Baryonic coherent state formation from small domain disoriented chiral condensates

S.M.H. Wong$^a$ * and J.I. Kapusta$^b$ †

$^a$Department of Physics, The Ohio State University, Columbus, Ohio 43210, U.S.A.

$^b$School of Physics and Astronomy, University of Minnesota, Minneapolis, MN 55455, U.S.A.

Rare hyperon yields such as the $\Omega$ and $\bar{\Omega}$ in heavy ion collision experiments are hard to be reproduced by numerical models. This, in combination with the thermal fit to SPS data, seems to call for a new production mechanism beyond the usual ones. Small domain disoriented chiral condensates (DCC) were proposed to be such a source of rare hyperons through skyrmion formation at the chiral phase transition. Here skyrmions are treated as coherent states of baryons on a compact manifold so that the distribution of baryons produced from a skyrmion can be known. From this more refined treatment, the number of topological defects produced are more than doubled to 30 or more and the domain size at the SPS is found to be even smaller than before at 1.1–1.5 fm. *It is imperative therefore not to use only pion distribution but other means for observing DCC.*

1. INTRODUCTION

Ever since the first proposal [1] of using strange hadrons as a probe of the formation of the quark-gluon plasma (QGP) in relativistic heavy ion collisions, the yields of strange hyperons remain a large part of the standard observables for verifying the detection of any enhancement. In this respect, it is also a goal of numerical models to simulate the amount of this yield in order to make sure that we have understood the underlying production mechanisms that may occur in these collisions. Data from the Super Proton Synchrotron (SPS), which came out gradually over a number of years, have allowed theorists to perform exactly this kind of comparison. It turned out that the $\Omega$ and $\bar{\Omega}$ had proved to be the most stubborn and challenging of the hyperons for numerical models. At the same time thermal fits can be done to most yields of the hadrons without too much of a problem with the exception of the $\Omega$ and $\bar{\Omega}$. In [2] it was shown that the fits improved dramatically by an order of magnitude as measured by the value of $\chi^2$ (one can of course fit even the $\Omega$ by lifting the requirement of the best fits should have small $\chi^2$ and allow for arbitrary large $\chi^2$). The general tendency is that there are more $\Omega$ and $\bar{\Omega}$ than expected even after taking into account of QGP formation (in which case thermal model should suffice). This indicates that another production mechanism is at work.

*This work was supported by the U.S. Department of Energy under grant no. DE-FG02-01ER41190.
†This work was supported by the U.S. Department of Energy under grant no. DE-FG02-87ER40328.
2. PRODUCING HYPERONS FROM DISORIENTED CHIRAL CONDENSATES

It has been pointed out in [3] that DCC could produce hyperons. Although they were more readily associated with pions, hyperon production is indeed possible when topological defects are created during the chiral phase transition. The remnants in this case are the so-called skyrmions from the Skyrme Model. Skyrmions are well known to be baryons, hence the connection of hyperons to DCC. To test this possibility against data, one has to have two pieces of information. One is the likelihood of a defect being created in the phase transition, and the other is the relative abundance to one another of all the observable baryons that will be produced from a skyrmion. The first piece of information was obtained in ref. [4]. It tells us that in terms of the correlation length $\xi$, there should be about 0.8 skyrmion and antiskyrmion per correlation volume $\xi^3$. By itself, this is not sufficient for verifying the hypothesis of hyperon production from DCC. In [3] one moved forward regardless by taking an assumption on the second piece of needed information, that is equal production probability of any octet and decuplet baryons from a skyrmion was assumed. From this one can deduce from the SPS data that $\xi \sim 2.0$ fm. This is within the range expected from theoretical considerations [5,6] and is too small for observing DCC using pion distribution [5]. Therefore the possibility is compatible with the existing data and all available information. It is now time for a refined treatment. Improvements can be done in a number of ways but shall be restricted to the removal the assumption of equal likelihood of any type of baryons being produced from a skyrmion.

3. BARYON DISTRIBUTION IN A SKYRMION

In experiments one cannot observe a skyrmion directly, since it is not a single observable hadron. It is essential to know exactly what the skyrmion is in terms of the well known baryons. In [7] it was argued that a skyrmion must necessarily be a coherent state of baryons. In other words it is a superposition of physical baryon states. Being faithful to the original Skyrme model and the set of wavefunctions derived from it using the collective coordinate approach [8], one can construct coherent states using these as ingredients. The baryon wavefunctions of [8] all live on $S^3$. Therefore the coherent states must also live in this compact space. Coherent states on compact manifolds may sound unfamiliar to a heavy ion physicist but not so in the field of quantum gravity and functional analysis. The basic method and related issues are outlined in a series of papers by different authors [9–12]. The method itself is not based explicitly on observable states of the Hamiltonian but rather on the position space of the manifold itself. It so happens that the collective coordinate quantization of the Skyrme model produces both spin one-half physical states and integral spin nonphysical states [8,13]. Therefore one has to be careful that no nonphysical states are actually present in the coherent states that represent the skyrmions.

The method of [9,10] requires the construction of annihilation operators and from there one obtains the coherent states as the simultaneous eigenstates of the operators. The annihilation operators in the case of the Skyrme model are of the form

$$\hat{A}_b = \exp \left( -\frac{1}{2\lambda\omega} J^2 \right) \hat{a}_b \exp \left( \frac{1}{2\lambda\omega} J^2 \right)$$ (1)
where $\lambda$ is a constant in the Skyrme Hamiltonian, $\omega$ is a fundamental energy scale of the problem, $\hat{a}_b$ ($b = 0, 1, 2, 3$) are the position operators on the compact manifold of $S^3$ and $\hat{J}_i$ are the spin angular momentum operators. One can easily see that the eigenstates must be given by

$$|\psi, a\rangle = \exp\left(-\frac{1}{2\lambda\omega}\hat{j}^2\right)|a\rangle$$

with the eigenvalues given by the position label $a$. This is, however, not sufficient because the real $a$ does not contain enough information to label a coherent state. It must carry information of both position and momentum. To qualify it has first to be analytically continued to complex $a^c$. One now has

$$|\psi, a^c\rangle = \exp\left(-\frac{1}{2\lambda\omega}\hat{j}^2\right)|a^c\rangle.$$  

(3)

To check that it has the required physical properties to be a coherent state of baryons, a complete set of spin and isospin states of the Skyrme Hamiltonian is inserted

$$|\psi, a^c\rangle = \sum_{j, m, n \in \mathbb{Z}/2} \exp\left(-\frac{1}{2\lambda\omega}\hat{j}^2\right)|j, m, n\rangle\langle j, m, n|a^c\rangle.$$  

(4)

As already mentioned the sum over states includes both integral and half-integral (iso)spin states which naturally presents a problem. The simplest solution seems to be to drop all the integral spin states from Eq. (4) but that apparently does not solve the problem. The Skyrme model is usually quantized using the $SU(2)$ collective coordinates [8], hence the coordinates operators are $\hat{a}_b$. The algebra of $SU(2)$ dictates that for example $[\hat{J}_3, \hat{a}] \sim [\hat{I}_3, \hat{a}] \sim \frac{1}{2}\hat{a}$, so $\hat{a}$ acting on $|j, m, n\rangle$ will change $(j, m, n)$ from half-integral numbers to integral numbers. Therefore a pure fermion version of Eq. (4) cannot stay within the fermionic space forever. Physical and unphysical states will mix.

A solution to this is to first map from $SU(2)$ to $SO(3)$ [7,14] using $A^i A^i = \tau_j R^i_j$ where $A = a_0 + i\tau_1 a_1$. The resulting nine $SO(3)$ coordinate operators $\hat{R}_{ij}$ keeps $|j, m, n\rangle$ a fermion state if it is one to begin with, because of the algebra $[\hat{J}_3, \hat{R}] \sim [\hat{I}_3, \hat{R}] \sim \hat{R}$. The solution is then to use the $SO(3)$ operators in combination with the original $SU(2)$ baryon states of [8]. The annihilation operators become

$$\hat{A}_{ij} = \exp\left(-\frac{1}{2\lambda\omega}\hat{j}^2\right) \hat{R}_{ij} \exp\left(\frac{1}{2\lambda\omega}\hat{j}^2\right).$$

(5)

So finally one can equate

$$|\psi, a^c\rangle = \sum_{j, m, n \in \mathbb{Z}/2} \exp\left(-\frac{1}{2\lambda\omega}\hat{j}^2\right)|j, m, n\rangle\langle j, m, n|a^c\rangle \equiv |S\rangle$$

(6)

with a skyrmion. The probability of the skyrmion being in any of the baryon states $|j, m, n\rangle$ is given by the modulus squared of the coefficient divided by the normalization.

Clearly that the assumption made in [3] of equal probability for any of the baryon states to emerge from a skyrmion isn’t exactly correct. Applying the above result at low energy
appropriate to DCC, using the octet and decuplet baryon mass difference and setting the fundamental energy scale at $\omega \sim T_c$ since this is the only scale available, one finds that the instead of 1:1, the octet to decuplet baryon ratio is between 5:1 to 14:1 depending on the exact value of $T_c$ used (150–200 MeV). Now proceeding again as in [3] but with the use of this new found information, one deduces that the domain size is now even smaller but not unreasonably small at $\xi \sim 1.1$–1.5 fm. Because both the SPS data and the probability of skyrmion formation per unit correlation volume do not change, there must be more skyrmions to compensate for this reduction in domain size. This increase is at least double that from the previous estimate of 14 to 30–65.

In conclusion even with the refined treatment of the skyrmions, this mechanism for the production of excess hyperons is still compatible with all the existing SPS data and the theoretical expectation on the small size of the DCC domains. In [5] it was mentioned that if the system was in equilibrium the domain size would be too small for the observation of DCC through pions. This is assuming that one is using pion distribution as the basis of observation. The mechanism discussed here does not rely on pions and the small size of the domains will not preclude them from being observed. In fact they thrive on small $\xi$ [3,4]. Details of the work done here will be reported elsewhere [15]. Another way to observe DCC also discussed in this conference [16], when the domain size is small, is by using pion/kaon fluctuations proposed in [17]. Although any attempts so far at detecting DCC have been unsuccessful, they are all based on the premise that the domain size has to be large. It is definitely about time that this old strategy be set aside and new ones be further developed to take into account of the possibility that DCC are formed only in small domains.

REFERENCES

15. S.M.H. Wong, work in progress.
16. S. Gavin’s talk in these proceedings.