG and the present one is the existence of SUSY. In the non-SUSY case the mass of PsNG is generated via residual gauge interaction which breaks the tree level symmetry and the order is estimated to be $m^2 = g^2 \Lambda^2$, with $\Lambda$ being a characteristic scale of the interaction responsible for the spontaneous breaking. In SUSY case, on the other hand, the masses of PsNG fields are protected until the SUSY breaking occurs. This ensures masses of PsNG very light of the order $\sim g M_{\text{SUSY}}$.

We would like to stress that $E_6$\textsuperscript{19) model has many advantages. Especially after the recent neutrino oscillation observations confirmed the remarkable fact of the neutrino large mixings, $E_6$ model became more attractive because we anyhow need some non-parallel (twisting) family structure in order to reproduce those large mixings.\textsuperscript{20)} $E_6$ provides us with the most natural scenario for realizing this twisting family structure.\textsuperscript{13),16) We have seen in this paper that this twisting structure is also required in the symmetry breaking pattern to assure the intersection $H_\phi \cap H_\Sigma$ reduces to the standard theory gauge group $G_S$.

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