Limits on cosmological variation of quark masses and strong interaction

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We discuss limits on variation of $(m_q/\Lambda_{QCD})$. The results are obtained by studying $n-\alpha$ interaction during Big Bang, Oklo natural nuclear reactor data and limits on variation of the proton $g$-factor from quasar absorption spectra.

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I. INTRODUCTION

Recent astronomical data suggest a possible variation of the fine structure constant $\alpha = e^2/\hbar c$ at the $10^{-5}$ level over a time-scale of 10 billion years, see \cite{1} (a discussion of other limits can be found in Ref. \cite{2} and references therein). The data motivated immediately more general discussions of possible variations of other constants. Unlike for the electroweak forces for the strong interaction there is generally no direct relation between the coupling constants and observable quantities. In a recent paper \cite{3} we presented general discussions of possible influence of the strong scale variation on primordial Big Bang Nucleosynthesis (BBN) yields, the Oklo natural nuclear reactor, quasar absorption spectra and atomic clocks.

Here we continue this work, concentrating on the properties of $^5\text{He}$ (BBN), Sm (Oklo natural nuclear reactor), $^{12}\text{C}$ (stars) and proton magnetic $g$-factor (quasar absorption spectra). Since one can measure only variation of the dimensionless quantities, we can extract from our results variation of the dimensionless ratio $m_q/\Lambda_{QCD}$ where $m_q$ is the quark mass (with the dependence on the normalization point removed) and $\Lambda_{QCD}$ is the strong interaction scale determined by a position of the pole in perturbative QCD running coupling constant. It is convenient to assume that $\Lambda_{QCD}$ is constant, and quark mass $m_q$ is variable.

A position of an energy level in a strong potential depends both on the parameters of the strong interaction and nucleon mass $M$. For example, in a simplest case of a particle in a deep square potential well the single-particle energy levels are

$$E_n \approx -V + \frac{\hbar^2 \pi^2 (n+1/2)^2}{2MR^2}, \quad (1)$$

where $V$ and $R$ are the depth and radius of the potential. The strong interaction is sensitive to the light quark mass because the $\pi$-meson mass $m_\pi \sim \sqrt{m_q\Lambda_{QCD}}$, where $m_q = m_u + m_d$. The nucleon mass is also sensitive to quark mass because $M$ has large “strange fraction” \cite{4}

$$\frac{\partial M}{\partial m_s} \approx N|\bar{s}s|N \approx 1.5 \quad (2)$$

Putting the strange quark mass $m_s = 130 \text{ MeV}$ one finds that about 1/5 of the nucleon mass comes from the “strange term”.

II. N-\alpha INTERACTION

The production of nuclei with $A > 5$ during BBN is strongly suppressed because of the absence of stable nuclei with $A = 5$. $^5\text{He}$ is unstable nucleus which is seen as a resonance in $n-\alpha$ elastic scattering at neutron laboratory energy around 1.1 MeV \cite{1}. The resonance corresponds to the ground state of unstable nucleus $^5\text{He}$ with mass excess $\Delta = 11386.234 \text{ KeV}$ \cite{5}. The ground state lies at 0.89 MeV above neutron threshold. This energy is rather small in nuclear scale, therefore, rather small variation of the strong interaction scale may influence considerably the position of the resonance making, for example, $^5\text{He}$ stable. Stable $^5\text{He}$ at the time of BBN would change strongly the primordial abundances of light elements. Basing on the fact that the standard BBN theory describes rather well the observed light element abundances we can put bounds on strong scale variation at the time of BBN.

A phenomenological potential describing $n-\alpha$ scattering phases at low energy has the Woods-Saxon shape \cite{6},

$$U(r) = -V (e^{r+1}+(\hbar/m_\pi c)^2 V_s (L \cdot \sigma) r^{-1} (d/dr)(e^{r+1}). \quad (3)$$

In Eq. \cite{7},

$$x = (r - R)/a, \quad x_s = (r - R_s)/a_s. \quad (4)$$

Here, $R$ and $R_s$ are the central and the spin-orbit radiiuses of the potential,

$$R = r_0 (M_\alpha/M_p)^{1/3}, \quad R_s = r_s (M_\alpha/M_p)^{1/3},$$

where $(M_\alpha/M_p) = 3.973$ is the ratio of the $\alpha$ and the proton masses. The other parameters of the potential are

$$V = 41.88 \text{ MeV}, \quad V_s = (3.0 + 0.1E_n) \text{ MeV}$$

$r_0 = (1.50 - 0.01E_n) \text{ fm}, \quad r_s = 1.0 \text{ fm}$

$a = a_s = 0.25 \text{ fm}$

The main contributions to this potential come from exchange of scalar and vector mesons. These contributions
are not related directly to the pion and we can expect them to be insensitive to variation of the pion properties. Pion exchange interaction gives zero contribution into the potential via direct term. The exchange term, however, gives nonzero contribution. If we isolate this contribution, we can study, at least approximately, the dependence of $^3$He ground state properties on pion mass and pion-nucleon coupling constant.

A. One pion exchange (OPE) contribution

We start from the standard OPE interaction

$$V_\pi(r_1-r_2) = -\frac{f^2}{\mu_\pi^2} (\tau_1 \cdot \tau_2) (\sigma_1 \cdot \nabla_1) (\sigma_2 \cdot \nabla_2) e^{-m_\pi r_{12}/r_{12}}.$$  \hspace{1cm} (5)

Here we distinguish between pion mass in the exponent and pion mass in the denominator. The last one is used for normalization of pion-nucleon coupling constant and should not be varied.

The exchange contribution into $n - \alpha$ potential can be written in the form

$$W(r_1, r_2) = -\frac{3}{4\pi} \frac{f^2}{\mu_0^2} m_0^2 R_0(r_1) \frac{1}{r_{12}} e^{-m_\pi r_{12}/r_{12}} R_0(r_2).$$  \hspace{1cm} (6)

In Eq. (5), $R_0(r)$ is 1s radial wave function of nucleon inside $\alpha$-particle. For our purpose it is sufficient to choose $R_0(r)$ as a simple gaussian

$$R_0(r) = \frac{1}{\rho_i^2/2} \left(\frac{2}{\pi}\right)^{1/4} \exp(-r^2/4\rho_i^2),$$  \hspace{1cm} (7)

where the parameter $\rho_i$ is related to mean-square size of the $\alpha$-particle

$$\overline{r^2} = 3\rho_i^2.$$

We omitted the contact term in Eq. (5). It does not depend on the pion mass and can be included into phenomenological part of the potential Eq. (5) together with other mesons contribution. With the potential Eq. (5) the Schrödinger equation for $n - \alpha$ system becomes non-local. For $p_{3/2}$ radial wave function $\chi_1(r)$ we obtain the equation

$$-\frac{\hbar^2}{2m} \frac{d^2}{dr^2} \chi_1(r) + \left(U(r) + \frac{\hbar^2}{2m} \frac{2}{r^2}\right) \chi_1(r) - 6 \frac{f^2}{\mu_0^2} \frac{m_0^2}{\rho_i^2/2} \frac{2}{\pi} \left[ r k_1(m_\pi r)e^{-r^2/4\rho_i^2} \int_0^r r' i_1(m_\pi r') e^{-r'^2/4\rho_i^2} \chi_1(r') dr' + r i_1(m_\pi r) e^{-r^2/4\rho_i^2} \int_r^\infty r' k_1(m_\pi r') e^{-r'^2/4\rho_i^2} \chi_1(r') dr' \right] = E \chi_1(r).$$  \hspace{1cm} (8)

In Eq. (6), $i_1(x)$, and $k_1(x)$ are the spherical Bessel functions of imaginary argument. Using subtraction of real terms Eq. (8) we can now adjust the phenomenological parameter $V$ putting the resonance in $p_{3/2}$ wave in its position $E = 0.89$ MeV. The procedure gives $V = 32.74$ MeV that should be compared with 41.88 MeV for full phenomenological potential Eq. (5). The difference between these two values can be treated as an effective depth of equivalent local potential corresponding to non-local exchange contribution of pion, Eq. (5). The Eq. (6) was obtained neglecting pion-nucleon formfactor. This is reasonable approximation since the cutoff parameter $\Lambda$ in the formfactor is large compared to nucleon momentum in $\alpha$-particle nucleus.

B. BBN bounds on pion, nucleon and quark mass variation

Using Eq. (6) we can study the dependence of $p_{3/2}$ resonance position on pion mass. The potential Eq. (5) is proportional to pion mass squared. Therefore, an increase in pion mass would lead to lowering of the resonance position. Simple numerical exercise leads to conclusion that the increase in pion mass on 21.9% would produce $^3$He nucleus with zero neutron binding energy.

Any further increase of pion mass would lead to stable $^5$He producing serious modifications in the process of nucleosynthesis. Using the relation for the $\pi$-meson mass $m_\pi \sim \sqrt{m_q QCD}$ we obtain the limit

$$\frac{\delta(m_\pi/L_{QCD})}{(m_\pi/L_{QCD})} < 0.4$$  \hspace{1cm} (9)

Note that this OPE path to obtain the limits does not look very reliable - see discussions in recent papers [3, 4]. Therefore, below we present the limits obtained in a different way.

Variation of the nucleon mass $M$ leads to change of the kinetic energy and change of the resonance position. We found that 5% increase of the nucleon mass produces $^5$He bound state with zero neutron binding energy. Using the relation (2) we find the limit for the variation of the strange quark mass

$$\frac{\delta(m_s/L_{QCD})}{(m_s/L_{QCD})} < 0.25$$  \hspace{1cm} (10)

These are limits on the variations of the light quark mass and strange quark mass from BBN to present time.

III. LIMITS FROM OKLO NATURAL NUCLEAR REACTOR

Similar limits follow from data on natural nuclear reactor in Oklo active about 2 bn years ago. The most sensitive phenomenon (used previously for limits on the variations of the electromagnetic $\alpha$) is disappearance of certain isotopes (especially $^{149}$Sm) possessing a neutron resonance close to zero [8]. Today the lowest resonance energy $E_0 = 0.0973 \pm 0.0002$ eV is larger compared to its width, so the neutron capture cross section $\sigma \sim 1/E_0^2$. 

The data constrain the ratio of this cross section to the non-resonance one which was used to find neutron flux. It therefore implies (under assumption that the same resonance was the lowest one at the time of Oklo reactor) that these data constrain the variation of the following ratio $\delta(E_0/E_1)$ where $E_1 \sim 1.0 \text{ MeV}$ is a typical single-particle energy scale which is determined mainly by $\Lambda_{\text{QCD}}$.

A very small value of $E_0$ appears as a result of the nearly exact cancellation of two large terms $E_0 = E_n - S_n$ where $S_n$ is the neutron separation energy in $^{150}\text{Sm}$ and $E_n$ is the energy of the many-body excited compound state relative to the ground state of $^{150}\text{Sm}$ (recall that we consider the reaction $n + ^{149}\text{Sm}$). The difference is very small because we deliberately selected the lowest compound resonance above neutron threshold. The neutron separation energy is $S_n = V - \epsilon_F$ where $V$ is the depth of the potential and $\epsilon_F = p_F^2/2M \sim (h^2 A^{2/3})/(MR^2)$ is the Fermi energy. The energy of compound state $E_n$ is approximately equal to the difference of several single-particle energies given by Eq. (1). Therefore, both $\epsilon_F$ and $E_n$ scale as $\hbar^2/(MR^2)$. We can present the resonance energy in the following form:

$$E_0 = E_c - S_n = E_c + \epsilon_F - V = K \frac{\hbar^2}{MR^2} - V \tag{11}$$

where $K$ is a numerical constant. We can find this constant from the present time condition $E_0 \approx 0$:

$$K \left( \frac{\hbar^2}{MR^2} \right)_{\text{present}} = V_{\text{present}} \tag{12}$$

Now we may consider change of $E_0$ under variation of $M, R, V$.

$$\delta E_0 = -K \frac{\hbar^2}{MR^2} \left( \frac{\delta M}{M} + \frac{2\delta R}{R} \right) - \delta V = -V \frac{\delta M}{M} - \left( \frac{2\delta R}{R} + \delta V \right) \tag{13}$$

Comparing with the $^5\text{He}$ calculation in the previous section we can say that two last terms in the brackets correspond to the variation of the energy level due to the variation of the pion mass $m_\pi$. This gives us the following expression

$$\delta E_0 = -V \frac{\delta M}{M} + \frac{dE}{dm_\pi} \delta m_\pi = \frac{dE}{dM} \delta M + \frac{dE}{dm_\pi} \delta m_\pi \tag{14}$$

We can find numerical values of the derivatives using $V = 50 \text{ MeV}$, $\delta M/M = 0.2\delta m_\pi/m_\pi$ (see Eq. (3)), $\delta m_\pi/m_\pi = 0.5\delta m_q/m_q$.

$$\delta E_0 = -0.05 \delta M - 0.03 \delta m_\pi = -10 \text{ MeV} \frac{\delta m_\pi}{m_\pi} - 2 \text{ MeV} \frac{\delta m_q}{m_q} \tag{15}$$

Note that in the $^5\text{He}$ calculation the derivative $\frac{dE}{dm_\pi}$ had smaller absolute value (-0.02 instead of -0.05 here). The difference may be related to the fact that the $p$-wave resonance wave function in $^5\text{He}$ was localised mainly outside the nucleus (where the coefficient $V$ in Eq. (14) is equal to zero). We extrapolated the derivative $\frac{dE}{dm_\pi}$ from the $^5\text{He}$ calculation (accuracy in this term is very low anyway).

The limit on the shift of the Sm resonance is $\delta E_0 < 0.02 \text{ eV}$. A comparison with Eq. (15) gives very stringent limits:

$$\left| \frac{\delta (M/\Lambda_{\text{QCD}})}{(M/\Lambda_{\text{QCD}})} + 0.6 \frac{\delta (m_\pi/\Lambda_{\text{QCD}})}{(m_\pi/\Lambda_{\text{QCD}})} \right| < 4 \times 10^{-10} \tag{16}$$

$$\left| \frac{\delta (m_\pi/\Lambda_{\text{QCD}})}{(m_\pi/\Lambda_{\text{QCD}})} + 0.2 \frac{\delta (m_q/\Lambda_{\text{QCD}})}{(m_q/\Lambda_{\text{QCD}})} \right| < 2 \times 10^{-9} \tag{17}$$

Note that the authors of the last work in $^3\text{He}$ found also the non-zero solution $\delta E_0 = -0.097 \pm 0.008 \text{ eV}$. This solution corresponds to the same resonance moved below thermal neutron energy. In this case

$$\delta E_0 \left( \frac{M/\Lambda_{\text{QCD}}}{M/\Lambda_{\text{QCD}}} \right) + 0.6 \left( \frac{m_\pi/\Lambda_{\text{QCD}}}{m_\pi/\Lambda_{\text{QCD}}} \right) = (2 \pm 0.2) \times 10^{-9} \tag{18}$$

$$\delta E_0 \left( \frac{m_\pi/\Lambda_{\text{QCD}}}{m_\pi/\Lambda_{\text{QCD}}} \right) + 0.2 \left( \frac{m_q/\Lambda_{\text{QCD}}}{m_q/\Lambda_{\text{QCD}}} \right) = (1 \pm 0.1) \times 10^{-8} \tag{19}$$

In principle, the total number of the solutions can be very large since $^{150}\text{Sm}$ nucleus has millions of resonances and each of them can provide two new solutions (thermal neutron energy on the right tale or left tale of the resonance). However, these extra solutions are probably excluded by the measurements of the neutron capture cross-sections for other nuclei since no significant changes have been observed there also, see $^3\text{He}$.

## IV. LIMITS FROM $^{12}\text{C}$ PRODUCTION IN STARS AND QUASAR ABSORPTION SPECTRA

In the previous sections we obtained limits on variation of $m_\pi/\Lambda_{\text{QCD}}$ during the interval between the Big Bang and present time and on shorter time scale from Oklo natural nuclear reactor which was active 1.8 billion years ago. It is also possible to obtain limits on the intermediate time scale. One possibility is related to position of the resonance in $^{12}\text{C}$ during production of this element in stars. This famous resonance at $E = 380 \text{ KeV}$ is needed to produce enough carbon and create life. According to Ref. $^3\text{He}$ the position of this resonance can not shift by more than 60 KeV (one can also find in Ref. $^3\text{He}$ the limits on the strong interactions and other relevant references). We can use Eq. (15) to provide some rough estimates only:

$$\left| \frac{\delta (M/\Lambda_{\text{QCD}})}{(M/\Lambda_{\text{QCD}})} + 0.6 \frac{\delta (m_\pi/\Lambda_{\text{QCD}})}{(m_\pi/\Lambda_{\text{QCD}})} \right| < 1.2 \times 10^{-3} \tag{20}$$

$$\left| \frac{\delta (m_\pi/\Lambda_{\text{QCD}})}{(m_\pi/\Lambda_{\text{QCD}})} + 0.2 \frac{\delta (m_q/\Lambda_{\text{QCD}})}{(m_q/\Lambda_{\text{QCD}})} \right| < 6 \times 10^{-3} \tag{21}$$
Unfortunately, these limits are relatively weak. Much stronger limits can be obtained from the measurements of quasar absorption spectra. Comparison of atomic H 21 cm (hyperfine) transition with molecular rotational transitions gave the following limits on $\alpha^2 g_p [1]$

$$\frac{\delta Y}{Y} = (-0.20 \pm 0.44) \times 10^{-5} \text{ for redshift } z=0.2467$$

and

$$\frac{\delta Y}{Y} = (-0.16 \pm 0.54) \times 10^{-5} \text{ for } z=0.6847.$$  

The second limit corresponds to roughly $t=6$ billion years ago. These results were used in [1] to obtain the limits on variation of $\alpha$. In Ref. [3] it was suggested to use $\delta Y/Y$ to estimate variation of $m_q/\Lambda_{QCD}$. Here we continue this work.

According to calculation in Ref. [1], dependence of the proton magnetic moment on $\pi$-meson mass $m_\pi$ can be approximated by the following equation

$$\mu_p(m_\pi) = \frac{\mu_p(0)}{1 + a m_\pi + b m_\pi^2} \tag{22}$$

where $a = 1.37/GeV, b = 0.452/GeV^2$. The proton $g$-factor is $g_p = \mu_p/(e\hbar/2Mc)$. Using Eq. (22) we obtain the following estimate:

$$\frac{\delta g_p}{g_p} = \frac{\delta M}{M} - 0.18 \frac{\delta m_\pi}{m_\pi} = 0.2 \frac{\delta m_s}{m_s} - 0.09 \frac{\delta m_q}{m_q} \tag{23}$$

Comparing with the limits on $\delta Y/Y$ we obtain for the red shift $z=0.2467$ (neglecting variation of $\alpha$):

$$\frac{\delta (M/\Lambda_{QCD})}{(M/\Lambda_{QCD})} - 0.18 \frac{\delta (m_\pi/\Lambda_{QCD})}{(m_\pi/\Lambda_{QCD})} = (-0.2 \pm 0.44) \times 10^{-5} \tag{24}$$

$$2 \frac{\delta (m_s/\Lambda_{QCD})}{(m_s/\Lambda_{QCD})} - \frac{\delta (m_q/\Lambda_{QCD})}{(m_q/\Lambda_{QCD})} = (-2.0 \pm 4.4) \times 10^{-5} \tag{25}$$

For the red shift $z=0.6847$:

$$\frac{\delta (M/\Lambda_{QCD})}{(M/\Lambda_{QCD})} - 0.18 \frac{\delta (m_\pi/\Lambda_{QCD})}{(m_\pi/\Lambda_{QCD})} = (-0.16 \pm 0.54) \times 10^{-5} \tag{26}$$

$$2 \frac{\delta (m_s/\Lambda_{QCD})}{(m_s/\Lambda_{QCD})} - \frac{\delta (m_q/\Lambda_{QCD})}{(m_q/\Lambda_{QCD})} = (-1.6 \pm 5.4) \times 10^{-5} \tag{27}$$

There is also a limit on $X \equiv \alpha^2 g_p m_e/M_p [12] \delta X/X = (0.7 \pm 1.1) \times 10^{-5} \text{ for } z=1.8$. This limit was interpreted as a limit on variation of $\alpha$ or $m_e/M_p$.

Using eq. (22) we obtain

$$\frac{\delta (g_p m_e/M_p)}{(g_p m_e/M_p)} = \frac{\delta m_e}{m_e} - 0.18 \frac{\delta m_\pi}{m_\pi} = \frac{\delta m_e}{m_e} - 0.09 \frac{\delta m_q}{m_q} \tag{28}$$

This gives us a limit for the red shift $z=1.8$

$$\frac{\delta (m_e/\Lambda_{QCD})}{(m_e/\Lambda_{QCD})} - 0.09 \frac{\delta (m_q/\Lambda_{QCD})}{(m_q/\Lambda_{QCD})} = (0.7 \pm 1.1) \times 10^{-5} \tag{29}$$

We should stress that in this paper we present experimental errors only. The theoretical errors are hard to estimate, therefore some of these limits, possibly, should be treated as order of magnitude estimates.

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