Large mixing angle oscillations as a probe of the deep solar interior

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ABSTRACT

We re-examine the sensitivity of solar neutrino oscillations to noise in the solar interior using the best current estimates of neutrino properties. Our results show that the measurement of neutrino properties at KamLAND provides new information about fluctuations in the solar environment on scales to which standard helioseismic constraints are largely insensitive. We also show how the determination of neutrino oscillation parameters from a combined fit of KamLAND and solar data depends strongly on the magnitude of solar density fluctuations. We argue that a resonance between helioseismic and Alfvén waves might provide a physical mechanism for generating these fluctuations and, if so, neutrino-oscillation measurements could be used to constrain the size of magnetic fields deep within the solar radiative zone.


1. Introduction

Current solar (Ahmad et al. 2002a,b; Fukuda et al. 2002; Cleveland et al. 1998; Abdurashitov et al. 2002; Kirsten 2002) and atmospheric (Fukuda et al. 1998) neutrino data give compelling evidence that neutrino conversions take place. For the simplest case of oscillations, the relevant parameters are two mass-squared differences $\Delta m^2_{\text{sol}}$ and $\Delta m^2_{\text{atm}}$, three angles $\theta_{12}$, $\theta_{23}$, $\theta_{13}$ plus a number of CP violating phases (Schechter & Valle 1980). One

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knows fairly well now that $\theta_{23}$ is nearly maximal (from atmospheric data) and that the preferred solar solution for $\theta_{12}$ is the so-called large mixing angle (LMA) solution (Gonzalez-Garcia et al. 2000), while the third angle $\theta_{13}$ is strongly constrained by the result of reactor experiments (Apollonio et al. 1999). The CP phases are completely unknown at present. A recent analysis of solar and atmospheric data in terms of neutrino oscillations is given in Maltoni et al. (2002a), and finds the currently-favored LMA solution of the solar neutrino problem has

$$\tan^2 \theta = 0.46, \quad \Delta m^2 = 6.6 \times 10^{-5} \text{ eV}^2$$

and corresponds to oscillations into active neutrinos. Other recent analyses of solar data can be found in: Fogli et al. (2002); Bahcall, Gonzalez-Garcia & Pena-Garay (2002); Bandyopadhyay et al. (2002a,b); Barger et al. (2002); de Holanda & Smirnov (2002); Creminelli, Signorelli & Strumia (2001).

The recent results from the KamLAND reactor experiment (Eguchi et al. 2002) have brought neutrino physics to a new stage. For the first time the solar neutrino anomaly has been probed using terrestrial neutrino sources. This is fundamental for two reasons. First, among the various proposed solutions of the solar neutrino problem, such as the possibility of neutrino spin-flavor-precession (Schechter & Valle 1981; Akhmedov 1988; Lim & Marciano 1988; Miranda et al. 2001a,b; Barranco et al. 2002), or non-standard neutrino matter interactions (Guzzo et al. 2001), which may arise in models of neutrino mass (Mohapatra & Valle 1986; Hall, Kostelecky, & Raby 1986), it singles out a unique “oscillation-type” solution: the LMA MSW solution. Second, it brings to fruition one of the initial motivations for studying solar neutrinos in the first place (Bahcall 1989): the use of solar neutrinos to infer the equilibrium properties of the solar core.

In this article we make the following points:

- We show how the determination of neutrino oscillation parameters from a combined fit of KamLAND and solar data shows a strong dependence on the magnitude of solar density fluctuations.
- We show that the fact that the KamLAND results largely support LMA neutrino oscillations, can be used to provide new information about fluctuations in the solar core on much shorter scales than those which existing constraints (like helioseismology) can presently probe.
- We propose a physical process which could arise in the core, that may produce fluctuations on the scales to which solar neutrinos are sensitive.
2. Effect of Fluctuations on Neutrino Propagation

The evolution of solar neutrinos in the presence of a fluctuating solar matter density has been considered previously by Balantekin, Fetter, & Loreti (1996); Bamert, Burgess, & Michaud (1997); Nunokawa et al. (1996); Burgess & Michaud (1996). In a nutshell, these studies show that neutrino oscillations in the Sun can be influenced by density fluctuations provided that two conditions are satisfied at the position of the MSW oscillation (Wolfenstein 1977; Mikheev & Smirnov 1985): (i) The fluctuation’s correlation length, $L_0$, is comparable to the local neutrino oscillation length, $L_{osc} \sim 100$ km and, (ii) the fluctuation’s amplitude, $\xi$, is at least roughly 1%. These conclusions are summarized in Fig. 1, where we show how the electron-neutrino survival probability depends on the fluctuation amplitude, $\xi$, given optimal choices for $L_0 = 100$ km and neutrino oscillation parameters fixed at the best fit in Eq. (1).

These early estimates can now be sharpened in view of our better understanding of neutrino-oscillation parameters. To illustrate this we have performed a global analysis of the solar data, including radiochemical experiments (Chlorine, Gallex-GNO and SAGE) as well as the latest SNO data in the form of 17 (day) + 17 (night) recoil energy bins (which include CC, ES and NC contributions, see Maltoni et al. (2002a)) (Ahmad et al. 2002a,b), the Super-Kamiokande spectra in the form of 44 bins (Fukuda et al. 2002) and the KamLAND
rates and spectra given in Eguchi et al. (2002).

The combined analysis of KamLAND and solar neutrino data with a fluctuating solar medium depends on four parameters: the two neutrino oscillation parameters $\Delta m^2$ and $\tan^2 \theta$ and the two parameters $\xi$ and $L_0$ characterizing solar noise. In what follows we present the results of two different analyses. First we display the allowed neutrino oscillation parameters for given assumption about the solar noise parameters. Later we present the limits on solar noise parameters for given assumptions about the solar neutrino oscillation parameters.

### 3. Constraints on Solar Noise

The sensitivity of the neutrino signal to the solar density fluctuations is shown in Fig. 2. The regions denote the bounds on $\xi$ as a function of the correlation length $L_0$ at different confidence levels for 2 d.o.f..

Taking into account only the current solar neutrino data as in (Maltoni et al. 2002a) with the neutrino oscillation parameters in the region $\Delta m^2 = 10^{-6} \div 10^{-3}$ eV$^2$ and $\tan^2 \theta = 0.1 \div 1$, we have found a very weak bound on the noise. For $L_0 < 100$ km or large $L_0 > 300$ km there is essentially no constraint. Only for intermediate $L_0$ values inside the region 100 km $< L_0 < 300$ km we find that $\xi < 8\%$ at 90\% C.L..

In contrast, the inclusion of the KamLAND reactor data constrains the level of the solar density noise, irrespective of the values of neutrino oscillation parameters in the above region. The increased sensitivity on the solar density noise parameters found in the global analyses of solar + KamLAND data is seen in the left panel of Fig. 2. If the solar neutrino parameters were known with higher accuracy one could use them to probe the solar noise level with even better sensitivity. For example, fixing the neutrino oscillation parameters at the current global solar + KamLAND best-fit LMA point given in Maltoni et al. (2002b), yields the bound given in the right panel of Fig. 2.

Another way to present the global analysis of solar+KamLAND results is illustrated in Fig. 3. Here we display the minimal $\chi^2$ versus noise amplitude before and after KamLAND, irrespective of neutrino parameters. All in all, for a density fluctuation scale $L_0$ of 100 km or so, one finds from this figure that the current limit to the noise amplitude is about 3\% at 95\% C.L.. Note that these bounds follow from the neutrino data themselves, no helioseismological argument can at present rule out their existence.
Conversely, for given assumptions on the solar noise parameters, the combined solar and KamLAND data can be used to determine the allowed region of neutrino mixing parameters, as shown in Fig. 4. First we give, in the left panel, the allowed LMA-MSW region for a smooth solar density profile (noiseless case), from Maltoni et al. (2002a). The effect of the noise is shown in the right panel, where we have fixed the spatial scale of fluctuation at its optimal value $L_0 = 100$ km and left the noise magnitude free. One sees that, taking into account solar neutrino data alone the allowed region of neutrino oscillation parameters becomes bigger. As seen from the right panel, the inclusion of KamLAND data implies the existence of a new region with substantially lower $\Delta m^2$ values, around $\Delta m^2 = 2 \times 10^{-5}$ eV$^2$, and somewhat lower $\tan^2 \theta$ around $\tan^2 \theta = 0.25$. This new region is present even at 90% C.L. with 2 d.o.f.. One finds that the global best fit point for neutrino parameters at $\xi = 8\%$ noise level and $L_0 = 100$ km becomes $\Delta m^2 = 2 \times 10^{-5}$ eV$^2$ and $\tan^2 \theta = 0.25$, a region excluded by the standard KamLAND + solar neutrino data analysis without noise. This illustrates how a precise knowledge of the solar interior (the solar noise level) is required in order to sharpen the determination of neutrino oscillation parameters.
Two objections were believed to limit this kind of analysis. First, on the observational side, the success of helioseismology seemed to preclude the existence of fluctuation amplitudes larger than 1% in size. Second, on the theoretical side, no known physics of the solar core could generate fluctuations large enough to be detected. Of these two, the first is the more serious, since our inability to guess a source of fluctuations in such a complicated environment is much less worrying than is a potential conflict with helioseismic data. Nonetheless, in the remainder of this letter we argue why neither of these objections can rule out the possibility of having large enough density fluctuations without undergoing more careful scrutiny.

5. Helioseismic Bounds and Fluctuation Mechanism

Helioseismology (Castellani et al. 1997; Christensen-Dalsgaard 2002) is rightfully celebrated as a precision tool for studying the inner properties of the Sun. Careful measurements have provided precise frequencies for numerous oscillation modes, and these may be compared with calculations of these frequencies given assumed density and temperature profiles for the solar interior. Constraints on solar properties arise because careful comparison between theory and measurements gives agreement only if the assumed profiles are within
Fig. 4.— Allowed regions for neutrino oscillation parameters $\Delta m^2$ and $\theta$, for the “standard” noiseless Sun (left panel) and for a noisy Sun with arbitrary density noise magnitude $\xi$ on a $L_0 = 100$ km spatial scale. The lines and shaded regions correspond to the analyses of solar and solar+KamLAND data respectively. The best-fit point is indicated by a triangle (solar) or a star (solar+KamLAND).

roughly 1% of the predictions of the best solar models.

For the present purposes, the weak link in this train of argument lies in the details of the inversion process which obtains the solar density profile given an observed spectrum of helioseismic frequencies. This inversion is only possible if certain smoothness assumptions are made about solar properties, due to the inevitable uncertainties which arise in the observed solar helioseismic oscillation pattern. As a result helioseismology severely constrains the existence of density fluctuations, but only those which vary over very long scales $\gg 1000$ km (Castellani et al. 1997; Christensen-Dalsgaard 2002). In particular, the measured spectrum of helioseismic waves is largely insensitive to the existence of density variations whose wavelength is short enough – on scales close to $L_{osc} \sim 100$ km, deep within the solar core – to be of interest for neutrino oscillations. In particular, we claim that such density variations with amplitudes as large as 10% cannot yet be ruled out by helioseismic data.
Fluctuation Mechanism

A mechanism which might produce density variations of the required amplitude and correlation length arises once helioseismology is reconsidered in the presence of magnetic fields, which are normally neglected in helioseismic analyses (Couvidat, Turck-Chieze, & Kosovichev 2002). Generally, the neglect of magnetic fields in helioseismology is very reasonable since the expected magnetic field energy densities, $B^2/8\pi$, are much smaller than are gas pressures and other relevant energies.

In (Burgess et al. 2003) we study helioseismic waves in the Sun, and find, contrary to naive expectations, that reasonable magnetic fields in the radiative zone (Boruta 1996; Parker 1979) can appreciably affect the profiles of helioseismic $g$-modes as a function of solar depth. In particular, density profiles due to these waves tend to form spikes at specific radii within the Sun, corresponding to radii where the frequencies of magnetic Alfvén modes cross those of buoyancy-driven ($g$-type) gravity modes. Due to this resonance, energy initially in $g$-modes is directly pumped into the Alfvén waves, causing an amplification of the density profiles in the vicinity of the resonant radius. This amplification continues until it is balanced by dissipation, resulting in an unexpectedly large density variation at the resonant radii. Furthermore, these level crossings only occur with $g$-modes, and so typically occur deep within the solar radiative zone. They also do not affect substantially the observed $p$-modes, which makes it unlikely that these resonances alter standard analyses of helioseismic data (which ignore solar magnetic fields), in any significant way.

Fig. 5 plots the positions of the Alfvén/$g$-mode resonant layers as a function of an integer mode label $n$, for various values of a hypothetical magnetic field. What this figure shows is that there are very many such level crossings, whose position varies most quickly with radius within the Sun near the solar center. The superposition of several different modes results in a series of relatively sharp spikes in the radial density profile at the radii where these resonances take place. From the point of view of exiting neutrinos, passage through these successive helioseismic resonances mimics the passage through a noisy environment whose correlation length is the spacing between the density spikes.

In (Burgess et al. 2003) we make several estimates of the size of these waves in the Sun. Fig. 6 shows how the spacing between the spikes varies as a function of their position within the Sun. Note that, while typically the width of the spikes is comparable to their separation, and therefore causes important effects in neutrino propagation, it is however considerably narrower than characteristic $g$-waves. As a result we find that the energy cost of producing them with amplitudes as large as 10% can be much less than the prohibitively large values which were required to obtain similar amplitudes for $g$-waves alone (Bamert, Burgess, & Michaud 1997). It is remarkable that the position at which this spacing is close to 100 km
lies near $r = 0.12 \, R_\odot$ for a very wide range of magnetic fields. On the other hand, we have observed (Burgess et al. 2003) that, for magnetic fields which are of order $10 \, kG$, the spacing of resonances is near $100 \, km$ for a very wide range of radii – including the neutrino resonance region, $r \sim 0.3 \, R_\odot$.

Are such large radiative-zone magnetic fields possible? Very little is directly known about magnetic field strengths within the radiative zone. The only generally-applicable bound there is due to Chandrasekar, and states that the magnetic field energy must be less than the gravitational binding energy: $B^2 / 8\pi < GM_\odot^2 / R_\odot^4$, or $B < 10^8 \, G$. A stronger bound is also possible if one assumes the solar magnetic field to be a relic of the primordial field of the collapsing gas cloud from which the Sun formed. In this case it has been argued that central fields cannot exceed around $30 \, G$ (Boruta 1996). (Still stronger limits, $B < 10^{-3} \, G$, are possible (Mestel & Weiss 1987) if the solar core should be rapidly rotating, as is sometimes proposed.) Since the initial origin of the central magnetic field is unclear, we believe any magnetic field up to the Chandrasekar bound should be entertained.
The above mechanism has many of the features required to produce density fluctuations in the Sun which may be relevant to the analysis of solar neutrino data. Although much more study is required to establish whether these resonances really occur and affect neutrinos, we believe their potential existence substantially reinforces the general motivation for using neutrino oscillations to directly probe short-wavelength density fluctuations deep within the solar core.

6. Summary and conclusion

We have re-examined the sensitivity of solar neutrino oscillations to fluctuations in the solar density profile, using the best current estimates of neutrino properties, especially the new reactor data from KamLAND. Our results show that the measurement of neutrino properties in the latter experiment provides new information about fluctuations in the solar environment on scales to which standard helioseismic constraints are largely insensitive. Conversely we have seen how the determination of solar neutrino parameters from a fit of
the data in the case of a noisy Sun differ from the quiet Sun case. We have also argued that a resonance between helioseismic and Alfvén waves can provide a physical origin for such fluctuations and, if so, neutrino-oscillation measurements could be used to constrain the size of magnetic fields deep within the solar radiative zone.

This work was supported by Spanish grants BFM2002-00345, by the European Commission RTN network HPRN-CT-2000-00148, by the European Science Foundation network grant N. 86, by Iberdrola Foundation (VBS) and by INTAS grant YSF 2001/2-148 and MECD grant SB2000-0464 (TIR). C.B.’s research is supported by grants from NSERC (Canada) and FCAR (Quebec). M.M. is supported by contract HPMF-CT-2000-01008. VBS, NSD and TIR were partially supported by the RFBR grant 00-02-16271. M.A.T. was supported by the MECD fellowship AP2000-1953.

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