Heavy to light vector meson semileptonic decays * †

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New (preliminary) results for the form factors relevant for the semileptonic decays of heavy pseudoscalar to a light vector meson are presented. In particular, we discuss the form factors for $D \to K^*$ and $B \to \rho$ modes.

The main source of uncertainty in the extraction of the CKM matrix elements from the simple decay processes is our incomplete knowledge of the non-perturbative dynamics that is necessary to compute the relevant hadronic matrix elements. In particular, to extract $|V_{cs}|$ and $|V_{ub}|$ from experimentally measured $D \to K^* \ell \nu$ and $B \to \rho \ell \nu$ decay rates requires a reliable QCD based computation of the following matrix elements:

$$\langle V(p', \varepsilon) | q \gamma_\mu Q | H(p) \rangle = i\epsilon_{\mu\nu\alpha\beta} \varepsilon^{* \nu} p^\alpha p^\beta \frac{2V(q^2)}{m_H + m_V},$$

$$\langle V(p', \varepsilon) | q \gamma_\mu \gamma_5 Q | H(p) \rangle = \frac{2m_V (\varepsilon^{* \mu} q) A_0(q^2) q_\mu}{q^2} + (m_H + m_V) A_1(q^2) \left( \varepsilon^{* \mu} q - \frac{\varepsilon^{* \mu} q}{q^2} q_\mu \right) - A_2(q^2) \frac{\varepsilon^{* \mu} q}{m_H + m_V} \left[ (p + p')_\mu - \frac{m_H^2 - m_V^2}{q^2} q_\mu \right],$$

where we consider a generic $H \to V \ell \nu$ decay and use the standard decomposition in terms of four Lorentz invariant form factors, $V, A_{1,2,0}$, which depend on $q^2 = (p - p')^2$. At $q^2 = 0$, the axial form factors satisfy

$$2m_V A_0(0) = (m_H + m_V) A_1(0) - (m_H - m_V) A_2(0).$$

We compute the above matrix elements on the lattice using the complete $\mathcal{O}(a)$ non-perturbatively improved Wilson quark action and operators, working in the quenched approximation. We generated two sets of independent gauge field configurations: 200 on $24^3 \times 64$ lattice at $\beta = 6.2$ ($a^{-1} = 2.7(1)$ GeV), and 100 on $32^3 \times 70$ at $\beta = 6.45$ ($a^{-1} = 3.7(1)$ GeV).

We compute the following two- and three-point correlation functions:

$$C_V^{(2)}(t) = \left( \sum_x e^{i(p'-\bar{q}) \cdot \bar{x}} \langle \bar{q} \gamma_\mu q \rangle_0 (\bar{q} \gamma_\mu q)_{\bar{x},t} \right),$$

$$C_H^{(2)}(t) = \left( \sum_x e^{i\bar{p} \cdot \bar{x}} \langle \bar{Q} \gamma_5 q \rangle_0 (\bar{Q} \gamma_5 q)_{\bar{x},t} \right),$$

$$C_{\mu_0}^{(3)}(t) = \left( \sum_{\bar{x},y} e^{i\bar{p} \cdot \bar{x}} - i \bar{q} \cdot \bar{y} \langle \bar{q} \gamma_\mu Q \rangle_{\bar{x},t,F} (\bar{q} \gamma_\mu Q)_{\bar{y},t} \langle \bar{q} \gamma_\alpha q \rangle_0 \right).$$

At $\beta = 6.2$ we have 3 light ($q$) and 4 heavy ($Q$) quark masses, whereas at $\beta = 6.45$ we work with 4 light and 6 heavy quarks. The directly simulated vector mesons lie in the range...
light quark mass (linearly) to zero, their masses are no large discretisation artefacts, when

$$m_V \in (0.9, 1.1) \text{ GeV},$$

which means that the $K^*$-meson is within the grasp of our lattice study, while for the $\rho$-meson an extrapolation is needed. As for the heavy-light mesons, after sending the light quark mass (linearly) to zero, their masses at $\beta = 6.2$ are $m_{H_d} \in (1.7, 2.6) \text{ GeV}$. On the finer lattice ($\beta = 6.45$) that interval extends to $m_{H_d} \in (1.7, 3.6) \text{ GeV}$.

In other words, the charm sector is simulated directly, while the beauty can be reached through an extrapolation (a normal feature of current lattice studies in which fully relativistic heavy quarks are used). The form factors are extracted from the ratios

$$R_{\mu \nu}(t) = \frac{C^{(3)}_{\mu \nu}(t)\sqrt{Z_V}Z_H}{C^{(2)}_{V}(t)C^{(2)}_{H}(t_F - t)} |V(p')\bar{q}H(Q)| \cdot \frac{1}{V(p)\bar{q}H(Q)|H(p)} .$$

To study the functional dependence of the form factors on $q^2$, we also consider 7 different combinations of three-momenta for the interacting hadrons (for more details, please see ref. [1]). Discrete symmetries have been used to average over the equivalent momentum configurations. A suitable kinematical situation for a comparison of the lattice data at two values of the lattice spacing is when both mesons are at rest (i.e. at $q^2 = q^2_{\text{max}}$), because only the form factor $A_1$ contributes. In fig. 1 we plot the signal for $A_1(q^2_{\text{max}})$ as extracted from the ratio (2) for both of our lattice spacings and for almost the same masses of mesons (in physical units). From this exercise we see that there are no large discretisation artefacts, when

$$|\vec{p}^*| = |\vec{p}| = 0.$$  

For each fixed heavy quark mass and combination of momenta $\vec{q}$ and $\vec{p}$, the leading dependence of each $F \in (V, A_1, A_2, A_0)$ on the light final vector meson mass is expected and seen to be linear, i.e.

$$F(m_V) = \alpha + \beta m_V ,$$

Fitting to this give the form factors for the transitions $H \rightarrow K^*$ and $H \rightarrow \rho$. The discussion of the dependence on the heavy quark (meson) mass is tightly related to the $q^2$-shapes of the form factors. We chose to fit our (directly computed) form factors to the pole/dipole forms

$$V(q^2) = \frac{V(0)}{(1 - q^2/M_V^2)^2} , \quad A_1(q^2) = \frac{A_1(0)}{(1 - q^2/M_V^2)^2},$$

$$A_2(q^2) = \frac{A_2(0)}{(1 - q^2/M^2)^2} , \quad A_0(q^2) = \frac{A_0(0)}{(1 - q^2/M^2)^2},$$

additionally constrained by the condition (1). The pole/dipole forms (3) reconcile the $t$-channel pole dominance with the HQET scaling laws, according to which (for small recoil momenta) the form factor $A_1 (V, A_{2,0})$ multiplied by $m_H^{-1/2}$ ($m_H^{-1/2}$) scales as a constant, up to $1/m_H$ corrections [2]. In addition, the forms (3) are consistent with the "$m_{H}^{−3/2}$" scaling law arising in the limit in which the light meson is very energetic (LEL) [3].

With the $H \rightarrow K^*$ and $H \rightarrow \rho$ form factors fitted to the pole/dipole forms (3) we can interpolate in the inverse heavy meson mass to reach $m_H = m_D$, and extrapolate to $m_H = m_B$. To that end, we use the HQET scaling laws, and for a fixed value of $v \cdot p' = (m_H^2 + m_K^2 - q^2)/(2m_H)$, we fit our data to

$$F(v \cdot p')m_H^{d/2} = a + b/m_H + c/m_H^2 .$$

where $d = +1$ for $F = A_1$, and $d = -1$ otherwise. The difference between this form and the linear one ($c = 0$) is used to estimate the systematic error (as in ref. [4]). Such a difference is completely negligible in the case of $D$-meson because $m_D$ is very close to the lightest of the heavy-light mesons directly simulated on the lattice. $D \rightarrow K^*$ transition form factors are shown.

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**Figure 1.** Plateau of the form factor $A_1$, the function of the time $t$, between two source operators (fixed at 0 and $t_F$, where $t_F = 27$ at $\beta = 6.2$ and $t_F = 34$ at $\beta = 6.45$). Illustration provided for $q^2_{\text{max}} = (m_H - m_V)^2$, with $m_H \simeq 1.8 \text{ GeV}$ and $m_V \simeq 1 \text{ GeV}$. 

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in fig. 2, and the result of the fit to the forms (3) is given in tab. 1. We also remind the reader that $A_0$ does not enter the expression for the decay rate (see eq. [5]). $A_2$, instead, enters the part describing the longitudinally polarised vector meson. Since the quality of our signals for $A_2$ is low (much worse than for $A_0$), we use the exact relation (1) to compute $A_2(0)$. From the results of tab. 1, for the integrated decay rate, we obtain

$$|V_{cs}|^{-2} \Gamma(D^- \rightarrow K^0 \ell \nu) = 0.066(14) \ p_{\beta=6.2}^{-1} \ 0.062(15) \ p_{\beta=6.45}^{-1},$$

which after comparison to the recently measured branching ratio [7] lead to $|V_{cs}| = 0.99(9)$ and $|V_{cs}| = 1.03(12)$, respectively.

An additional comparison with the experimental data is provided for the ratios of the form factors at $q^2 = 0$. Our results

$$\frac{V}{A_1} = 1.48(12)_{\beta=6.2}, 1.46(11)_{\beta=6.45},$$

$$\frac{A_2}{A_1} = 0.6(3)_{\beta=6.2}, 1.0(2)_{\beta=6.45},$$

agree very well with $(V/A_1)^{exp.} = 1.50(7)$ [6]. The agreement with $(A_2/A_1)^{exp.} = 0.88(9)$ [6] is only marginal, as discussed above.

We next discuss the $B \rightarrow \rho$ form factors. The results of (quadratic) extrapolation (4), for each fixed $(v \cdot p')$, are shown in fig. 3. We observe the standard effect that after the heavy quark extrapolation the form factors fall into the region $q^2 > 10 \text{ GeV}^2$. As compared to the benchmark calculation by UKQCD [8], our results have larger errors (in spite of the fact that our statistics is higher). In particular, the errors for our $A_2^{B \rightarrow \rho}(q^2)$ are of $O(100\%)$.

To compute the decay rate we have to integrate over the entire phase space. Therefore we combine the lattice results for $(d\Gamma/dq^2)_{q^2>10\text{ GeV}^2}$ with lightcone sumrule results for $(d\Gamma/dq^2)_{q^2<10\text{ GeV}^2}$ [5] (which are expected to

Table 1

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$F(0)$</th>
<th>$M_F(\text{GeV})$</th>
<th>$\beta = 6.2$</th>
<th>$M_F(\text{GeV})$</th>
<th>$\beta = 6.45$</th>
<th>$M_F(\text{GeV})$</th>
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<tr>
<td>$V$</td>
<td>0.91(10)</td>
<td>2.3(3)</td>
<td>0.90(11)</td>
<td>2.0(2)</td>
<td></td>
<td></td>
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<tr>
<td>$A_1$</td>
<td>0.62(5)</td>
<td>2.6(5)</td>
<td>0.61(6)</td>
<td>2.2(3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_2$</td>
<td>0.37(14)</td>
<td>3(23)</td>
<td>0.64(14)</td>
<td>2(1)</td>
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<td></td>
</tr>
</tbody>
</table>

Figure 3. $B \rightarrow \rho$ form factors on our lattices are accessed for $q^2 > 10 \text{ GeV}^2$. For the decay rate they are combined with the lightcone sumrule (LCSR) results in the $q^2 < 10 \text{ GeV}^2$ region (which are also shown in the figure).
be reliable for low values of $q^2$).

To obtain $(dI/dq^2)|_{q^2>10\text{GeV}^2}$, we use the pole/dipole forms (3), eliminating $F(0)$ in favour of one of our two $F(q_0^2)$. We choose $q_0^2 = 14.6\text{GeV}^2$ in the middle of the region covered by our results and fit to

$$
F(q^2) = \frac{1 - q^2/M_B^2}{1 - q^2/M_F^2},
$$

where $p = 1$ for the case $F = A_1$, and $p = 2$ otherwise. Results of this single parameter interpolation procedure are listed in tab. 2. Notice that we neglect the slope of $A_2$, for which a flat $q^2$-form with $O(100\%)$ of error on the central value should be conservative enough. We finally obtain

$$
|V_{ub}|^{-2}\Gamma(B^0 \rightarrow \rho^+ \ell \nu) = (17 \pm 3) \frac{ps^{-1}_{[\beta=6.2]}}{[\beta=6.45]},
$$

which we then match with the experimental branching ratio (as measured by CLEO, BaBar and Belle [9]) to extract $|V_{ub}|$. We find

$$
|V_{ub}| = 0.0034(6),
$$

where we added all the errors in quadrature.

As a final exercise, we check the relation among form factors which holds true in the LEL (for a recent discussion see refs. [3,10]), namely

$$
\frac{A_1(q^2)}{V(q^2)} = \frac{2E_p m_B}{(m_B + m_\rho)^2},
$$

This relation is verified in the LCSR approach (note that $E_p \simeq m_B/2$ for $q^2 \approx 0$ in the $B$-meson rest frame). In fig. 4, we plot the ratio of our $B \rightarrow \rho$ form factors (computed on the lattice), and compare them to the r.h.s. of eq. (6). Interestingly, we do not see deviations from that relation (6) in spite of the fact that the lattice results are produced at large $q^2$.

### Table 2

<table>
<thead>
<tr>
<th>$F$</th>
<th>$F(q_0^2)$</th>
<th>$M_F$[GeV]</th>
<th>$F(q_0^2)$</th>
<th>$M_F$[GeV]</th>
</tr>
</thead>
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<tr>
<td>$V$</td>
<td>0.84(26)</td>
<td>5.4(5)</td>
<td>0.93(31)</td>
<td>5.2(4)</td>
</tr>
<tr>
<td>$A_1$</td>
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<td>5.9(1.2)</td>
<td>0.46(9)</td>
<td>5.3(4)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0.7(7)</td>
<td>–</td>
<td>0.9(7)</td>
<td>–</td>
</tr>
</tbody>
</table>

### REFERENCES

1. A. Abada et al. [SPQcdR], in preparation.
6. J. M. Link et al. [FOCUS], [hep-ex/0207049].
7. G. Brandenburg et al. [CLEO], [hep-ex/0203030].