current status of neutrino masses and mixings
sensitivity to $\nu_\mu$ and $\nu_e$ ($\sigma_{\nu_\mu,\nu_e}^{\text{ES}} \simeq \sigma_{\nu_\mu}^{\text{ES}} / 6$), measured in Super-Kamiokande [11] and SNO [12].

Let me emphasize also the importance of the Homestake experiment [7], in which the solar neutrino problem was discovered, and the Gallium SAGE [8], GALLEX [9] and GNO [10] experiments, which are sensitive to the fundamental flux of $pp$ neutrinos (see Ref. [13]). Moreover, the results of all neutrino experiments are necessary to get information on the neutrino mixing parameters.

The global analysis of all solar neutrino data in terms of $\nu_e \to \nu_\mu$, performed in Ref. [14] yielded

\begin{equation}
0.24 < \tan^2 \theta_5 < 0.89, \\
2.3 \times 10^{-5} < \Delta m^2_5 / \text{eV}^2 < 3.7 \times 10^{-4},
\end{equation}

at 99.73% C.L. (3σ), where $\Delta m^2_5$ is the relevant neutrino squared-mass difference and $\theta_5$ is the effective mixing angle in two-generation analyses of solar neutrino data. In Eq. (2.1) I reported only the boundaries of the so-called LMA region (see Ref. [3]), which is currently favored, because it is much larger than other regions (a LOW region and three VAC regions appear at 99% C.L.) and it contains the minimum of the $\chi^2$ (best fit) at

\begin{equation}
\tan^2 \theta_5 \simeq 0.42, \quad \Delta m^2_5 \simeq 5.0 \times 10^{-5} \text{ eV}^2.
\end{equation}

The limits in Eq. (2.1) show that the mixing relevant for solar neutrino oscillations is large. However, maximal mixing seems strongly disfavored from the analysis of solar neutrino data in Ref. [14]. This conclusion is supported by the results of some other authors [11, 15, 16, 18], whereas the authors of Refs. [17, 19–21] found slightly larger allowed regions, with maximal mixing marginally allowed. Therefore, it is not clear at present if maximal mixing in solar neutrino oscillations is excluded or not.

In particular, the authors of Refs. [20, 21] found a larger LMA region and several allowed LOW+QVO and VAC regions, where QVO indicates the region at $\Delta m^2_5 \sim 10^{-5} \text{ eV}^2$ where both matter effect and vacuum oscillations are important.

The authors of Ref. [19] found that taking into account only the recent Gallium data (taken in the period 1998–2001, with improved systematic uncertainty with respect to the earlier data) the LOW region is less disfavored with respect to the LMA region.

In any case, there are good reasons to believe that the SNO experiment has solved the solar neutrino problem proving its neutrino physics origin and the presence of $\nu_e \to \nu_\mu, \nu_\tau$ transitions, and that the current data indicate a large mixing. The final confirmation of the true allowed region will hopefully come from the results of the KamLAND [22] or BOREXINO [23] experiments.

\[ \text{FIG. 1: The two possible types of three-neutrino schemes.} \]

**III. ATMOSPHERIC NEUTRINOS**

In 1998 the Super-Kamiokande experiment found model independent evidence of disappearance of atmospheric $\nu_\mu$'s [24], which is supported by the results of the K2K long-baseline experiment [25], of the Soudan 2 [26] and MACRO [27] atmospheric neutrino experiment. The atmospheric neutrino data of the Super-Kamiokande experiment are well fitted by $\nu_\mu \to \nu_\tau$ transitions with large mixing (fact (F4) above):

\begin{equation}
1.2 \times 10^{-3} < \Delta m^2_\Delta / \text{eV}^2 < 5.0 \times 10^{-3}, \\
\sin^2 2\theta_A > 0.84,
\end{equation}

at 99% C.L. [28], where $\Delta m^2_\Delta$ is the relevant neutrino squared-mass difference and $\theta_A$ is the effective mixing angle in two-generation analyses of atmospheric neutrino data. The best fit is at

\begin{equation}
\sin^2 2\theta_A = 1.0, \quad \Delta m^2_\Delta \simeq 2.5 \times 10^{-3} \text{ eV}^2.
\end{equation}

Furthermore, other oscillation channels ($\nu_\mu \equiv \nu_e$ or $\nu_\mu \to \nu_\tau$, where $\nu_\tau$ is a sterile neutrino) as well as other mechanisms (as $\nu$ decay) are disfavored.

In the future, the K2K experiment will improve its results taking more data and the MINOS [29] long-baseline experiment will measure with better accuracy the mixing parameters in Eq. (3.1). The long-baseline experiment ICARUS [30] and OPERA [31] will be aimed at direct observation of $\nu_\mu \to \nu_\tau$ transitions.

**IV. THREE-NEUTRINO MIXING**

Solar and atmospheric neutrino data can be well fitted in the framework of three-neutrino mixing (see
Ref. [3]) with the hierarchy of squared-mass differences
\[ \Delta m^2 \equiv \Delta m^2_{12} \ll |\Delta m^2_{23}| \simeq \Delta m^2_\lambda, \]  
(4.1)
where \( \Delta m^2_{k,j} \equiv m^2_k - m^2_j \) for \( k, j = 1, 2, 3 \), and \( m_1, m_2, m_3 \) are the three neutrino masses.

Figure 1 shows the two possible types of three-neutrino schemes: a “normal” type (a) and an “inverted” type (b). The absolute scale of these schemes (mystery (M1) above) is not fixed by neutrino oscillation experiments, which can measure only squared-mass differences. The normal three-neutrino scheme allows the natural mass hierarchy
\[ m_1 \ll m_2 \ll m_3, \]  
(4.2)
whereas the inverted scheme allows the so-called “inverted hierarchy” \( m_3 \ll m_2 \simeq m_1 \).

The absolute scale of neutrino mass is bounded from above by the results of Tritium \( \beta \)-decay experiments, which found [32]
\[ m_\nu \ll 2.2 \text{ eV} \quad (95\% \text{ C.L.}). \]  
(4.3)
Since in the case of neutrino mixing \( \nu_e \) does not have a definite mass, but is a superposition of massive neutrinos, the limit in Eq. (4.3) is a bound on the masses of the main massive neutrino components of \( \nu_e \). In the three-neutrino scheme in Fig. 1 all the neutrino masses are constrained by the limit in Eq. (4.3). An independent confirmation of this limit comes from the study of the formation of large-scale structures in the Universe [33, 34]. In the future the KATRIN experiment [35] will probe the effective electron neutrino mass in Tritium decay down to about 0.3 eV. Unfortunately, in the case of the natural scheme in Fig. 1a with the neutrino mass hierarchy (4.2), Eqs. (3.1) and (4.1) imply that \( m_3 \lesssim 7 \times 10^{-2} \text{ eV} \), which is beyond the reach of the KATRIN experiment.

Let us discuss now the current information on the neutrino mixing matrix \( U \). Since the solar neutrino transitions of \( \nu_e \) in \( \nu_x \) or \( \nu_x \) cannot be experimentally distinguished because the energy is well below the threshold for charged-current reactions with production of \( \mu^- \) or \( \tau^- \), solar neutrino oscillations depend only on the elements \( U_{e1}, U_{e2}, U_{e3} \) of the mixing matrix. On the other hand, the hierarchy (4.1) of squared-mass differences in the two possible three-neutrino schemes in Fig. 1 implies that atmospheric (and long-baseline) neutrino oscillations depend only on the elements \( U_{\mu 3}, U_{\tau 3}, U_{\nu 3} \) (see Ref. [3]). Since \( \Delta m^2_\odot = \Delta m^2_{12} \) and \( \Delta m^2_\lambda \simeq |\Delta m^2_{23}| \) are independent, the only quantity that connects solar and atmospheric neutrino oscillations is \( U_{\nu 3} \). Therefore, any information on the value of \( U_{\nu 3} \) is of crucial importance.

From the results of the CHOOZ long-baseline reactor neutrino experiment [36], it is known that the element \( U_{\nu 3} \) of the three-generation neutrino mixing matrix is small [24]:
\[ |U_{\nu 3}|^2 < 5 \times 10^{-2} \quad (99.73\% \text{ C.L.}). \]  
(4.4)
\[ \text{The results of the CHOOZ experiment have been confirmed by the Palo Verde experiment [37], and by the absence of } \nu_e \text{ transitions in the Supernakande atmospheric neutrino data [28].} \]

An important consequence of the smallness of \( U_{\nu 3} \) is the practical decoupling of solar and atmospheric neutrino oscillations [38], which can be analyzed in terms of two-neutrino oscillations with the effective mixing angles \( \theta_S \) and \( \theta_A \) given by
\[ \sin^2 \theta_S = \frac{|U_{\nu 3}|^2}{1 - |U_{\nu 3}|^2}, \quad \sin^2 \theta_A = \frac{|U_{\mu 3}|^2}{1 - |U_{\nu 3}|^2}. \]  
(4.5)
Neglecting a possible small value of \( |U_{\nu 3}|^2 \), the mixing matrix can be written as
\[ U \simeq \begin{pmatrix} c_{\theta_A} & s_{\theta_A} & 0 \\ -s_{\theta_A} c_{\theta_A} & c_{\theta_A} c_{\theta_A} & s_{\theta_A} \\ s_{\theta_A} s_{\theta_A} & -c_{\theta_A} s_{\theta_A} & c_{\theta_A} \end{pmatrix}, \]  
(4.6)
with \( c_{\theta} = \cos \theta \) and \( s_{\theta} = \sin \theta \). In this case, solar neutrino transitions occur from \( \nu_e = c_{\theta} \nu_1 + s_{\theta} \nu_2 \) to the orthogonal state \( -s_{\theta} \nu_1 + c_{\theta} \nu_2 = c_{\theta} \nu_3 + s_{\theta} \nu_\tau \). Hence, the fractions of \( \nu_e \rightarrow \nu_\mu \) and \( \nu_e \rightarrow \nu_\tau \) transitions are determined by the value of the atmospheric effective mixing angle \( \theta_A \).

From the experimental best-fit value \( \theta_A = \pi/4 \) in Eq. (3.2), we see that solar \( \nu_e \) transform approximately into \( (\nu_\mu - \nu_\tau) / \sqrt{2} \), which is an equal superposition of \( \nu_\mu \) and \( \nu_\tau \). Since the suppression factor of solar \( \nu_x \) measured in the charged-current SNO reaction is about 1/3 [12], the flux of \( \nu_\mu \), \( \nu_\tau \) and \( \nu_e \) on Earth is approximately equal. The mixing matrix is approximately given by
\[ U \simeq \begin{pmatrix} c_{\theta_{\text{eff}}} & s_{\theta_{\text{eff}}} & 0 \\ -s_{\theta_{\text{eff}}} / \sqrt{2} & c_{\theta_{\text{eff}}} / \sqrt{2} & 1 / \sqrt{2} \\ s_{\theta_{\text{eff}}} / \sqrt{2} & -c_{\theta_{\text{eff}}} / \sqrt{2} & 1 / \sqrt{2} \end{pmatrix}. \]  
(4.7)
Let us now consider the best fit value in Eq. (2.2) of the effective solar mixing angle, which implies that \( \theta_{\text{eff}} \simeq \pi/6 \), leading to the approximate bijective mixing matrix
\[ U \simeq \begin{pmatrix} \sqrt{3} / 2 & 1 / 2 & 0 \\ -1 / 2 \sqrt{3} & 1 / 2 \sqrt{3} & 1 / 2 \\ 1 / 2 \sqrt{3} & -1 / 2 & 1 / 2 \sqrt{3} \end{pmatrix}. \]  
(4.8)
However, it is widely hoped that \( |U_{\nu 3}|^2 \) is not much smaller than the current upper limit in Eq. (4.4), because a non-vanishing value of \( U_{\nu 3} \) is essential for the
The possibility of measuring CP violation in neutrino oscillations (mystery (M5) above, see Ref. [39]) and for the measurement of the sign of $\Delta m^2_{11}$, which distinguishes the normal and inverted schemes in Fig. 1. The sign of $\Delta m^2_{11}$ can be measured in long-baseline neutrino experiments through matter effects in the Earth, which need the participation to the oscillations of $\nu_e$ through $U_{e3}$ ($\nu_{\mu}$ and $\nu_{\tau}$) have the same neutral-current interaction with matter, whereas $\nu_e$ has also charged-current interactions; neutrino oscillations are affected only by the difference of interactions of different flavors.

The matrix in Eq. (4.8) can be considered as the approximate current best-fit mixing matrix. The limits on $\theta_{23}$, $\theta_{13}$ and $U_{e3}$ in Eqs. (2.1), (3.1) and (4.4), respectively, allow to derive the following allowed intervals for the absolute values of the elements of the mixing matrix (the intervals are correlated, because of unitarity):

$$|U| \approx \begin{pmatrix}
0.71 & -0.90 & 0.43 - 0.69 & 0.00 - 0.22 \\
0.24 & -0.66 & 0.40 - 0.81 & 0.53 - 0.84 \\
0.24 & -0.66 & 0.40 - 0.81 & 0.53 - 0.84
\end{pmatrix},$$

(4.9)

The best known way to investigate the Majorana nature of neutrinos (mystery (M2) above) is the search for neutrinoless double-$\beta$ decay, whose amplitude is proportional to the effective Majorana mass

$$|\langle m \rangle| = |\sum_k r_{ek}^2 m_k|,$$

(4.10)

for which the current experimental upper limit is between 0.3 and 1 eV, taking into account a theoretical uncertainty of about a factor 3 in the calculation of the nuclear matrix element (see Ref. [40]). In general, since the elements $U_{ek}$ of the mixing matrix are complex, cancellations among the contributions of the three massive neutrinos are possible.

The current allowed interval for $|\langle m \rangle|$ in the case of the two three-neutrino mixing schemes in Fig. 1 can be found in Ref. [41]. Here let me consider the case of the natural scheme in Fig. 1a with the neutrino mass hierarchy (4.2). In this case, the limits in Eqs. (2.1) and (3.1) imply that the contribution of $m_1$ to $|\langle m \rangle|$ is negligible and the absolute values of the contributions of $m_2$ and $m_3$ are limited by

$$1.5 \times 10^{-3} \text{eV} \lesssim |U_{e2}^2 m_2| \lesssim 9.5 \times 10^{-3} \text{eV},$$

$$|U_{e3}^2 m_3| \lesssim 3.5 \times 10^{-3} \text{eV}.$$  

(4.11)

(4.12)

Therefore, it is possible that the dominant contribution to $|\langle m \rangle|$ comes from $m_2$, and not from the largest mass $m_3$. If $|U_{e3}^2 m_3| \ll |U_{e2}^2 m_2|$, there are no cancellations in Eq. (4.10) [42] and the effective Majorana mass is expected in the interval in Eq. (4.11). Unfortunately, such values of $|\langle m \rangle|$ are beyond the reach of proposed experiments (see Ref. [40]).

V. CONCLUSIONS

The results of all neutrino experiments, except LSND, are in agreement with the hypothesis of three-neutrino mixing. If future data will confirm this scenario the most important phenomenological task will be to refine the measurement of the neutrino mixing parameters, especially the measurement of $U_{e3}$, whose value is related to the possibility of distinguishing the two schemes in Fig. 1 and of measuring CP violation in neutrino oscillations. The measurements of the absolute scale of neutrino masses and of the effective Majorana mass in neutrinoless double-$\beta$ decay seem to be more difficult tasks (beyond the reach of existing projects if the neutrino masses satisfy the natural hierarchy (4.2)).
[22] T. Matsui, these proceedings.