Energy and momentum losses in the process of neutrino scattering on plasma electrons with the presence of a magnetic field.

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Abstract

The neutrino-electron scattering in a dense degenerate magnetized plasma under the conditions $\mu^2 > 2eB \gg \mu E$ is investigated. The volume density of the neutrino energy and momentum losses due to this process are calculated. The results we have obtained demonstrate that plasma in the presence of an external magnetic field is more transparent for neutrino than non-magnetized plasma. It is shown that neutrino scattering under conditions considered does not lead to the neutrino force acting on plasma.

Key words: neutrino-electron processes, plasma, magnetic field, supernova envelope

PACS numbers: 13.15.+g, 95.30.Cq, 97.60.Bw

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1 Introduction.

It is known that the neutrino physics plays an unique role in astrophysics and cosmology. In particular, these light weakly interacting particles are special for astrophysical phenomena like a supernova explosion when a large number of neutrinos is produced in a collapsing stellar core (Raffelt 1996). The compact core with the typical radius $R \sim 10$ km, the supranuclear density $\rho \sim 10^{14}$ g/cm$^3$ and the high temperature $T \sim 30$ MeV, is opaque for neutrinos. While a rather rarified remnant envelope with the typical density $\rho \sim 10^{10} - 10^{12}$ g/cm$^3$ and temperature of the order of few MeV, becomes partially transparent for the neutrino flux.

Notice that in investigations of neutrino processes in medium not only dense substance, but also a magnetic field should be taken into account. We stress that a magnetic field can play the role of additional component of an active medium, and influence substantially on particle properties. This influence becomes especially important in the case when the magnetic field strength reaches the critical, Schwinger value $B_e = m_e^2/e = 4.1 \times 10^{14}$ G. 1

According to modern astrophysical models, very strong magnetic fields up to $10^{17}$ G could be generated, for example, in a rapidly rotating supernova remnant (Duncan and Thompson 1992; Bisnovatyi-Kogan 1993; Mathews et.al. 1997).

Previously, in the studies of neutrino interactions with a dense stellar medium the main attention was given to the neutrino – nucleon processes. This is due to the fact that the Urca-processes and the neutrino - nucleon scattering defined the major contribution into the energy balance of the collapsing core, and were considered as a main source of neutrino opacity. The neutrino - electron processes were less investigated. However, as it was pointed in studies Mezzacappa and Bruenn (1993), taking account of the neutrino - electron scattering in a detail analysis of the supernova dynamics is physically justified indeed. In particular, the neutrino - electron processes can contribute significantly into the asymmetry and provide a competition with the neutrino-nucleon processes. For example, in the paper Kuznetsov and Mikheev (2000) the total set of neutrino-electron processes ($\nu e^\pm \leftrightarrow \nu e^\pm$, $\nu e^- e^+ \leftrightarrow \nu$, $e^\pm \leftrightarrow \nu \bar{\nu} e^\pm$) was investigated in a strong magnetic field limit, when electrons and positrons occupied the lowest Landau level. It was shown that the neutrino force action on plasma along the magnetic field turns out

\[ \text{We use natural units in which } c = \hbar = 1, e > 0 \text{ is the elementary charge.} \]
to be of the same order and, what is essential, of the same sign as the one caused by the $\beta$-processes (Gvozdev and Ognev 1999).

By this means, the investigations of the neutrino-electron processes under extreme conditions of a high density and/or temperature of matter and also of a strong magnetic field are the subject of a great interest.

In this study we investigate the neutrino – electron processes in a dense magnetized plasma. In contrast to (Kuznetsov and Mikheev 2000) we consider the physical situation when the magnetic field is not so strong, whereas the density of plasma is large. Thus the chemical potential of electrons, $\mu$, is the dominating factor:

$$\mu^2 > 2eB \gg T^2, E^2 \gg m_e^2, \quad (1)$$

where $T$ is the plasma temperature, $E$ is the typical neutrino energy. Under the conditions (1) plasma electrons occupy the excited Landau levels. At the same time it is assumed that the magnetic field strength being relatively weak, (1), is simultaneously strong enough, so that the following condition is satisfied:

$$eB \gg \mu E. \quad (2)$$

In the present astrophysical view, the conditions (1), (2) could be realized, as an example, in a supernova envelope, where the electron chemical potential is assumed to be $\mu \sim 15$ MeV, plasma temperature $T \sim 3$ MeV. The magnetic field could be as high as $10^{15} - 10^{16}$ G. Under the conditions considered the approximation of ultrarelativistic plasma is a good one, so we will neglect the electron mass wherever this causes no complications.

As it was shown in paper Mikheev and Narynskaya (2000), under the conditions (1), (2) the total set of neutrino - electron processes reduces to the process of neutrino scattering on plasma electrons. Moreover, both initial and final electrons occupy the same Landau level.

The neutrino – electron scattering in dense magnetized plasma was investigated by Bezchastnov and Haensel (1996). Numerical calculations of the differential cross – section of this process in the limit of a weak magnetic field ($eB < \mu E$) were performed. The purpose of our work is to calculate analytically not only the probability of this process, but also the volume density of the neutrino energy and momentum losses under the conditions (1),(2).
2 Probability of the neutrino-electron scattering.

We start from the effective local Lagrangian of the neutrino – electron interaction in the framework of the Standard Model:

\[ L_{\text{eff}} = \frac{G_F}{\sqrt{2}} \bar{\nu}_\alpha (c_v - c_a \gamma_5) e j^\alpha, \]  

(3)

where \( j^\alpha = \bar{\nu}_\gamma \alpha (1 - \gamma_5) \nu \) is the current of massless left neutrinos, \( c_v = \pm 1/2 \pm 2\sin^2\theta_W \), \( c_a = \pm 1/2 \). Here upper signs correspond to the electron neutrino \( (\nu = \nu_e) \) when both Z and W boson exchange takes part in a process. The lower signs correspond to \( \mu \) and \( \tau \) neutrino \( (\nu = \nu_\mu, \nu_\tau) \), when the Z boson exchange is only presented in the Lagrangian (3).

In order to impart a physical meaning to the probability of the neutrino - electron scattering per unit time, it is necessary to integrate not only over the final but also over the initial electron states as well:

\[ W(\nu_e^- \to \nu_e^-) = \sum_{n=0}^{n_{\text{max}}} \frac{1}{T} \int \sum_{s,s'} | S |^2 \ dn_{e^-} \ dn'_{e^-} \frac{d^3 k'}{(2\pi)^3} V (1 - f(E')). \]  

(4)

Here \( n_{\text{max}} \) corresponds to the maximal possible Landau level number, which is defined as the integer part of the ratio \( \mu^2 / 2eB \geq 1 \), \( T \) is the total interaction time, \( | S |^2 \) is the S-matrix element squared of the process considered, \( V \) is the normalization volume, \( f(E') \) is a distribution function of final neutrinos, \( f(E') = [e^{(E' - \mu_\nu) / T_\nu} + 1]^{-1} \), \( E' \) is the final neutrino energy, \( \mu_\nu \) and \( T_\nu \) are the effective chemical potential and the spectral temperature of the neutrino gas correspondingly. In a general case the neutrino spectral temperature \( T_\nu \) can differ from the plasma temperature \( T \) (we do not assume the equilibrium between neutrino gas and plasma). The phase-space elements of the initial and final plasma electrons in the presence of a magnetic field are defined by the following way: \(^2\)

\[ dn_{e^-} = \frac{dp_y dp_z}{(2\pi)^2} L_y L_z f(\varepsilon_n), \quad dn'_{e^-} = \frac{dp'_y dp'_z}{(2\pi)^2} L_y L_z (1 - f(\varepsilon'_n)), \]

where \( p_z \) is the electron momentum along the magnetic field, \( p_y \) is the generalized momentum which defines the position of the center of a Gaussian

\(^2\)we use the gauge \( A^\nu = (0, 0, Bx, 0) \), the magnetic field is directed along the z axis.
packet along the $x$ axis, $x_0 = -p_y/eB$, while $\varepsilon_n \simeq \sqrt{p_x^2 + 2eBn}$ is the energy of an ultrarelativistic plasma electron occupying the $n$-th Landau level, $f(\varepsilon_n)$ is a distribution function of electrons, $f(\varepsilon_n) = [e^{(\varepsilon_n-\mu)/T} + 1]^{-1}$.

The details of integration over the phase space of particles had been published in our previous paper (Mikheev and Narynskaya 2000). The result of calculation of the probability (4) can be presented in the relatively simple form:

$$W(\nu_e \rightarrow \nu_e^-) = \frac{G_F^2 (c_v^2 + c_a^2) eB T^2 E}{4 \pi^3} \sum_{n=0}^{n_{\text{max}}} \frac{1}{z^2} \times$$

$$\left\{ \left[ ((1 + z^2)(1 + u^2) - 4uz) \int_{-a}^{b} \Phi(\xi) d\xi \right]ight. +$$

$$\left. \left[ \frac{1}{zt\tau}(z^2 - 1)(z - u) \int_{-a}^{b} \xi \frac{\Phi(\xi)}{\xi^2} d\xi \right] \right\} + (u \rightarrow -u),$$

where $z = \sqrt{1 - 2eBn/\mu^2}$, $\Phi(\xi) = \xi[(e^{\xi} - 1)(e^{\eta}/\tau - \xi/\tau + 1)]^{-1}$, $a = r\tau z(1 + u)/(1 + z)$ and $b = r\tau z(1 - u)/(1 - z)$, $r = E/T_\nu$, $\tau = T_\nu/T$, $\eta_\nu = \mu_\nu/T_\nu$, $u = \cos\theta$, $\theta$ is the angle between the initial neutrino momentum $\vec{k}$ and the magnetic field direction. The variable $\xi$ defines the spectrum of the probability (5) on the final neutrino energy, $\xi = (E' - E)/T$.

In the limit of a very dense plasma ($\mu^2 \gg eB$), when a great number of Landau levels are occupied by plasma electrons, one can change the summation over $n$ by integration over $z$:

$$[\mu^2/2eB] \sum_{n=0}^{n_{\text{max}}} F(z) \simeq \frac{\mu^2}{eB} \int_{0}^{1} F(z) \frac{dz}{z}.$$ 

In this case the contribution from the lowest Landau levels turns out to be negligibly small, so the main contribution into the probability arises from the highest Landau levels. In this limit the probability (5) can be rewritten in the following form:

$$W(\nu_e \rightarrow \nu_e^-) = \frac{G_F^2 (c_v^2 + c_a^2) \mu^2 T^2 E}{4 \pi^3} \left[ \int_{0}^{1} \frac{dz}{z} \right]$$

$$\times \left\{ \left[ ((1 + z^2)(1 + u^2) - 4uz) \int_{-a}^{b} \Phi(\xi) d\xi \right] + (u \rightarrow -u) \right\}$$

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$$\times \left\{ \left[ ((1 + z^2)(1 + u^2) - 4uz) \int_{-a}^{b} \Phi(\xi) d\xi \right] + (u \rightarrow -u) \right\}$$

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As one can see, the probability (6) does not depend on the value of the magnetic field strength, but is not isotropic. The dependence on the angle $\theta$ manifests this anisotropy of the neutrino-electron process in the presence of a magnetic field. In the limit of a rare neutrino gas when $f(E') \ll 1$, the result has a more simple form:

$$W(\nu e^{-} \rightarrow \nu e^{-}) \approx \frac{G_F^2}{15\pi^3} \left( \frac{c_v^2 + c_a^2}{\mu^2} \right) E^3 I(u),$$

(7)

$$I(u) = \int_0^1 \frac{z d\xi}{(1 + z)^2} \left( u^4 (3z^2 + 2z + 1) - 12u^2 z + z^2 + 2z + 3 \right).$$

For comparison we present here the probability of the neutrino-electron scattering without field in the same limit of the rare neutrino gas:

$$W_{vac} = \frac{G_F^2}{15\pi^3} \left( \frac{c_v^2 + c_a^2}{\mu^2} \right) E^3.$$

(8)

The numerical estimation of the ratio of the probabilities (7) and (8) is presented in Fig.1. It is seen that the probability in a magnetized plasma exceeds the vacuum probability in the vicinity of a point $\theta = \pi/2$ only.

3 Integral neutrino action on a magnetized plasma.

In this section we will calculate the volume density of neutrino energy and momentum losses per unit time in a medium, which could be defined by the following way:

$$\langle \dot{\epsilon}, \vec{F} \rangle = \frac{1}{(2\pi)^3} \int \frac{(q_0, \vec{q}) d^3k}{e^{(E_{\nu} - \mu_\nu)/T_{\nu}} + 1} dW,$$

(9)

where $q_\alpha$ is the difference between the momenta of the initial and final neutrinos, $q_\alpha = k_\alpha - k'_\alpha$. The zeroth component, $\dot{\epsilon}$, determines the neutrino energy loss in unit volume per unit time. In general, a neutrino propagating
Figure 1: The relative probability of the neutrino-electron scattering in a magnetized plasma as a function of the angle between the initial neutrino momentum and the magnetic field direction. $W_{\text{vac}}$ is the probability in a non-magnetized plasma.

through plasma can both lose and capture energy. So, we will mean the "loss" of energy in the algebraic sense.

The vector $\vec{F}$ in Eq. (9) is associated with the volume density of the neutrino momentum loss in unit time, and therefore it defines the neutrino force acting on plasma. Because of the isotropy of plasma without a magnetic field, in the presence of a magnetic field one would expect to obtain the neutrino force action along the magnetic field only. However, as it was shown above, the probability of the neutrino-electron scattering (6) is the symmetric functions with respect to the substitution $u \rightarrow -u$ (or $\theta \rightarrow \pi - \theta$). This means that the neutrino scattering on excited electrons does not give a contribution into the neutrino force acting on plasma along the magnetic field. Thus, under the conditions (1), (2) there is no neutrino force action on plasma at all. Therefore, this force is caused by a contribution of neutrino interactions with ground Landau level electrons only, and the result obtained by Kuznetsov and Mikheev (2000) has a more general applicability in fact. It may be used even in the limit of dense plasma when chemical potential is considerably greater than the magnetic field strength ($\mu^2 \gg eB$).

For the neutrino energy loss in unit volume per unit time in the limit of
a very dense plasma we obtain the following result:

$$\dot{\varepsilon}_B = \frac{G_F^2 (c_v^2 + c_a^2)}{\pi^3} \mu^2 T^4 n_\nu J_B(\tau),$$  \hspace{1cm} (10)

$$J_B(\tau) = \frac{\tau^4}{2} \int_{0}^{1} \frac{dz}{z^2} \int_{0}^{\infty} dy \ y^2 \left[ y (1 - z^2) + 4 z (1 + z^2) \right]$$

$$\times \frac{1 - e^{y(1-\tau)}}{1 - e^{-y\tau}} e^{-y(1+z)/2z},$$  \hspace{1cm} (11)

where \( n_\nu \) is the concentration of initial neutrinos, the parameter \( \tau \) has a meaning of a relative neutrino spectral temperature, \( \tau = T_\nu/T \). It is interesting to compare this result with the one in a non-magnetized plasma which can be presented in a similar form:

$$\dot{\varepsilon}_{B=0} = \frac{G_F^2 (c_v^2 + c_a^2)}{\pi^3} \mu^2 T^4 n_\nu J_{B=0}(\tau),$$  \hspace{1cm} (12)

$$J_{B=0}(\tau) = 4\tau^4 \int_{0}^{\infty} d\xi \ \xi^2 \frac{e^{\xi(\tau-1)} - 1}{e^{\xi\tau} - 1}.$$  \hspace{1cm} (13)

The functions \( J_B(\tau) \) and \( J_{B=0}(\tau) \) define the dependence of the neutrino energy losses on the relative neutrino spectral temperature in a magnetized plasma and in a plasma without field correspondingly. In the limit of a sufficiently large neutrino spectral temperature \( (e^\tau \gg 1) \) they reduce to the power functions:

\[ J_B(\tau) \simeq 4.35 \ \tau^4, \quad J_{B=0}(\tau) \simeq 8 \ \tau^4. \]

The graphs of the functions (11) and (13) are presented in Fig.2. As one would expect, at neutrino spectral temperature smaller than the plasma one (\( \tau < 1 \)) the values of the functions \( J_B(\tau) \) and \( J_{B=0}(\tau) \) are negative. It implies that neutrino propagating via medium captures energy from the plasma. At the \( T_\nu \) more then \( T (\tau > 1) \), neutrino gives up energy to the plasma. At the point \( \tau = 1 \) there is a thermal equilibrium when there is no energy exchange between neutrino and electron-positron plasma. It can be seen that the neutrino energy loss in a magnetized plasma is less than the one in non-magnetized plasma. By this means, under the conditions (1), (2) the magnetized plasma becomes more transparent for neutrinos than plasma without field.
Figure 2: The functions $J_B(\tau)$ (solid line) and $J_{B=0}(\tau)$ (dashed line) versus the relative spectral neutrino temperature.

4 Conclusions.

In this paper we have investigated the neutrino-electron scattering in a dense magnetized plasma. We have considered the physical situation when the plasma component is the dominating one of the two components of the active medium. At the same time, a magnetic field was assumed to be not too small ($\mu^2 > 2 eB \gg \mu E$). The probability and the volume density of the neutrino energy-momentum losses have been calculated.

It is found, that the neutrino scattering on excited electrons does not give a contribution into the neutrino force acting on plasma. This force is caused by the neutrino-electron processes when plasma electrons occupy the lowest Landau level only. Thus the result for this force, obtained in paper Kuznetsov and Mikheev (2000), has a more wide area of application. It may be used under a condition $\mu^2 \gg eB$ as well.

It is shown that under the conditions (1), (2) the combine effect of plasma and strong magnetic field leads to a decrease of the neutrino energy loss in comparison to the one in a pure plasma. Therefore, the complex medium, plasma + strong magnetic field, is more transparent for neutrinos than non-magnetized plasma.

One would believe that the result obtained will be useful for the detail
analysis of astrophysical cataclysms like supernova explosions.

Acknowledgments

This work was supported in part by the Russian Foundation for Basic Research under the Grant No. 01-02-17334 and by the Ministry of Education of Russian Federation under the Grant No. E00-11.0-5.

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